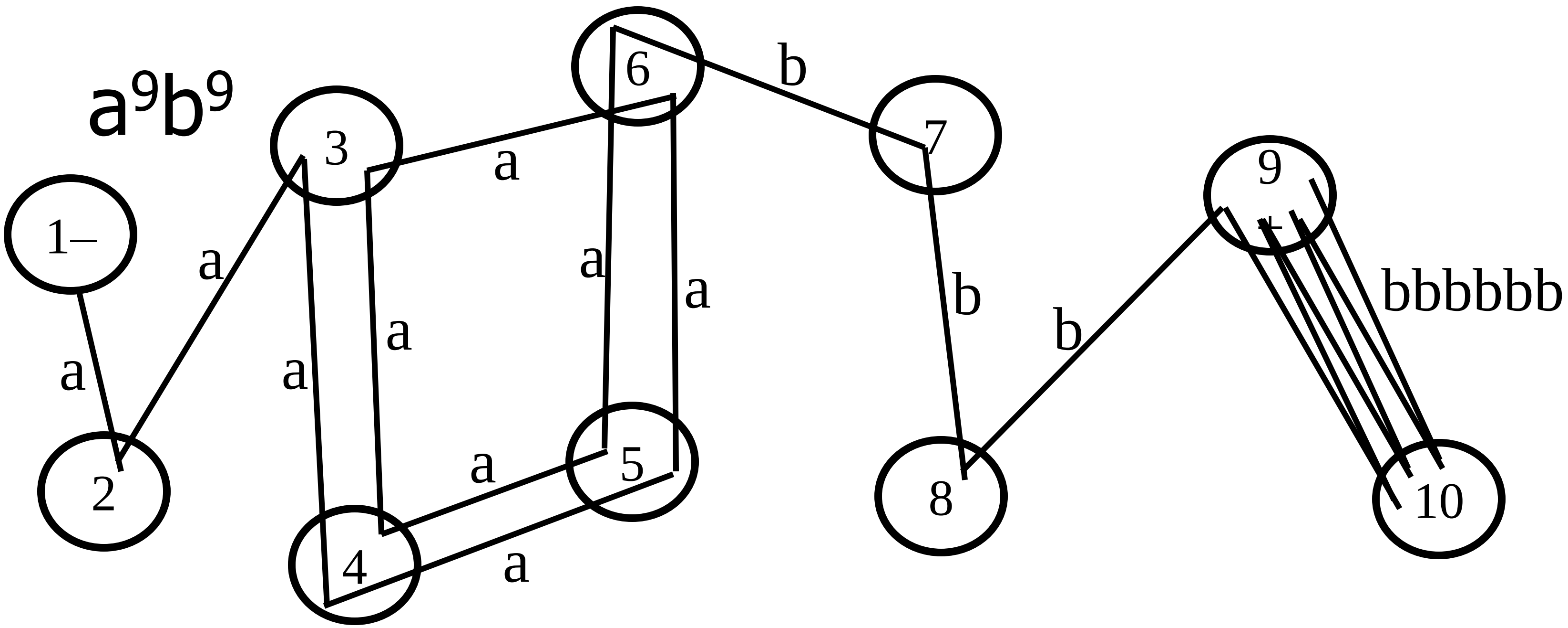
Chapter 10: Nonregular Languages

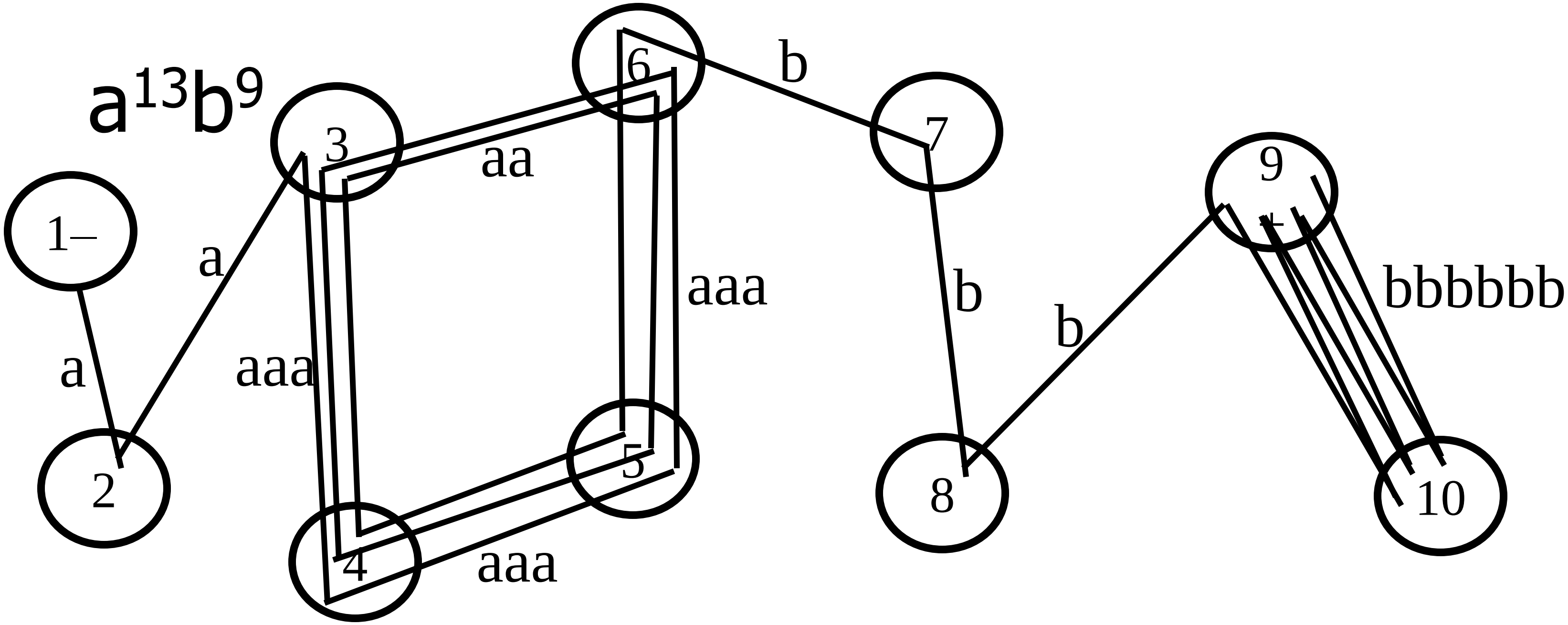
We show several examples of nonregular languages: those that cannot be defined by regular expressions.

Example:

- L = {Λ, ab, aabb, aaabbb, aaaabbbb, ...}
- L = {$a^n b^n$ | n=0,1,2,3,4,...}
- L = {$a^n b^n$}
- L ⊂ language(**a*b***)


