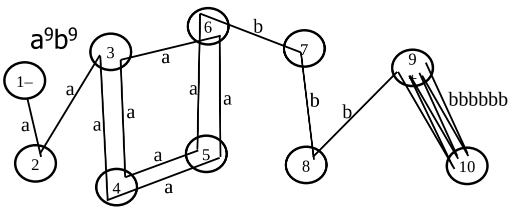
Chapter 10: Nonregular Languages

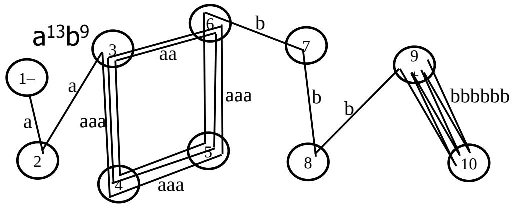
We show several examples of nonregular languages: those that cannot be defined by regular expressions.

Example:

- L = {Λ, ab, aabb, aaabbb, aaaabbbb, …}
- L = {$a^n b^n$ | n=0,1,2,3,4,…}
- L = {$a^n b^n$}
- L ⊂ language(**a*b***)

a^9b^9





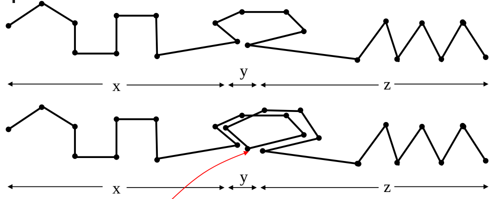
$a^5(a^4)^*b^9$

The pumping lemma:

Let L be any regular language that has infinitely many words. Then there exist three words x,y,z (where y is not the empty word) such that all words of the form:

$$xy^nz \quad n=1,2,3,4,\ldots$$

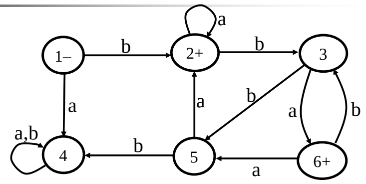
are in L.



first state that is visited more than once

$xz \in L$, $xyz \in L$, $xyyz \in L$, $xyyyz \in L$, …, $xy^nz \in L$.

Example



w = bbbababa

$$\underbrace{1 \xrightarrow{b} 2}_{x} \underbrace{\xrightarrow{b} 3 \xrightarrow{b} 5 \xrightarrow{a} 2}_{y} \underbrace{\xrightarrow{b} 3 \xrightarrow{a} 6 \xrightarrow{b} 3 \xrightarrow{a} 6}_{z}$$

Theorem: L = $\{a^n b^n \mid n=0,1,2,3,4,\ldots\}$ is not regular.

EQUAL = all words with the same number of a's and b's.
EQUAL = {Λ, ab, ba, aabb, abab, abba, baab, baba, bbaa, …}.

Theorem: EQUAL is not regular.

Theorem: L = $\{a^n b a^n \mid n=0,1,2,3,4,\ldots\}$
= {b, aba, aabaa, …} is not regular.

Pumping Lemma: (version 2)

Let L be any regular language that has infinitely many words that is accepted by a finite automaton with N states. All words w in L that have more than N letters can be decomposed into words x,y,z such that:

1. y is not the empty word
2. length(x) + length(y) $\leq$ N
3. w = xyz
4. for all n $\geq$1, $xy^n z$ is in L.

Theorem: PALINDROME is not regular.

Theorem: PRIME $= \{a^p \mid p \text{ is a prime number}\}$
$= \{aa, aaa, aaaaa, aaaaaaa, \ldots\}$
is not regular.