

# A Factor Graph Approach to Link Loss Monitoring in Wireless Sensor Networks

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**Abstract**—The highly stochastic nature of wireless environments makes it desirable to monitor link loss rates in wireless sensor networks. In a wireless sensor network, link loss monitoring is particularly supported by the *data aggregation* communication paradigm of network traffic: the data collecting node can infer link loss rates on all links in the network by exploiting whether packets from various sensors are received, and there is no need to actively inject probing packets for inference purposes. In this paper, we present a low complexity algorithmic framework for link loss monitoring based on the recent modeling and computational methodology of factor graphs. The proposed algorithm iteratively updates the estimates of link losses upon receiving (or detecting the loss of) recently sent packets by the sensors. The algorithm exhibits good performance and scalability, and can be easily adapted to different statistical models of networking scenarios. In particular, due to its low complexity, the algorithm is particularly suitable as a long-term monitoring facility.

**Index Terms**—Factor graphs, link loss monitoring, network tomography, sensor networks, sum-product algorithm.

## I. INTRODUCTION

RECENT technological advances have made it feasible to deploy large-scale sensor networks using low-cost sensor nodes. However, as the scale of sensor networks becomes larger, two key challenges potentially arise. First, *node failures*. Due to their inherent instability and energy constraints, sensor nodes are prone to failures. It would, thus, be useful to determine which set of nodes or which geographical areas within the network are experiencing high loss rates. Such information is potentially valuable to the design of fault-tolerant protocols or monitoring mechanisms, so that the problem areas may be redeployed, and critical data may be rerouted to avoid these failure-prone areas suffering high loss rates. Second, *resource constraints*. Since sensor nodes have limited computational resources, any algorithms developed for wireless sensor networks must not rely on the assumption of unlimited resources, and must sparingly use the limited resources that do exist. Further, due to wireless characteristics such as fading and interference, wireless sensor networks are subject to stringent bandwidth resource constraints. One cannot rely on the use of active

acknowledgments—which are not scalable or bandwidth-efficient—in the design of sensor network algorithms.

Motivated by the needs (fault-tolerance and reliability) and constraints (bandwidth and computational power) illustrated above, in this paper, we concentrate on the problem of efficiently determining link loss rates in wireless sensor networks. Particularly, we attempt to efficiently determine link loss rates based on the *data aggregation* communication paradigm in sensor networks. Due to the obvious need of centralized sensor data processing and monitoring, the paradigm of data aggregation, also referred to as *data fusion*, has been critical to the effective operation of sensor networks. Work in this area has been previously presented (refer to [1] as an example) and continues to be actively researched. In the process of data aggregation, a subset of nodes in the network attempts to forward the sensor data they have collected back to a *center* (or *sink*) node via a *reverse multicast tree*.

More specifically, in the process of data aggregation, before a node sends its data to the next node in the path to the *sink*, it waits to receive data from all of its child nodes in the reverse multicast tree (or until a specified period of time has elapsed). The node then aggregates its own data with the data it has received from its child nodes, and forward this aggregated data to the *sink* via the reverse multicast tree. Information about which nodes' data is present in the aggregated data must also be sent to the *sink*. Thus, data fusion saves communication overhead at the cost of additional computation and memory resources. Fig. 1 depicts a simple example of a sensor network using the data aggregation paradigm.

In wireline networks, the field of network inference, also referred to as network tomography, involves the estimation of network performance parameters using measurements. In wireline networks, inferring link losses requires either network multicast support (which is not always the case), or sending series of back-to-back packet pairs with unicast (see, e.g., [3] and [4] and the references included therein). In the case of multicast-based link loss inference, a center node sends out a batch of multicast probing packets to all terminals and, upon receiving the acknowledgments from the terminals whether these probing packets are received, performs statistical inference on the link loss rates. In addition to requiring the multicast protocol to be supported by the network, such a strategy perturbs the network by sending out extra packets solely for the purpose of probing. Similar observations can also be made in the case of inference with unicast-based packet-pair measurements, though it does not require specific support from the network layer. Injecting additional traffic will further aggravate the link losses at the loaded

Manuscript received December 15, 2003; revised July 30, 2004.

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Digital Object Identifier 10.1109/JSAC.2005.843557

