Chapter 5: Finite Automata

We introduce the simplest deterministic theoretical machines: Finite Automata.

- A finite automaton (FA) is the following 3 things:
  1. a finite set of states, one of which is designated as the start state, and some (maybe none) of which are designated the final states (or accepting states)
  2. an alphabet Σ of input letters
  3. a finite set of transitions that indicate, for each state and letter of the input alphabet, the state to go to next

$$\text{state} \xrightarrow{\text{letter}} \text{state}$$

- The language defined or accepted by a finite automaton is the set of words that end in a final state.
- If w is in the language defined by a finite automaton, then we also say that the finite automaton accepts w.

Example: $\Sigma = \{a,b\}$
states = {x,y,z}
start state : x
final states: {z}

transitions:

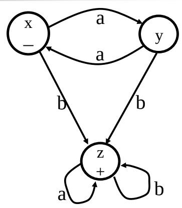
	a	b
x	y	z
y	x	z
z	z	z

aaa: $x \xrightarrow{a} y \xrightarrow{a} x \xrightarrow{a} y$ y is not final; aaa is not accepted

aaba: $x \xrightarrow{a} y \xrightarrow{a} x \xrightarrow{b} z \xrightarrow{a} z$ z is final; aaba is accepted

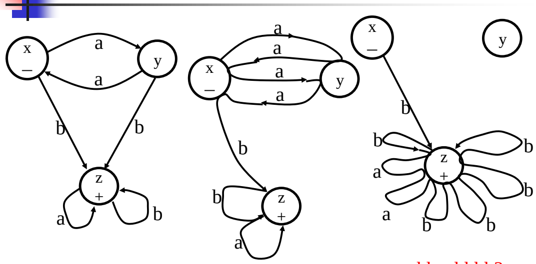
Transition Diagram

	a	b
x	y	z
y	x	z
z	z	z

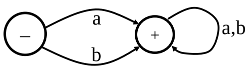


Regular expression: **(a+b)*b(a+b)***

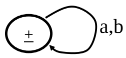
aaaabba?
bbaabbbb?



aaaabba? bbaabbbb?

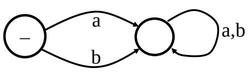


(a+b)(a+b)* = (a+b)$^+$

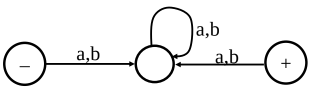


(a+b)*

Finite Automata that Accept No Words



No final state



The middle state is not a final state and all transitions that go into this state do not exit.

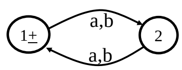
Two Ways to Study Finite Automata

1. Starting with a finite automaton (FA), analyze it to determine the language it accepts.
2. Starting from a language, build an FA.

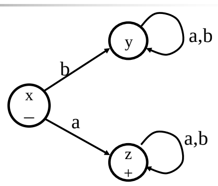
Example with 2 states: The language of all words with an even number of letters over the alphabet {a,b}:

- Two states: 1 – even number, 2 – odd number
- Start state: 1
- Final state: 1
- The transitions:

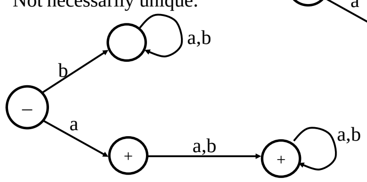
	a	b
1	2	2
2	1	1



a(a+b)*



Not necessarily unique:



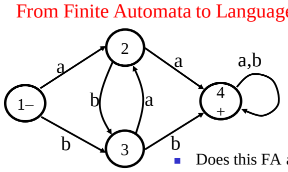
Remark: 2 final states

From Languages to Finite Automata

- There is not necessarily a unique FA that accepts a given language.
- Is there always at least one FA:
  - that accepts each possible language?
  - that defines a language associated with a given regular expression?

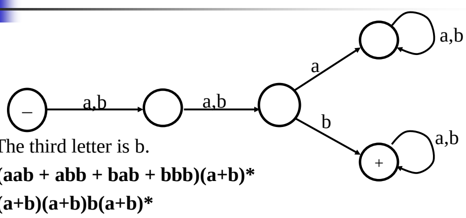
- <u>Example</u>: Build an FA that accepts all words containing a triple letter (either aaa or bbb).
  1. Build an FA that accepts aaa
  2. Add a path that accepts bbb.
  3. Add paths for words that contain a's and b's before or after the aaa or bbb.

From Finite Automata to Languages

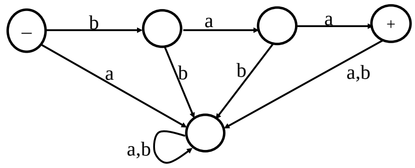


This example shows that it is possible to characterize states by the purposes they serve.

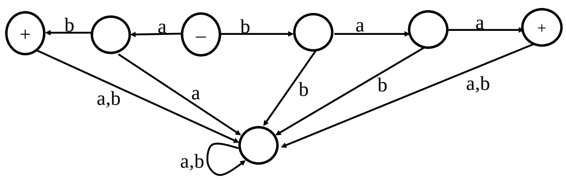
- Does this FA accept the word ababa?
- The word babbb?
- 2 ways to get to state 4
- The only way to get to state 2 is by reading an input a.
- The only way to get to state 3 is by reading an input b.
- What language?



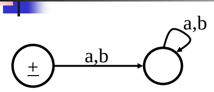
- The third letter is b.
  (aab + abb + bab + bbb)(a+b)*
  (a+b)(a+b)b(a+b)*
- The regular expression is not unique.
- Is there always at least one regular expression defining the language accepted by an FA?



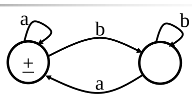
- Regular expression: **baa**
- A "collecting bucket" state for all other words.



- Regular expression: **baa + ab**

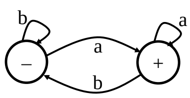


Λ



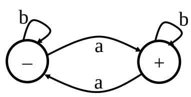
(a+b)*a + Λ

Words that do not end in b.

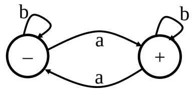


(a+b)*a

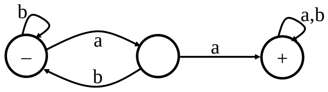
Words that end in a.



?

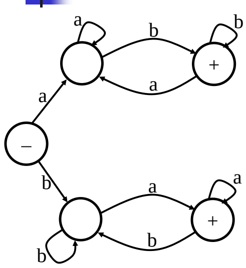


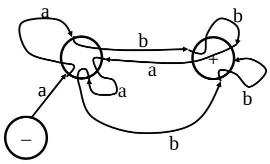
- The only letter that can change state is a.
  (b*ab*)(ab*ab*)*
- an odd number of a’s



Words that contain a double a
(a+b)*aa(a+b)*

- Start state: the previous letter (if there is one) was not an a.
- Middle state: we have just seen an a that was not preceded by an a.
- Final state: we have seen a double a.





aabbaabb

The Language EVEN-EVEN