Chapter 8: Finite Automata with Output

Two equivalent theoretical machines with output are introduced.

Finite automata as mathematical models of computers

letters of words ⟷ input

transitions ⟷ execution of instructions (a program)

state ⟷ contents of memory

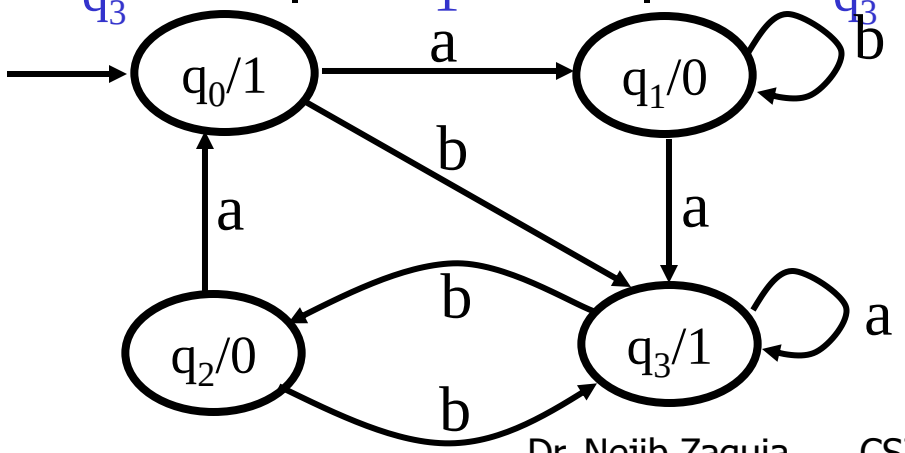
? ⟷ output

- A Moore machine is a collection of 5 things:
  1. a finite set of states $q_0, q_1, q_2, \ldots$, where q_0 is designated the start state
  2. an alphabet $\sum$ of input letters
  3. an alphabet Γ of output characters
  4. a transition table that shows for each state and each input letter what state to go to next.
  5. an output table that shows what character is output (or printed) when entering a state.

$$(\text{state, letter from } \sum) \xrightarrow{\text{transition}} \text{state}$$

Example: states = $\{q_0,q_1,q_2,q_3\}$ $\Sigma=\{a,b\}$ $\Gamma=\{0,1\}$

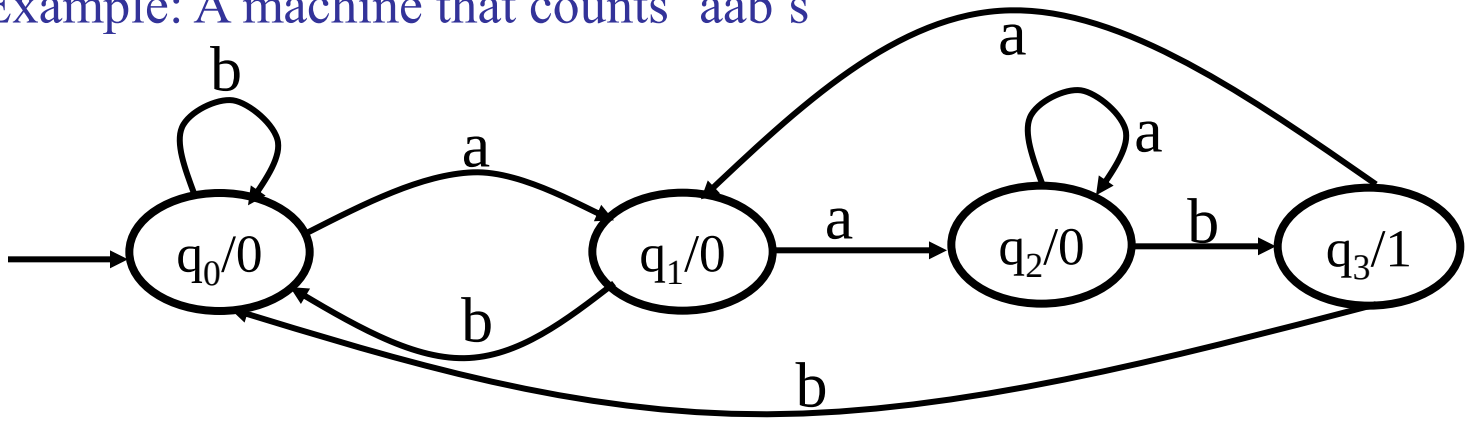
Old state	Output by the old state	New state: After input a	New state: After input b
$\rightarrow q_0$	1	q_1	q_3
q_1	0	q_3	q_1
q_2	0	q_0	q_3
q_3	1	q_3	q_2