a^9b^9
1–
a
2
a
3
a
a
4
a
a
5
a
a
6
a
b
7
b
8
b
9
+
bbbbbb
10

$a^{13}b^9$



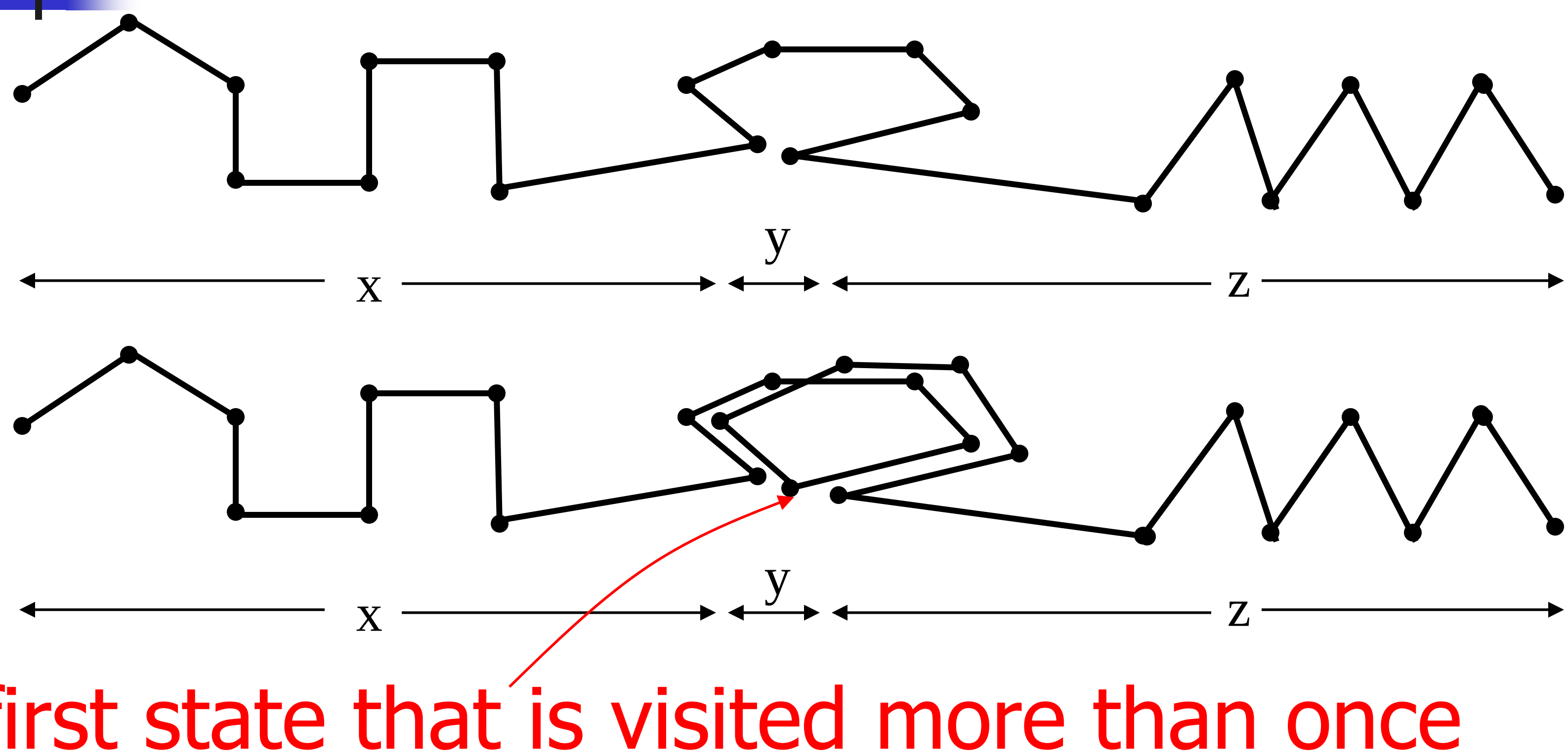
$a^5(a^4)^*b^9$

The pumping lemma:

Let L be any regular language that has infinitely many words. Then there exist three words x,y,z (where y is not the empty word) such that all words of the form:

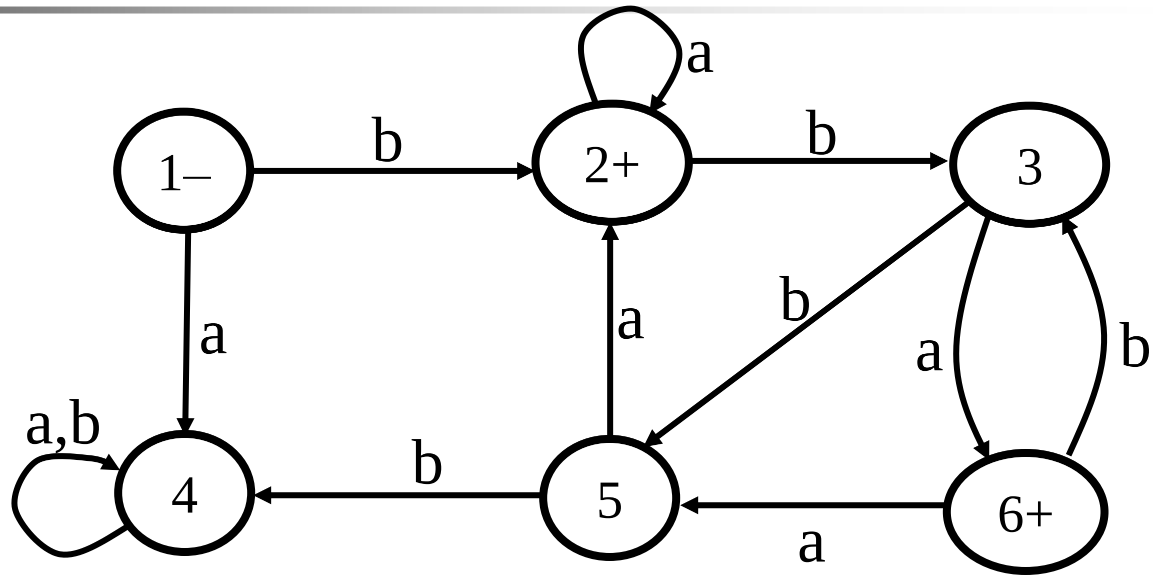
$$xy^nz \quad n=1,2,3,4,\ldots$$

are in L.

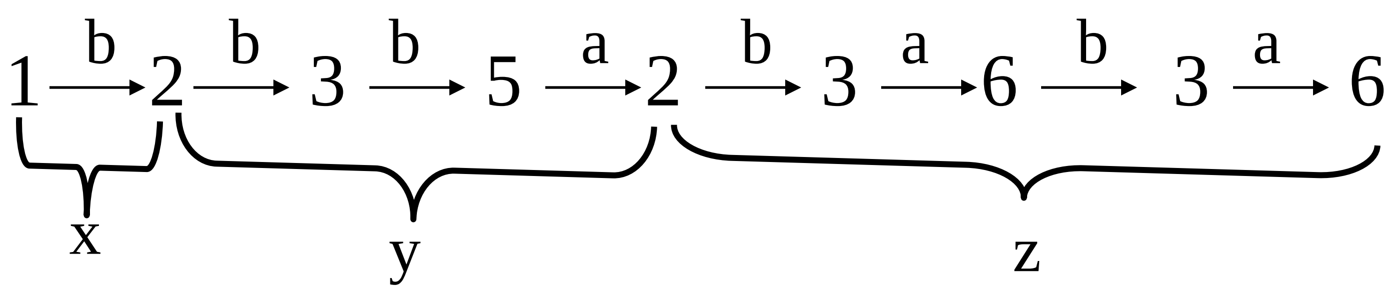


xz ∈L, xyz ∈L, xyyz ∈L, xyyyz ∈L, …, xy^nz ∈L.

Example



w = bbbababa



Theorem: $L = \{a^n b^n \mid n=0,1,2,3,4,\ldots\}$ is not regular.

EQUAL = all words with the same number of a's and b's.

EQUAL = {Λ, ab, ba, aabb, abab, abba, baab, baba, bbaa, …}.

Theorem: EQUAL is not regular.

Theorem: $L = \{a^n b a^n \mid n=0,1,2,3,4,\ldots\}$

$= \{b, aba, aabaa, \ldots\}$ is not regular.

Pumping Lemma: (version 2)

Let L be any regular language that has infinitely many words that is accepted by a finite automaton with N states. All words w in L that have more than N letters can be decomposed into words x,y,z such that:

1. y is not the empty word
2. $length(x) + length(y) \leq N$
3. $w = xyz$
4. for all $n \geq 1$, xy^nz is in L.

Theorem: PALINDROME is not regular.

Theorem: PRIME $= \{a^p \mid p \text{ is a prime number}\}$

$= \{aa, aaa, aaaaa, aaaaaaa, \ldots\}$

is not regular.