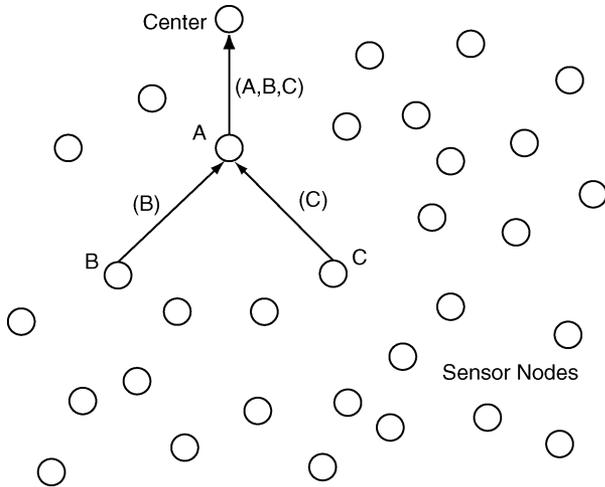


Fig. 1. Data aggregation (fusion) in wireless sensor networks: an example. Node B sends its data (B) destined for the center node, to node A. Node C similarly sends its data (C) destined for the center node, to node A. Node A then aggregates its own data (A) with that of nodes B and C, and sends the fused data (A, B, C) to the center node. With data aggregation, each node is only required to transmit once per data collection round. However, without data aggregation, node A would have to transmit three times per data collection round: once to send its own data to the center node, once to forward node B's data, and once to forward node C's data.

links, which makes it impractical as a long-term monitoring daemon in sensor networks, given existing resource constraints.

In this paper, we propose and examine a new and efficient mechanism to monitor link losses in wireless sensor networks. In a wireless sensor network, a set of terminal (sensor) nodes send data, concerning some measurements of the physical world, to a center (data-collecting) node via a set of wirelessly connected links. We take advantage of the data aggregation communication paradigm in sensor networks, where the data-carrying traffic flows from the terminals to the center via a reverse multicast tree. Such a characteristic potentially enables the implementation of simple protocols and algorithms for constant link loss monitoring at virtually no cost. Our original contributions are twofold. First, we present a novel algorithm for the purpose of link loss inference, based on the recent methodology of factor graphs and the sum-product algorithm [2]. We show that this algorithm has very low complexity, and demonstrate by simulations its excellent performance and scalability. Second, we are one of the first to consider network inference exploiting reverse multicast trees for data aggregation in sensor networks. Most existing research in this area has dealt with the traditional multicast and unicast communication paradigms in wireline networks, where probes are sent from a single source to one or several receivers (see, e.g., [3], [4], [13]–[16], [19], and [26]).

The remainder of this paper is organized as follows. We will first present the formulation of the problem in Section II, and introduce the algorithm in Section III. We will then give the simulation results and some discussion in Section IV and close this paper with a brief conclusion in Section V.

## II. MODEL AND PROBLEM FORMULATION

We consider a sensor network as a directed graph, where each node represents either a terminal (sensor), a router, or the center

(data collecting) node, each directed edge represents the link between these nodes, and the direction of an edge indicates the direction of the data flow on the link. Based on the data aggregation paradigm, we consider a reverse multicast tree rooted at the data collecting node, where messages are sent from leaf nodes to the data collecting node. We will not allow, except for the data collecting node, degree-two nodes in the graph; that is, if a degree-two node is not the data collecting node, it is suppressed in the graph. In this setting, what we refer to as a “link” is not necessarily in its physical sense, since a “link” can be a path consisting of several connected physical links as long as no other paths are branched from an intermediate node in the path. It can be verified that such a notion of “link” is defined without loss of generality, as far as loss rates are concerned. In this paper, the term “path” of a network refers to a path that starts from a terminal node and ends at the center node.

We assume that all terminal nodes send packets constantly, in a synchronous manner, to the center node along the tree (in Section IV, we will briefly discuss the possibility of relaxing this assumption). Each intermediate node in the tree, upon receiving the packets from its children, creates an aggregated packet and forwards it to its parent. Here, the notion of “packet” is also more conceptual than implementational. For example, we will not require packets sent by different nodes to have the same size, and rather assume that the aggregated packet sent by any intermediate node is large enough to “bundle” all the information contained in its children's packets. Throughout this paper, a packet sent from a sensor is said to be “received” or to “have arrived” if the data contained in the packet is received by the data collecting node in the aggregated packet. As part of the transmission protocol, for every packet transmitted by a terminal node, the center node expects it to arrive within a certain time frame; and if the packet is not received within that time frame, then a packet loss is suggested to have occurred on one of the links along the path. Based on successive observations on whether packets from each terminal have arrived, the center node can infer the link loss rates on all links in the network.

Formally, we will use  $E$  to denote the set of links of the network of interest, and  $W$  to denote all the paths in the network. Associate to each link  $e \in E$  a state  $x_e$ , taking values from  $\{0, 1\}$ ; when link  $e$  is at state 0 (“bad state”), no packet will pass through  $e$ , and when link  $e$  is at state 1 (“good state”), all packets can pass through  $e$ . For each path  $w \in W$ , let its state  $x_w$  be the logic AND of all the links consisting of  $w$ , for which we write

$$x_w = \bigoplus_{e \in w} x_e$$

where  $\bigoplus$  denotes the logic AND operator and “ $e \in w$ ” reads “ $e$  is a link contained in  $w$ .” For example, in the toy example of a sensor network in Fig. 2, there are three links  $a$ ,  $b$ , and  $c$ , and two paths  $\{a, b\}$  and  $\{c, b\}$ ; and the links states and path states are related by  $x_{\{a,b\}} = x_a \oplus x_b$ , and  $x_{\{c,b\}} = x_c \oplus x_b$ . Clearly, if and only if when the state  $x_w$  of path  $w$  is 1 can packets pass through  $w$ . Then, whether a packet will be received essentially indicates the state of the path along which the packet is to travel.