abab

10010

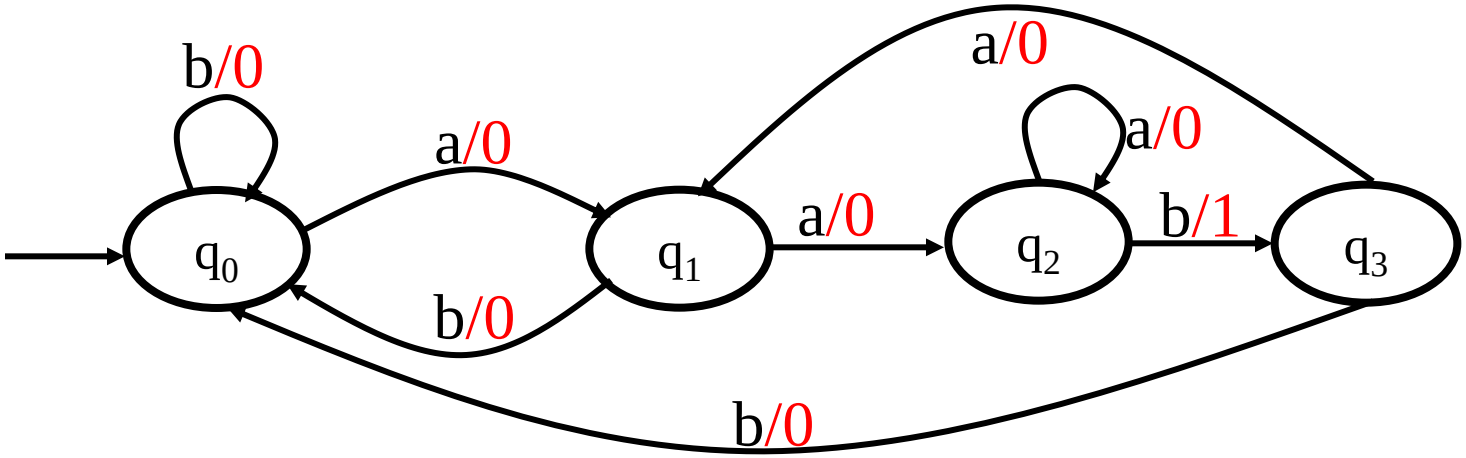
Example: A machine that counts 'aab's

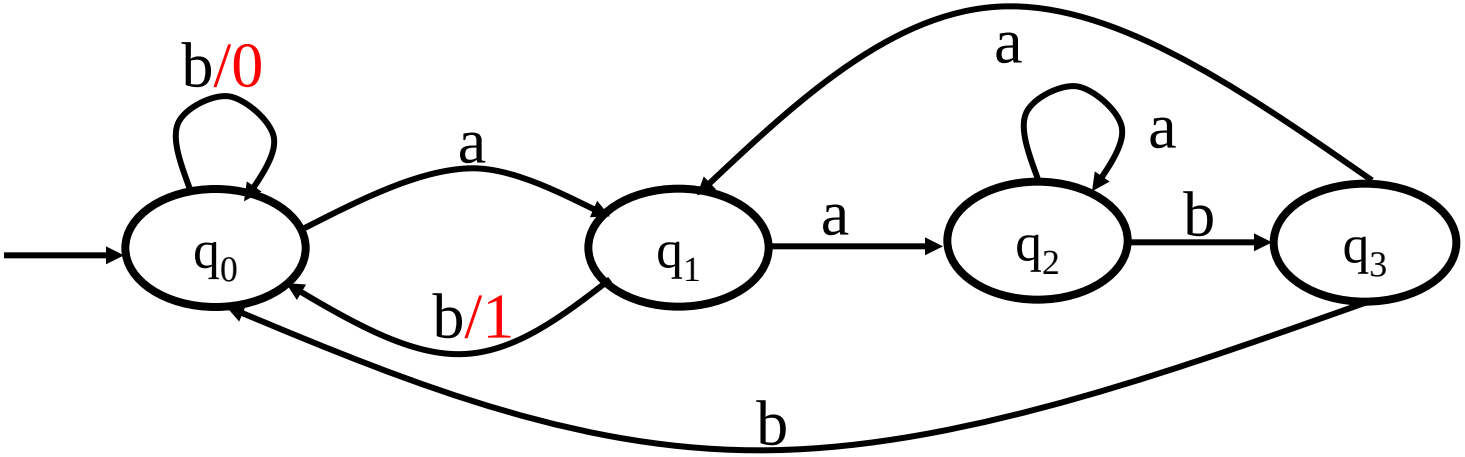


Input		a	a	a	b	a	b	b	a	a	b	b
State	q_0	q_1	q_2	q_2	q_3	q_1	q_0	q_0	q_1	q_2	q_3	q_0
Output	0	0	0	0	1	0	0	0	0	0	1	0

