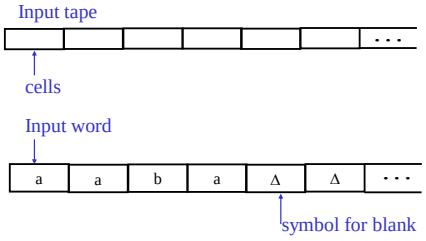
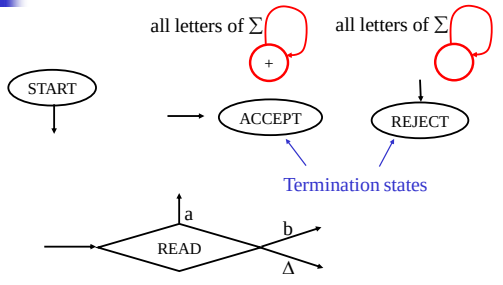
Chapter 14: Pushdown Automata

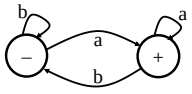
I. Theory of Automata
→ II. Theory of Formal Languages
III. Theory of Turing Machines …

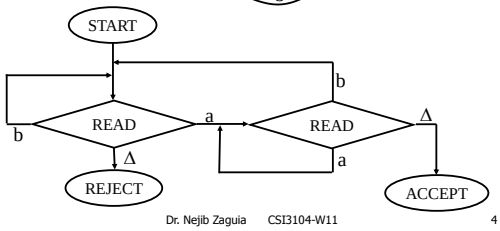
Pushdown Automata : A new model for theoretical machines.



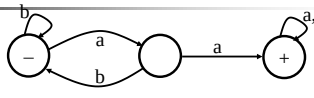


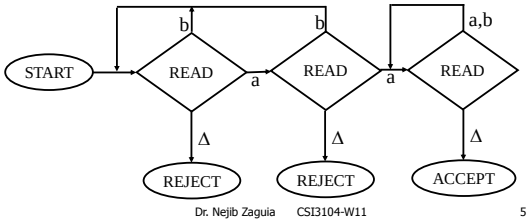
Example





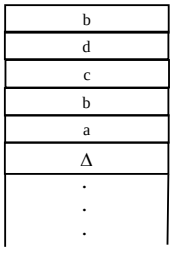
Example



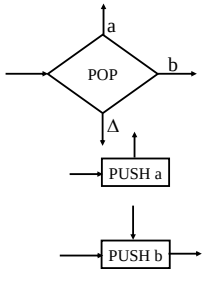


Adding a pushdown stack

Operations POP and PUSH

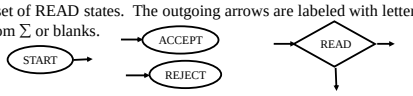


push a
push b
push c
push d
push b
push c
pop



A pushdown automaton (PDA) is:

1. an alphabet Σ of input letters
2. an input tape, finite on the left but infinite to the right and separated into cells. The input tape initially contains the sequence of input letters starting in the first cell. The rest of the tape contains only blanks Δ.
3. one start state with at least one outgoing edge and no incoming edges
4. a set of halt states with incoming edges only
5. a set of READ states. The outgoing arrows are labeled with letters from Σ or blanks.

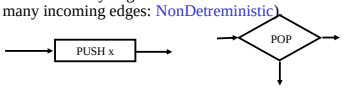


A pushdown automaton (PDA) is:

…

6. an alphabet Γ for stack characters
7. a pushdown stack, infinite in one direction. The stack is initially empty.
8. a finite set of PUSH states that put characters onto the top of the stack
9. a finite set of POP states. Outgoing edges are labeled with letters of Γ and blanks.

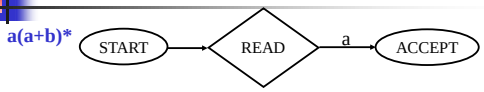
(There can be none or many edges labeled with the same character. There can be many incoming edges: NonDetreministic)



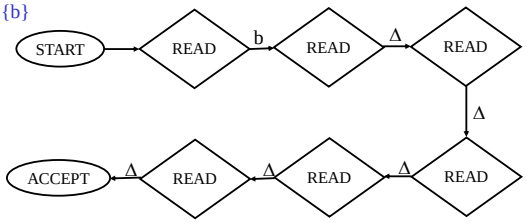
- The language accepted or recognized by a pushdown automaton is the set of words whose paths end in an ACCEPT state.
- Theorem. For every regular language L, there is some PDA that accepts it.