At any time instant, the state  $x_e$  of every link  $e$  can be regarded as a Bernoulli random variable with probability  $\alpha_e$

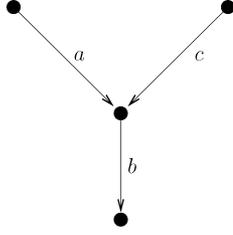


Fig. 2. Toy example of a sensor network.

taking value 1 and with probability  $1 - \alpha_e$  taking value 0. In this paper, we use

$$B(x, \alpha) = \begin{cases} \alpha, & x = 1 \\ 1 - \alpha, & x = 0 \end{cases}$$

to denote the probability mass function of a Bernoulli random variable parametrized by  $\alpha$ .

We will assume that  $\alpha_e$  at each link is quasi-static, namely, over a relatively small time window in which hundreds or thousands of packets may be sent by any terminal,  $\alpha_e$  stays as a constant. Suppose that during a time window, there are  $n$  batches of synchronized packets transmitted from each terminal to the center, where the  $i$ th batch of packets are transmitted at time  $t_i$ ,  $i = 1, 2, \dots, n$ , from all the terminals synchronously. Let  $x_e^{(i)}$  be the link state of  $e$  at time  $t_i$ , then the path state  $x_w^{(i)}$  of path  $w$  at time  $t_i$  is

$$x_w^{(i)} = \bigoplus x_e^{(i)}$$

experienced by the packet transmitted at  $t_i$  and traveling along  $w$ ; the observation whether the packets in the  $i$ th batch have arrived indicates the states of all paths at time  $t_i$ .

Collectively, we denote  $X_E^{(i)} := \{x_e^{(i)} : e \in E\}$ ,  $X_W^{(i)} := \{x_w^{(i)} : w \in W\}$ ,  $X_E^{(l,m)} := \{X_E^{(i)} : l \leq i \leq m\}$ ,  $X_W^{(l,m)} := \{X_W^{(i)} : l \leq i \leq m\}$ , and  $\alpha_E := \{\alpha_e : e \in E\}$ . The problem of link loss inference in the network is then the problem of estimating  $\alpha_E$  based on the observation of  $X_W^{(1,n)}$ . In this paper, we set up the problem as, for each  $e \in E$ , finding  $\hat{\alpha}_e$  that maximizes the posterior probability  $P[\alpha_e | X_W^{(1,n)}]$  of  $\alpha_e$  conditioned on the observation  $X_W^{(1,n)}$ . In addition, it is important to realize that  $\alpha_e$  changes with time. Thus, we ideally desire an algorithm to have a sufficiently low complexity so as to serve as a daemon tracking the link loss rates constantly.

### III. FACTOR GRAPH APPROACH

Link loss inference belongs to the relatively recent area of networking research, network tomography (see [3], [4], [13]–[22], [26], [27], etc.). Typical network tomography problems include the inference of link loss or delay characteristics from end-to-end measurements [3], [13]–[19], [22], [26], [27], the estimation of origin-destination traffic intensities from link measurements [23]–[25], and the inference of network topology [20], [21].

Prior to this work, most of the literature on link loss inference concerns wireline networks, where multicast packets are sent actively to probe the network. When multicast addressing is not supported by the network, there have been alternative proposals on link loss inference based on sending unicast probing packets, where clever protocols (for example, using back-to-back packets) are incorporated which essentially turns the inference problem to a multicast problem (see, for example, [13] and [14]).

For the multicast link loss inference problem, Caceres *et al.* [3] present an maximum-likelihood (ML) estimator for the link loss rates that is *asymptotic optimal* (namely, approaching the true ML estimator for asymptotically large number of probes). The expectation-maximization algorithm is also presented as a solution to this problem [15], [26]. It may be arguable that these techniques can be applied to link loss inference in wireless sensor networks, a main perspective of this paper is, however, the concern of the algorithm complexity when it is used as a long-term monitoring daemon. Comparing with previous works, the factor graph approach we present, although suboptimal, demonstrates good performance and most importantly, very low complexity.

In this section, we will first give a brief introduction on the factor-graph framework as an modeling inference methodology, and then proceed to introduce our algorithm. The performance of the algorithm will be shown in the next section.

#### A. Factor Graphs and the Sum-Product Algorithm

Recently, the notion of factor graphs has attracted intense research interest in areas of electrical engineering and computer science, since it was recognized that the framework of factor graphs and an iterative algorithm, called the sum-product algorithm—which operates on factor graphs—unify a variety of previously discovered important algorithms, such as the Viterbi algorithm, BCJR algorithm, Kalman filtering, FFT, belief propagation, forward-backward algorithm, etc. In particular, in the community of error correction coding, it is shown that factor graphs and the sum-product algorithm underlie the methodology of the most celebrated error control coding schemes, turbo codes [8] and low-density parity-check codes [9]–[11].