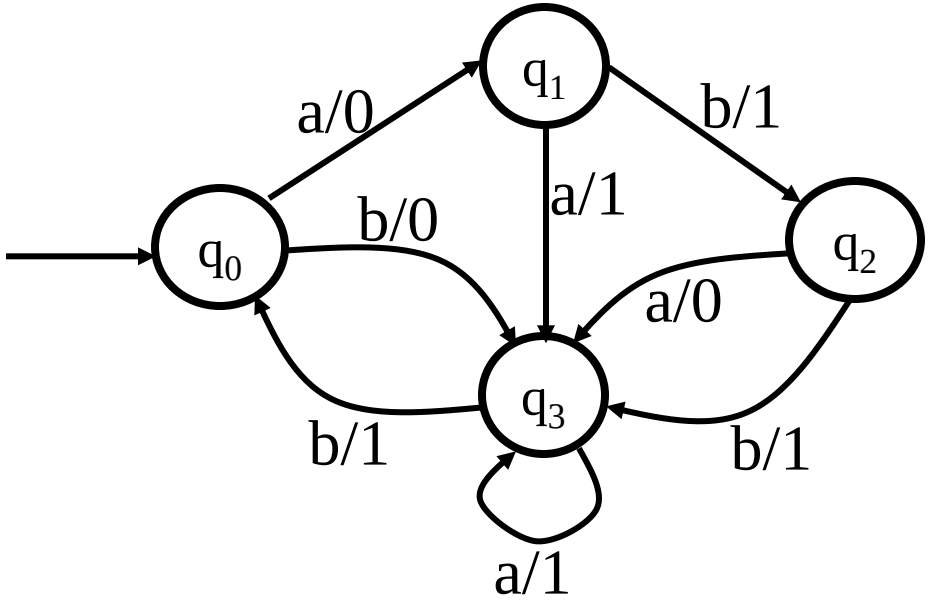
Printing 1 ≈ final state words that end in aab

- A Mealy machine is:
  1. a finite set of states $q_0,q_1,q_2,\ldots$, where q_0 is designated the start state
  2. an alphabet Σ of input letters
  3. an alphabet Γ of output characters
  4. a finite set of transitions that indicate, for each state and letter of the input alphabet, the state to go to next and the character that is output (printed).


b/0
a/0
a/0
a/0
a/0
b/1
b/0
b/0
q0
q1
q2
q3


b/0
a
a
a
a
b
b/1
b
q0
q1
q2
q3

Example

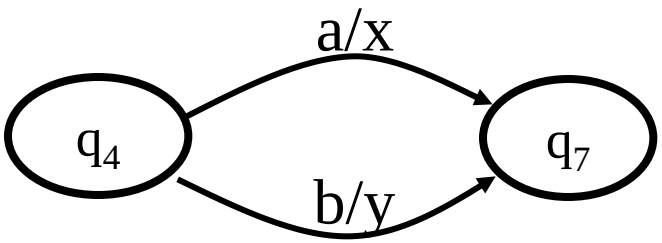


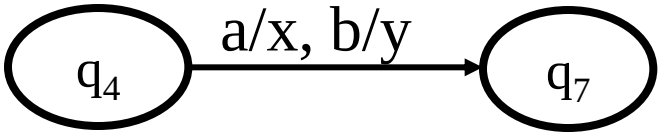
baaba

| | | | | |

01110

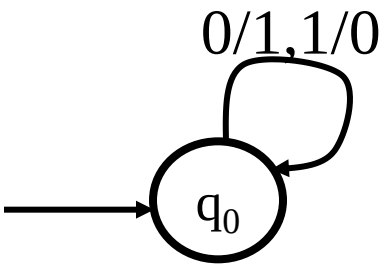
Abbreviation:





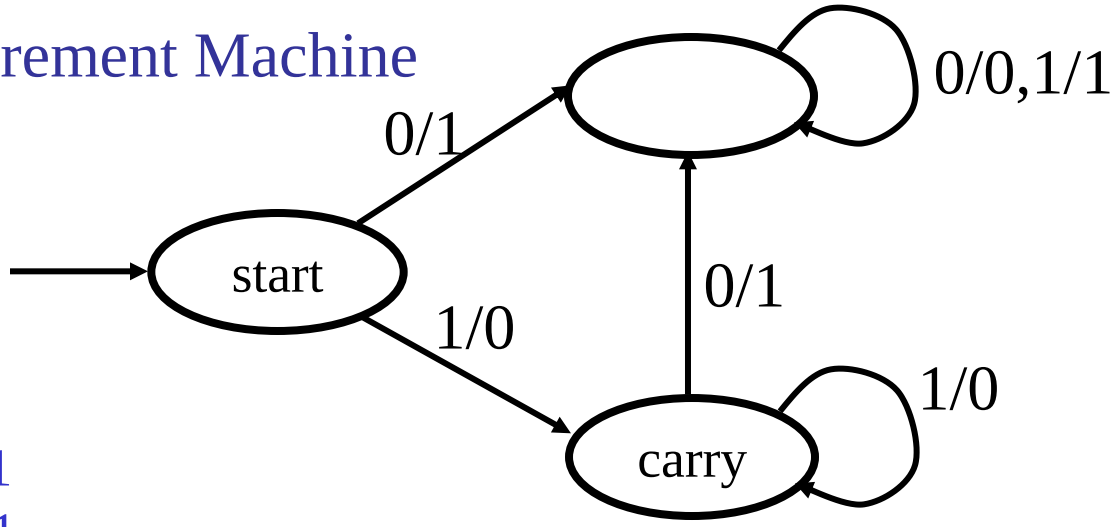
Example: binary complement

$\Sigma=\{0,1\}$ $\Gamma=\{0,1\}$



101	001010
010	110101

Example: an Increment Machine



decimal: 11
binary : 1011
reverse: 1101

00 11

reverse: 1100
decimal: 12

1111

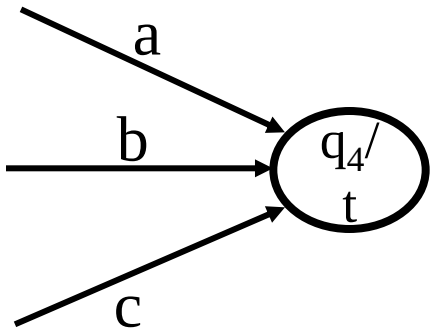
(overflow)

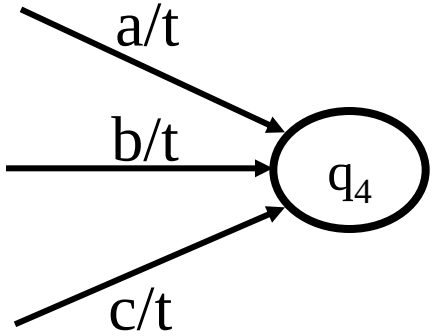
0000

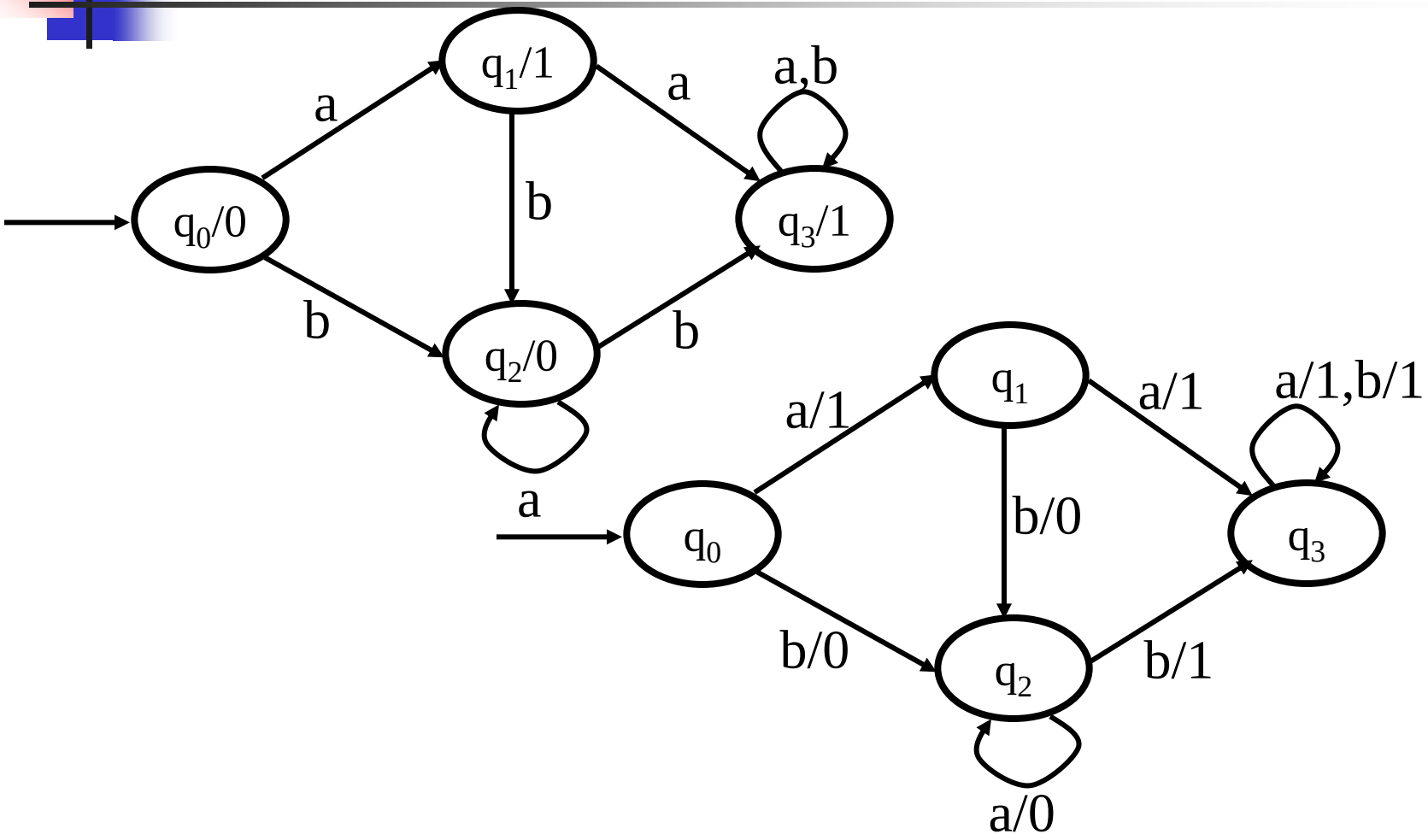
Definition: Let Mo be a Moore machine that prints x in the start state. Let Me be a Mealy machine. The two machines are equivalent if for every input, whenever the output from Me is w, the output from Mo is xw.