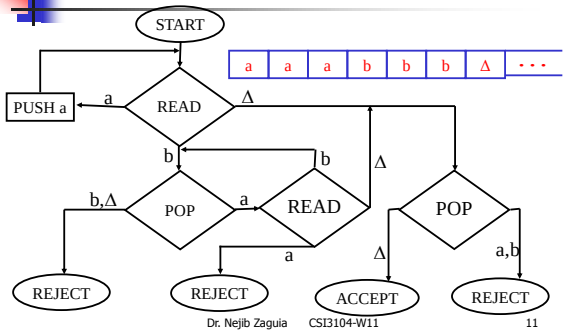
  Proof: There is an FA that accepts L because it is regular. We transform the FA into a PDA using the constructive algorithm already discussed.

a(a+b)*

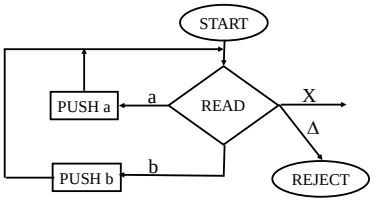


{b}

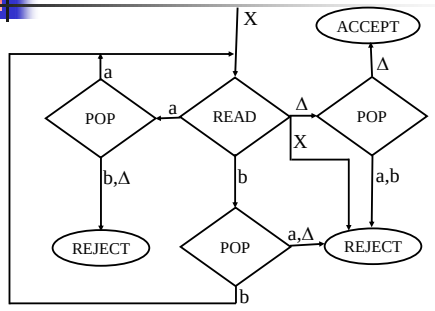


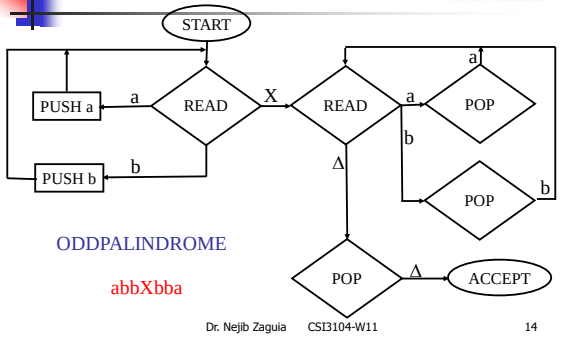


PALINDROMEX = {X,aXa,bXb,aaXaa,abXba,baXab,…}

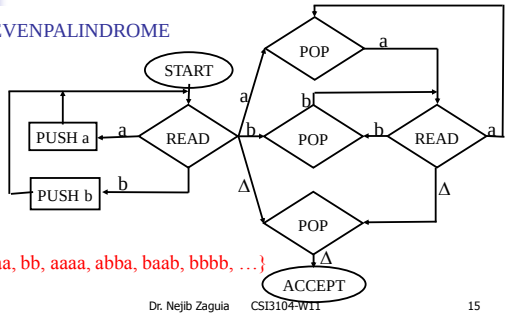


Cont …

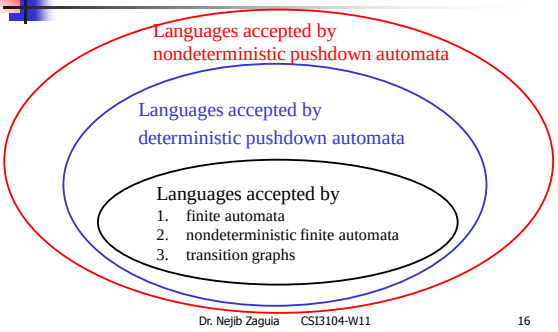


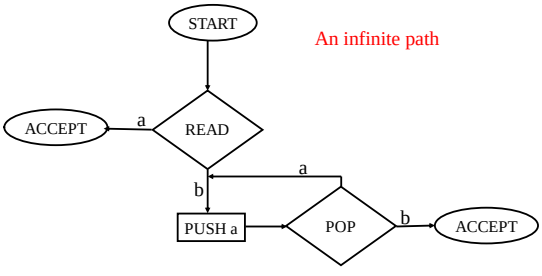


EVENPALINDROME

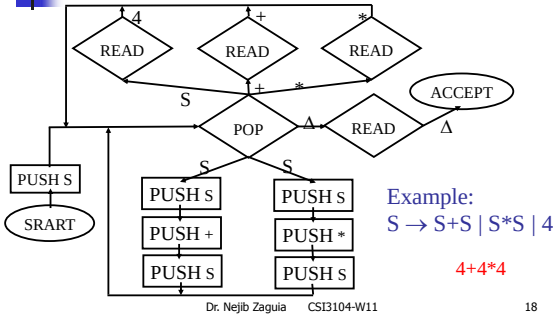


{Λ, aa, bb, aaaa, abba, baab, bbbb, …}





An infinite path



Example:
$S \rightarrow S+S \mid S*S \mid 4$

4+4*4

- Theorem. Let L be a language accepted by a PDA. There is another PDA that accepts L such that whenever a path leads to ACCEPT, the stack and the input tape contain only blanks.

It is enough to replace each

ACCEPT

BY