To date, the notion of factor graphs includes multiplicative factor graphs and convolutional factor graphs [2], [5], [6]. In this paper, we will mainly deal with multiplicative factor graphs, referred to as factor graphs from here on, for simplicity.

A factor graph is a bipartite graph representing the factorization structure of a multivariate function into a product of functions (factors), each involving only a subset of the variables. There are two types of vertices in the graph, variable vertices, representing the variables of the global multivariate function, and function vertices, representing the factors in the factorization; a variable vertex is connected to a function vertex by an edge if the variable is an argument of the factor. For example, Fig. 3 is a factor graph representing the factorization  $f_1(x_1, x_2)f_2(x_2, x_3)f_3(x_2, x_4)$ , where each square box is a function vertex representing factor  $f_1$ ,  $f_2$ , or  $f_3$ , and each circle is a variable vertex representing variable  $x_1$ ,  $x_2$ ,  $x_3$ , or  $x_4$ .

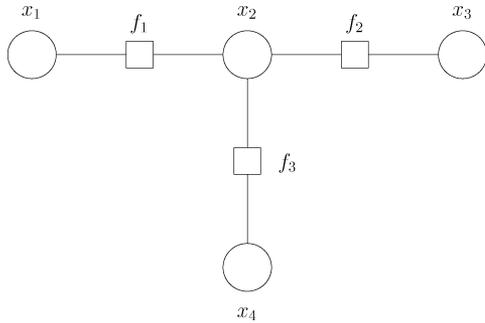


Fig. 3. Factor graph representing  $f_1(x_1, x_2)f_2(x_2, x_3)f_3(x_2, x_4)$ .

A factor graph can be used as a probabilistic graphical model which represents a joint probability mass function (PMF)<sup>1</sup> of random variables. In this case, each variable node represents a random variable, and each factor represents either the joint or conditional joint PMF (PDF) of a subset of random variables, and conditioned upon any subset of random variables corresponding to a cut-set of the graph, the separated two subgraphs (induced by removing the cut-set vertices) are independent. For example, let  $\mathbf{X}_1$ ,  $\mathbf{X}_2$ ,  $\mathbf{X}_3$ , and  $\mathbf{X}_4$  be a set of random variables where conditioned on  $\mathbf{X}_2$ , random variables  $\mathbf{X}_1$ ,  $\mathbf{X}_3$ , and  $\mathbf{X}_4$  are independent of each other. Then, the joint PMF of the four random variables may be represented by the factor graph in Fig. 3, where we may interpret  $f_1(x_1, x_2)$  as the joint PMF of  $\mathbf{X}_1$  and  $\mathbf{X}_2$ ,  $f_2(x_2, x_3)$  as the conditional PMF of  $\mathbf{X}_3$  given  $\mathbf{X}_2$ , and  $f_3(x_2, x_4)$  as the conditional PMF of  $\mathbf{X}_4$  given  $\mathbf{X}_2$ . We note that the interpretation of  $f_1$ ,  $f_2$ , and  $f_3$  is in general not unique; for example,  $f_1(x_1, x_2)$  may represent the conditional PMF of  $\mathbf{X}_1$  given  $\mathbf{X}_2$ ,  $f_2(x_2, x_3)$  may represent the conditional PMF of  $\mathbf{X}_3$  given  $\mathbf{X}_2$ , and  $f_3(x_2, x_4)$  may represent the joint PMF of  $\mathbf{X}_2$  and  $\mathbf{X}_4$ . That is, as a probabilistic model, a factor graph representing a joint PMF fundamentally specifies a set of conditional independence relationships and the functions (factors) represented by the function vertices may take an arbitrary scale, subject to the constraint that the product of the functions satisfy as a PMF or PDF (the sum or integral of the product over all variables equals to 1).

A useful function for representing a deterministic constraint in a factor graph is the constraint indicator function: let  $C(x)$  be a constraint (a boolean proposition) on a possibly vector-valued variable  $x$ , then the constraint indicator function of  $C(x)$  is defined as

$$\delta[C(x)] := \begin{cases} 1, & \text{if } C(x) \\ 0, & \text{otherwise} \end{cases}.$$

That is, the constraint indicator function evaluates to 1 if the constraint is satisfied, and to 0, otherwise.