Theorem: For every Moore machine, there exists a Mealy machine that is equivalent to it.

Proof: By constructive algorithm



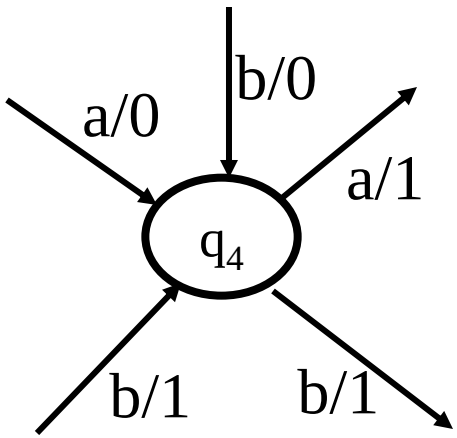


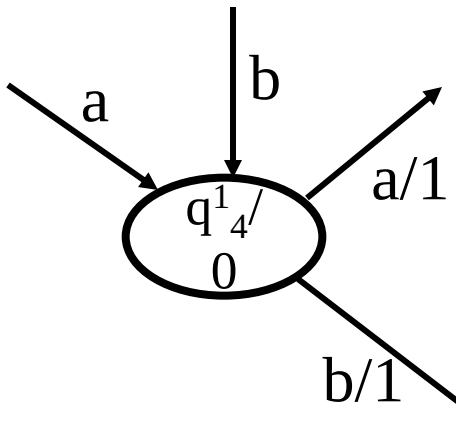

$q_1/1$
a
a
a,b
$q_0/0$
b
$q_3/1$
b
b
$q_2/0$
a
q_1
a/1
a/1
a/1,b/1
q_0
b/0
q_3
b/0
b/1
q_2
a/0

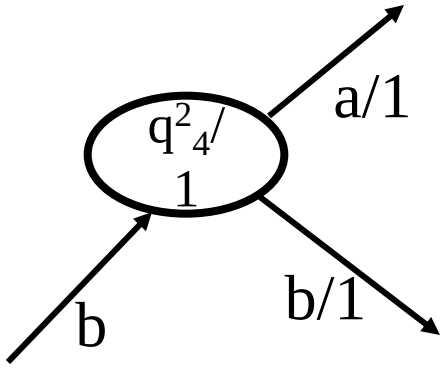
Theorem: For every Mealy machine, there exists a Moore machine that is equivalent to it.

Proof: By constructive algorithm

One copy of the state for each letter in Γ that labels an arrow entering it.

