University of Ottawa
School of Information Technology and Engineering

CSI 3104, Winter 2011 **Assignment #3 Solution**

Exercise # 1(v) page 203

(v) Using the stronger Pumping Lemma, $|xy|<N$ (number of states), consider $xyz = a^N b^N a$. Hence y must be contained in the first set of a's and when pumped the a's would not balance the b's. So $xy^n z$ is not in the language.

Exercise # 3 page 204

For this we need the Pumping Lemma with length condition. Let the FA have N states. Consider $xyz = a^N b a^N b$ a word in DOUBLEWORD. $|xy|<N$ so $xy = a^N$ and when pumped, it is no longer in DOUBLEWORD.

Exercise # 2 page 217

FA_1	a	b
$-q_1$	q_2	q_1
$+q_2$	q_2	q_1

FA_2	a	b
$-r_1$	r_2	r_3
$+r_2$	r_2	r_3
r_3	r_2	r_3

FA_1/FA_2		a	b
$-s_1$	q_1/r_1	s_2	s_3
s_2	q_2/r_2	s_2	s_3
s_3	q_1/r_3	s_2	s_3

$FA_1 \cap FA_2{}'$ accepts at $q_2 r_1$, $q_2 r_3$

$FA_1{}' \cap FA_2$ accepts at $q_1 r_2$

Exercise # 13 (iv) page 220 **INFINITE**

Exercise # 11 page 256

$S \rightarrow SS \mid EXE$
$X \rightarrow aX \mid a$
$E \rightarrow EQUAL$

Exercise # 17 page 257

(i) $S \Rightarrow \underline{X}aX \Rightarrow a\underline{X}aX \Rightarrow a\Lambda a\underline{X} \Rightarrow a\Lambda a = aa$

$S \Rightarrow Xa\underline{X} \Rightarrow a\underline{X}\Lambda \Rightarrow aa\underline{X} \Rightarrow aa\Lambda = aa$

(ii) $S \Rightarrow a\underline{S}X \Rightarrow aa\underline{S}XX \Rightarrow aa\Lambda X\underline{X} \Rightarrow aa\underline{X}a \Rightarrow aaaa = aaaa$

$S \Rightarrow a\underline{S}X \Rightarrow a\Lambda\underline{X} \Rightarrow aa\underline{X} \Rightarrow aaa\underline{X} \Rightarrow aaaa = aaaa$

(iii) $S \Rightarrow a\underline{S} \Rightarrow aa\underline{S} \Rightarrow aa\Lambda = aa$

$S \Rightarrow aa\underline{S} \Rightarrow aa\Lambda = aa$

(iv) (i) $S \rightarrow bS \mid aX$
$X \rightarrow aX \mid bX \mid \Lambda$
(ii) $S \rightarrow aX$
$X \rightarrow aX \mid a$
(iii) $S \rightarrow aS \mid bS \mid \Lambda$

(v) (i) $S \rightarrow bS \mid aX \mid a$
$X \rightarrow aX \mid bX \mid a \mid b$
(ii) same as above
(iii) $S \rightarrow aS \mid bS \mid a \mid b$

Exercise # 14(iv) page 287

$E \to EP$	$E \to 7$
$E \to ET$	$X \to +$
$E \to LR$	$Y \to *$

Exercise # 6 page 314

(i) Consider the machine in sections.

1. The machine accepts $\Lambda = a^0 s$ where length(s) = 0.
2. The first letter of any other string must be a. That a is stored. As long as the machine continues to read a's they continue to be stored. No word consisting only of a's can be accepted.
3. At the first b, control passes to the second loop. For each letter read (including that b) the STACK is popped once. If there are fewer a's in the STACK than letters from the first b to the end of the word the string crashes. When there are no more letters on the TAPE (read Δ) the string crashes if there are a's left on the STACK. Only those words are accepted that have the same number of letters from the first b to the end of the word as initial a's.

(ii) $S \to aSb \mid aSa \mid ab$

(iii) Use the strong version of the Pumping Lemma. Assume a machine that has n states, and take the counterexample of $\{a^{2n} b a^{2n-1}\}$. There must be a circuit within the first clump of a's, and this can displace the b that initiates the second half.

Exercise # 15 page 349

(i) $S \to AR \mid AB$

$R \to SB$

$A \to a$

$B \to b$

(ii)