Unifying various algorithms, the sum-product algorithm is an algorithm that operates iteratively on a factor graph by “passing messages” between function vertices and variable vertices. The “messages” are essentially functions (for continuous-valued variables) or tables (for discrete valued variables) computed in the intermediate steps of the algorithm. If  $f(x_1, x_2, \dots, x_m)$  is a function that factors according to a

<sup>1</sup>PMF or probability density function (PDF). For simplicity, we will often omit mentioning the term PDF.

factor graph having no cycles, it is known that the sum-product algorithm can simultaneously compute  $\sum_{\sim x_1} f(x_1, \dots, x_m)$ ,  $\sum_{\sim x_2} f(x_1, \dots, x_m)$ ,  $\dots$ , and  $\sum_{\sim x_m} f(x_1, \dots, x_m)$  in parallel, where  $\sum_{\sim x_i}$  refers to summation<sup>2</sup> over all variables except  $x_i$ . For a concrete understanding of the sum-product algorithm, the reader is referred to [2] and [7]. In essence, what underlies the sum-product algorithm is the distributive law between multiplication and summation (or the generalized distributive law on any semiring, with arbitrarily defined multiplication and summation [7]).

When the function  $f(x_1, \dots, x_m)$  represented by the factor graph is a joint (or conditional joint) PMF, then the objective of the sum-product algorithm coincides with the objective of many inference problems, i.e., finding the maximizing configuration for marginal PMF  $\sum_{\sim x_i} f(x_1, \dots, x_m)$ .<sup>3</sup> Clearly, our formulation for the link loss inference problem in the previous section is such an example.

When the factor graph representing the function  $f(x_1, \dots, x_m)$  contains cycles, it has been shown in various recent works that the sum-product algorithm can still be used as an excellent approximation algorithm, particularly when the objective is to find the maximizing configuration for  $\sum_{\sim x_i} f(x_1, \dots, x_m)$  and not the maximum itself. In fact, the decoding methods for turbo codes [8] and low-density parity-check codes (see, e.g., [11]) are precisely the sum-product algorithm applied on factor graphs with cycles, and the performance of the sum-product algorithms enables these codes to achieve the Shannon limit of digital communications. An intuitive explanation on why the algorithm works so well is that the factor graphs used in these schemes are large and sparse, and the effects of cycles fades away after a few iterations. Also, due to the fact that the graphs are sparse, the complexity of the algorithm is essentially linear in the average vertex degree, which make the sum-product algorithm highly scalable.

In the cases when the factor graph contains cycles, the passing of messages in the factor graph may be carried out in various orders, typically referred to as the *schedules* of the algorithm [2], [28]. For example, a popular schedule, known as the “flooding” schedule, is that in each iteration, all variable vertices first pass messages and then all function vertices pass messages, where the message-passing rules (the definitions of the messages) stay the same. Summary messages may also be computed at each iteration for the purpose of identifying convergence. The algorithm is usually terminated upon convergence of the summary messages or upon reaching a preset number of iterations.

It should be noted that when the factor graph contains cycles, any schedule of the sum-product algorithm will lead to a sub-optimal solution to the maximization problem. However, it has been reported that by adjusting the schedule of the algorithm there can be extra gain in the suboptimality, i.e., the found solution can be closer to the true optimal configuration [28].

<sup>2</sup>In fact, the summation operation here can be made more general. In particular, if the summation operation is the max operation, the sum-product algorithm is referred to as the max-product algorithm. See also footnote 3.

<sup>3</sup>In some inference problems, the objective is to find the maximizing configuration for the joint PMF  $f(x_1, \dots, x_m)$ ; this can be solved using the max-product algorithm on the factor graph representing  $f$ ; see [2].

### B. Proposed Algorithm

We assume that  $\alpha_e$  for each  $e$  takes discrete values from set  $S := \{1/L, 2/L, \dots, 1\}$ , where  $L$  is a positive integer. With no *a priori* knowledge, we assign to each  $\alpha_e$  a uniform prior over  $S$ .

Let the joint PMF of  $X_E^{(i)}$  parametrized by  $\alpha_E$  be denoted by  $B_E(X_E, \alpha_E)$ . Upon the assumption that each link suffers from independent link loss rates, we have

$$B_E(X_E, \alpha_E) = \prod_{e \in E} B(x_e, \alpha_e). \quad (1)$$

Then, the joint PMF of  $\alpha_E$ ,  $X_E^{(1,n)}$  and  $X_W^{(1,n)}$  factors as

$$P(\alpha_E, X_E^{(1,n)}, X_W^{(1,n)}) \propto \prod_{i=1}^n B_E(X_E^{(i)}, \alpha_E) P_{W|E}(X_W^{(i)}, X_E^{(i)}) \quad (2)$$

where  $P_{W|E}(X_W^{(i)}, X_E^{(i)})$  is a the conditional PMF of  $X_W^{(i)}$  given  $X_E^{(i)}$ , and in fact

$$P_{W|E}(X_W^{(i)}, X_E^{(i)}) = \prod_{w \in W} \delta[x_w^{(i)} = \bigoplus_{e \in w} x_e^{(i)}]. \quad (3)$$