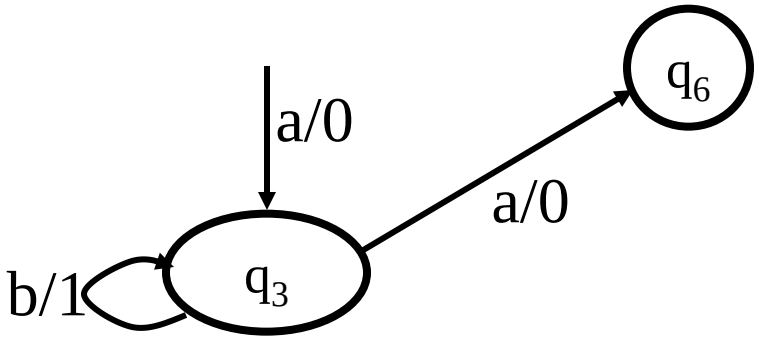


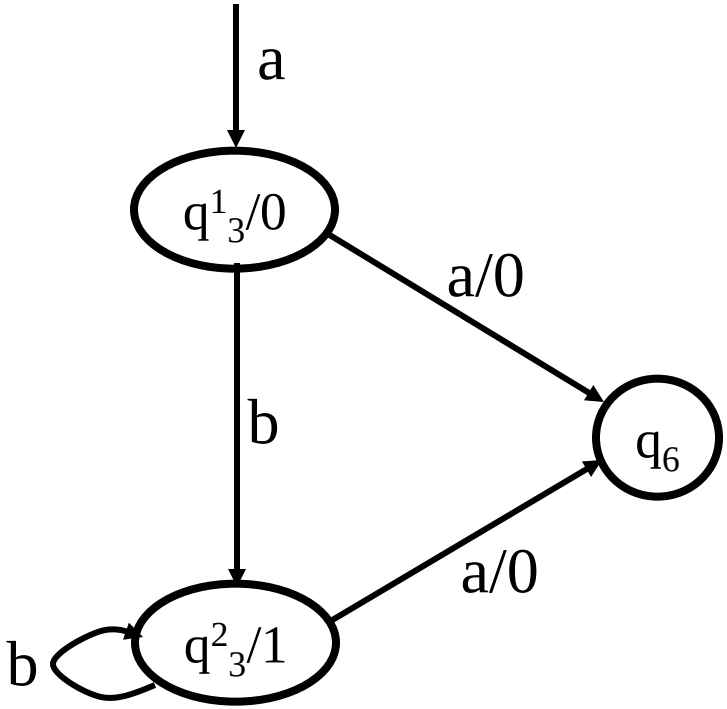




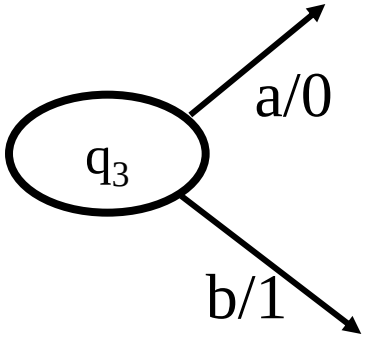
We must consider

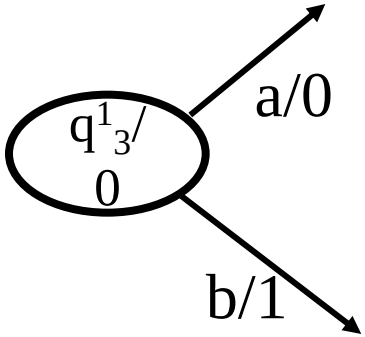
1. incoming edges
2. outgoing edges
3. loops