Then, our objective becomes finding  $\hat{\alpha}_e$  for each  $e \in E$  that maximizes

$$P[\alpha_e | X_W^{(1,n)}] \propto \sum_{\sim \alpha_e} P[\alpha_E, X_E^{(1,n)}, X_W^{(1,n)}] \propto \sum_{\sim \alpha_e} \prod_{i=1}^n B_E(X_E^{(i)}, \alpha_E) P_{W|E}(X_W^{(i)}, X_E^{(i)}). \quad (4)$$

Notice that the objective of finding for each  $e \in E$ , the maximizing  $\hat{\alpha}_e$  for function

$$\sum_{\sim \alpha_e} \prod_{i=1}^n B_E(X_E^{(i)}, \alpha_E) P_{W|E}(X_W^{(i)}, X_E^{(i)})$$

precisely coincides with the objective of the sum-product algorithm,<sup>4</sup> and function

$$\prod_{i=1}^n B_E(X_E^{(i)}, \alpha_E) P_{W|E}(X_W^{(i)}, X_E^{(i)}) \quad (5)$$

is also ready to be represented by a factor graph as of Fig. 4(a). That is, the sum-product algorithm can be used to simultaneously find  $\hat{\alpha}_e$  for all  $e \in E$  in parallel.

If we further express the factors  $P_{W|E}(\cdot)$  and  $B_E(\cdot)$  in (5) according to (1) and (3), the factor graph can be expanded to a form similar to Fig. 4(b). Note that in the factor graph of Fig. 4(b), we consider the case where  $n = 2$  and the network takes the topology in Fig. 2 to simplify the factor graph, merely for illustration purpose.

<sup>4</sup>More rigorously speaking, the objective of the sum-product algorithm is to find the marginals, not the maximum of the marginals.

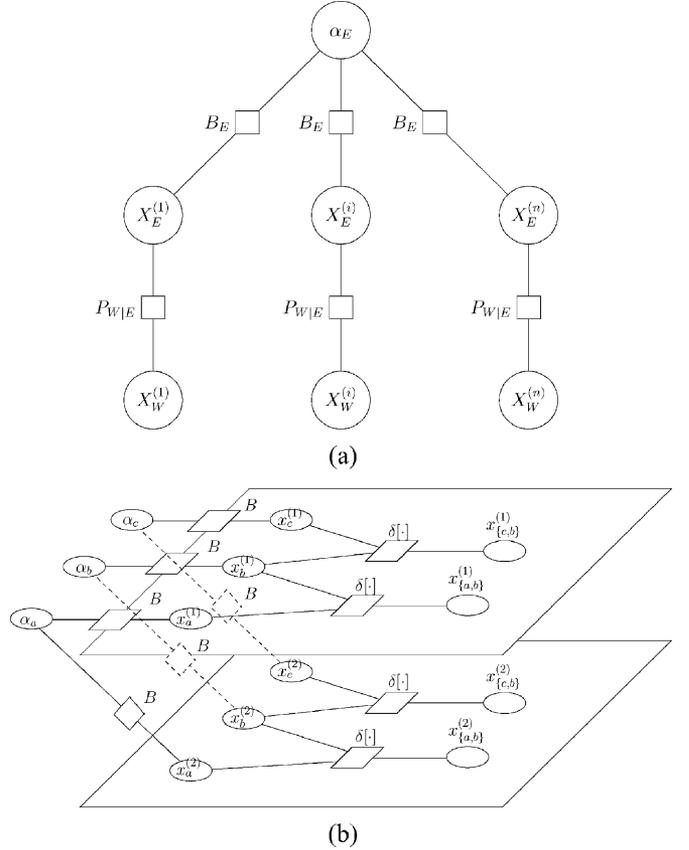


Fig. 4. (a) Factor graph representing the function  $\prod_{i=1}^n B_E(X_E^{(i)}, \alpha_E) P_{W|E}(X_W^{(i)}, X_E^{(i)})$  and (b) its expanded form, by letting  $n = 2$  and taking the network as of Fig. 2.

The sum-product algorithm will be applied on the factor graph in Fig. 4(b) to obtain the estimate  $\hat{\alpha}_e$  of  $\alpha_e$  for each  $e$ , which maximizes (4). Although the factor graph in Fig. 4(a) is cycle-free, typically its expanded form as of Fig. 4(b) contains cycles. This makes the application of the sum-product algorithm an approximation algorithm. Notice in Fig. 4(b), the factor graph consists of “layers” of subgraphs, each corresponding to a time instant  $t_i$ . This allows a natural schedule for message-passing in the sum-product algorithm, i.e., first passing messages in each layer, which we refer to as “intra-layer” message passing, and upon convergence, each layer of the graph passes messages to the vertices representing  $\alpha_e$ , which we refer to as “interlayer” message passing. With such a schedule, the processing of the path state at each time instant can be carried out independently, this will serve to significantly reduce the complexity in long-term monitoring, as will be addressed in Section IV.

We now present the algorithm.

*Step 1—Intralayer Message Passing:* The goal of this step can be understood as obtaining the posterior distribution for each link state at time instant  $t_i$ , conditioned upon observation on the  $i$ th batch of packets. In each layer, we use a “flooding” schedule similar to what was explained in Section III-A. One may follow the recipe discussed in [2] for the derivation the message-passing rule, where for the  $i$ th layer, the involved vertices are variable vertices representing  $x_e^{(i)}$  and  $x_w^{(i)}$ , and the function vertices representing  $\delta[\cdot]$ . Following the derivation, one should

see that the message-passing rule can be in fact made more compact by, in each layer, disregarding the vertices representing  $x_w^{(i)}$  and passing messages only between the  $\delta[\cdot]$  function vertices and the  $x_e^{(i)}$  vertices—note that the messages passed from the  $x_w^{(i)}$  vertices stay as constant from iteration to iteration; this is simply because the  $x_w^{(i)}$  vertices are leaf vertices. We will then use  $w$  to denote function vertex  $\delta[\cdot]$  that connects to  $x_w^{(i)}$ ,  $e$  to denote variable vertex  $x_e^{(i)}$ .

The message-passing rule for Step 1 is summarized as follows, where each message is a single number representing the (posterior) probability of a link taking state 1 (this is possible since each message is originally a function defined on  $\{0, 1\}$ , but since the values of the function at 0 and at 1 are dependent, i.e., summing to 1, one can reformulate the message as a single number).

In the initialization phase, each vertex  $e$  passes message  $\mu_{e \rightarrow w}$  representing the uniform distribution over the link state  $x_e$  to every adjacent  $\delta[\cdot]$  function vertex  $w \in \mathcal{N}(e)$ . That is,  $\mu_{e \rightarrow w} = 1/2$ .

In the propagation phase, then messages are passed iteratively between the variable vertices  $e \in E$  and function vertices  $w \in W$ . Similar to the “flooding” schedule in Section III-A, each iteration begins with every function vertex  $w$  passing messages to all its adjacent variable vertices  $\mathcal{N}(w)$ ; then every variable vertex  $e$  passes messages to its adjacent vertices  $\mathcal{N}(e)$ ; the message sent from any vertex  $u$  to any of its neighbor vertex  $v$  is calculated using only the incoming messages from  $\mathcal{N}(u) \setminus \{v\}$ . The message passed from a function vertex  $w$  to a variable vertex  $e$  is given by

$$\mu_{w \rightarrow e} = \begin{cases} 1, & \text{if } x_w^{(i)} = 1 \\ \frac{1 - \prod_{e' \in \mathcal{N}(w) \setminus \{e\}} \mu_{e' \rightarrow w}}{2 - \prod_{e' \in \mathcal{N}(w) \setminus \{e\}} \mu_{e' \rightarrow w}}, & \text{if } x_w^{(i)} = 0 \end{cases} \quad (6)$$

and the message passed from a variable vertex  $e$  to a function vertex  $w$  is given by

$$\mu_{e \rightarrow w} = \frac{\prod_{w' \in \mathcal{N}(e) \setminus \{w\}} \mu_{w' \rightarrow e}}{\prod_{w' \in \mathcal{N}(e) \setminus \{w\}} \mu_{w' \rightarrow e} + \prod_{w' \in \mathcal{N}(e) \setminus \{w\}} (1 - \mu_{w' \rightarrow e})}. \quad (7)$$

At the end of each iteration, a summary message  $\mu_e$  is computed for each variable vertex  $e$  using all incoming messages to  $e$ , as

$$\mu_e = \frac{\prod_{x_{w'} \in \mathcal{N}(e)} \mu_{w' \rightarrow e}}{\prod_{x_{w'} \in \mathcal{N}(e)} \mu_{w' \rightarrow e} + \prod_{x_{w'} \in \mathcal{N}(e)} (1 - \mu_{w' \rightarrow e})}. \quad (8)$$

The iterative process is terminated when the summary messages converge to a steady state or when a preset maximum number of iterations is reached.

We will denote the computed value  $\mu_e$  in the  $i$ th layer of the graph by  $\mu_e^{(i)}$ . One may verify that when each layer of the factor graph is cycle-free, then the algorithm converges definitely with

TABLE I  
AVERAGE NUMBER OF LINKS THAT A PATH CONTAINS AND THE AVERAGED NUMBER OF PATHS THAT A LINK JOINS

network size	links/path	paths/link
5000	2.97	2.72
10000	3.71	3.46
20000	7.41	7.10
50000	6.33	6.16

a finite number of iterations, and the computed  $\mu_e^{(i)}$  is precisely the posterior of  $x_e$  given the observation  $X_W^{(i)}$ .