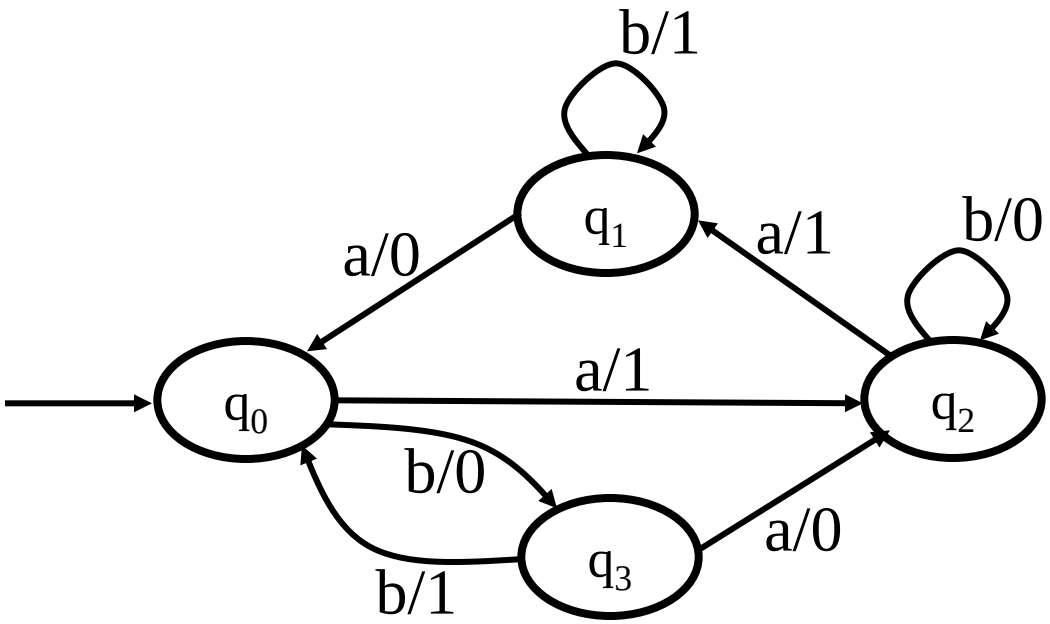


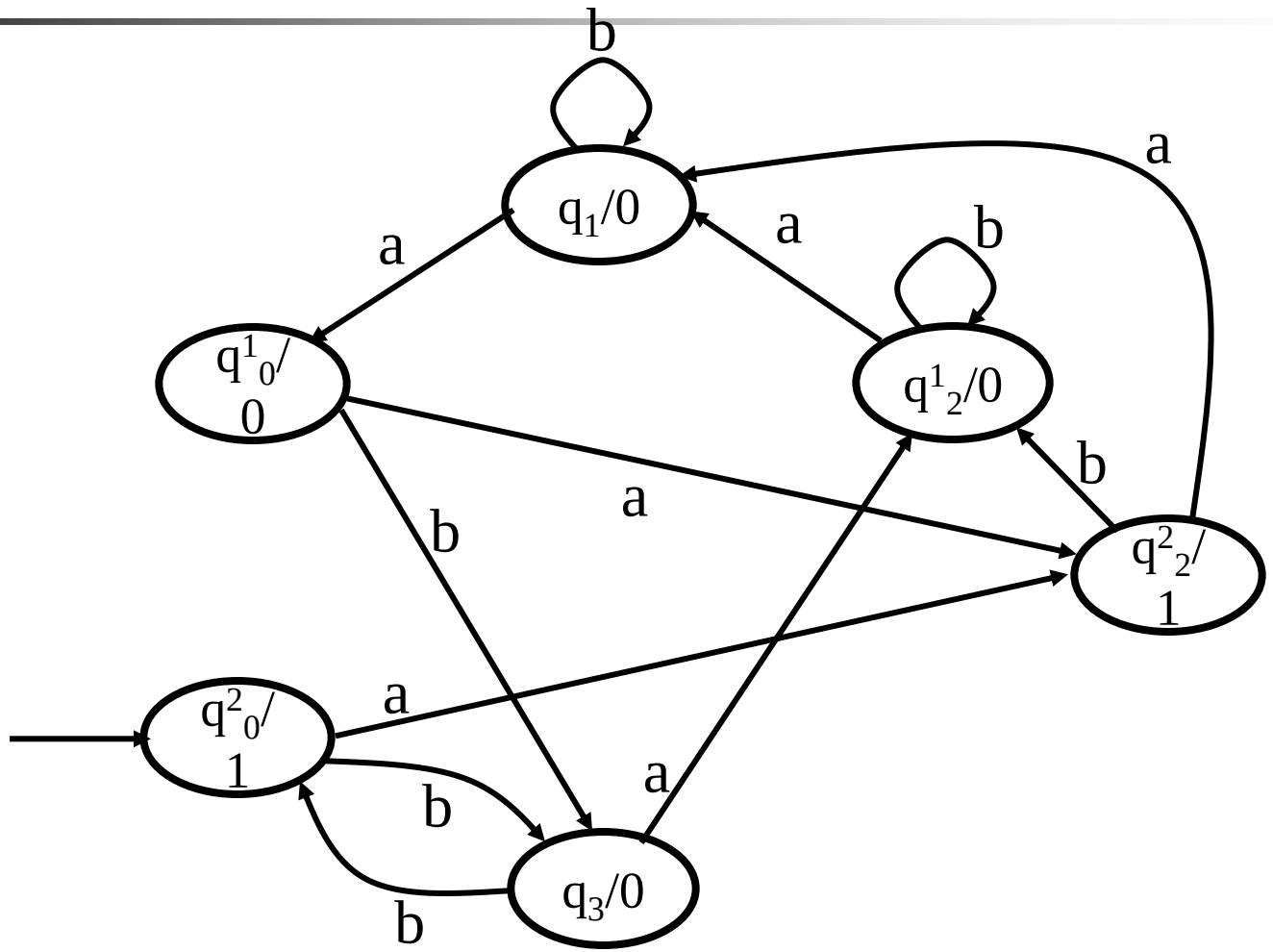
If there is no entering arrow, choose any letter from Γ.



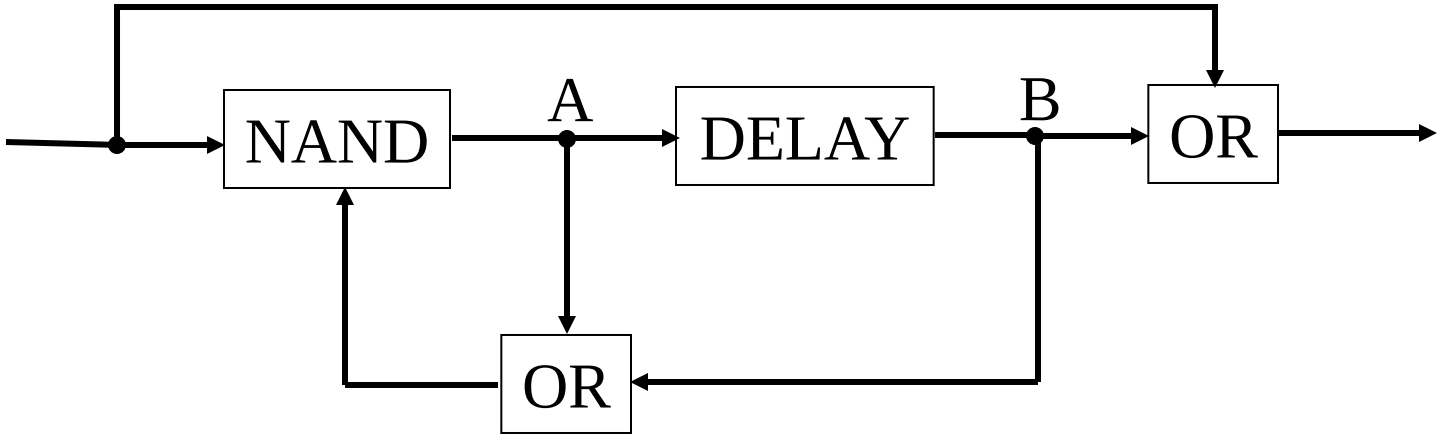


Choose any copy of q_0 as the start state.


b/1
q1
a/0
a/1
b/0
q0
a/1
q2
b/0
q3
a/0
b/1


b
a
$q_1/0$
a
a
b
$q^1_0/0$
$q^1_2/0$
b
a
b
$q^2_2/1$
a
$q^2_0/1$
a
b
$q_3/0$
b

Example: Mealy Machine Describing a Circuit



q_0: A=0,B=0 q_1: A=0,B=1 q_2: A=1,B=0 q_3: A=1,B=1

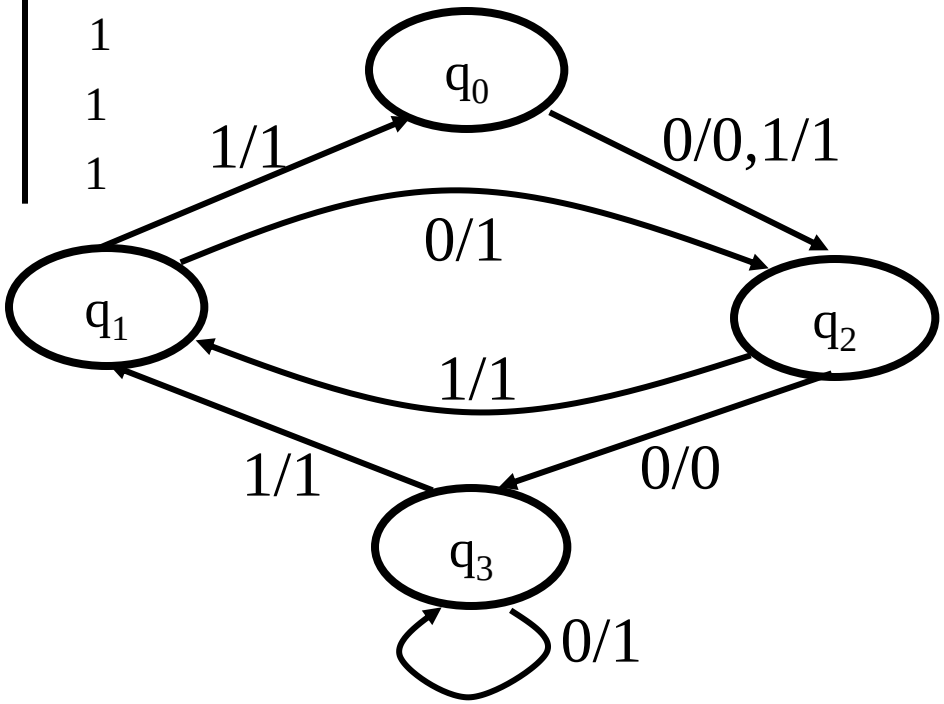
Transitions:

New B = old A

New A = input NAND (old A OR old B)

Output = input OR (old B)

Old state	Input 0		Input 1	
	new state	output	new state	output
q_0	q_2	0	q_2	1
q_1	q_2	1	q_0	1
q_2	q_3	0	q_1	1
q_3	q_3	1	q_1	1



Chapter 8: Finite Automata with Output
Automata Summary

	Finite Automata	Tr. Graphs	Generalized Tr. Graphs	non-determ automata	Moore Machines	Mealy Machines
start states	1	≥1	≥1	1	1	1
final states	≥0	≥0	≥0	≥0	0	0
labels on arrows	letters of Σ	words of Σ^*	Reg. Expr. over Σ	letters of Σ	letters of Σ	i/o, i$\in\Sigma$, o$\in\Gamma$
# of trans. from each state	1 transition for each letter of Σ	≥0	≥0	≥0	1 transition for each letter of Σ	1 transition for each letter of Σ
Deterministic?	Yes	No	No	No	Yes	Yes
Output?	No	No	No	No	Yes	Yes