*Step 2—Interlayer Message Passing:* In this step, we apply the sum-product algorithm by passing messages from every layer of the graph to vertices  $\{\alpha_e : e \in E\}$  [through the vertices representing function  $B(\cdot)$ ], to compute the posterior  $P[\alpha_e | X_W^{(1,n)}]$ . Due to the simple cycle-free graph structure at this level [that shown in Fig. 4(a)], this step of the sum-product algorithm can be in fact formulated in the following closed form:

$$P[\alpha_e | X_W^{(1,n)}] \propto \prod_{i=1 \dots n} (\mu_e^{(i)} \alpha_e + (1 - \mu_e^{(i)})(1 - \alpha_e)). \quad (9)$$

We then choose  $\hat{\alpha}_e$  that maximizes (9) as the estimate of  $\alpha_e$ . This can be done by evaluating the function for every element of  $S = \{1/L, 2/L, \dots, 1\}$  numerically and finding the maximizing value  $\hat{\alpha}_e \in S$ .

## IV. SIMULATION AND DISCUSSION

### A. Simulation Setup

To investigate the performance and the scalability of the proposed algorithm, we generate sensor networks with random tree topologies, consisting of 5000, 10 000, 20 000, and 50 000 nodes. Table I lists the averaged number of links that a path contains in a network and the averaged number of paths that a link joins.

To each edge (link)  $e$  in the network, a random loss rate  $(1 - \alpha_e)$  is assigned, where  $\alpha_e$  is drawn from distribution with probability density function

$$f(\alpha) = \lambda \alpha^{(\lambda-1)}, \alpha \in (0, 1].$$

Easy to generate and tune, this random variable has the expected value  $\lambda/(1 + \lambda)$ .

At time instant  $t_i$ , the state  $x_e^{(i)}$  of edge  $e$  is generated from the Bernoulli distribution parametrized by  $\alpha_e$ , and the state  $x_w^{(i)}$  of each path  $w$  is observed by the center node via the  $i$ th batch of packets. In each simulation, the number  $n$  of packet batches is chosen as 50, 100, 200, 500, 1000, and 2000, and the average value  $\bar{\alpha}$  of  $\alpha_e$  over all the links of each network is chosen as 0.9, 0.92, 0.94, 0.96, and 0.98.

For Step 1 of the algorithm, we conservatively set the maximum number of iterations to 30 (we observe in simulation that in the majority of cases, the algorithm converges within 10–15 iterations). In the second step of the algorithm, we discretize  $\alpha_e$  to  $L = 100$  levels. Estimation error is computed for each link

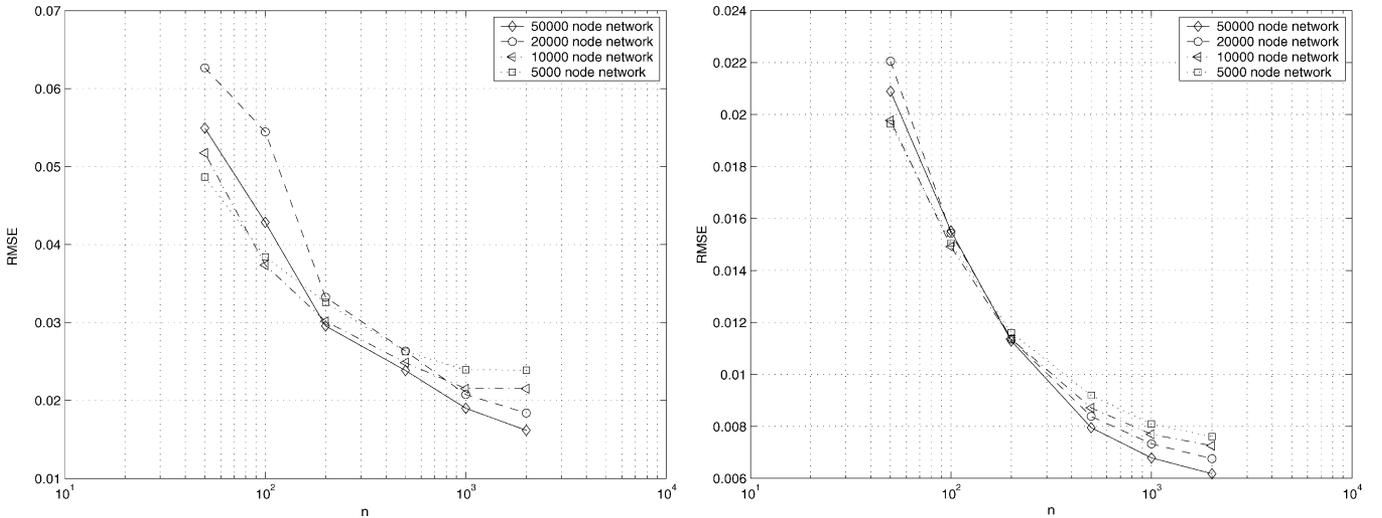


Fig. 5. RMSE for  $\bar{\alpha} = 0.9$  (left) and for  $\bar{\alpha} = 0.98$  (right), as a function of the number  $n$  of batches.

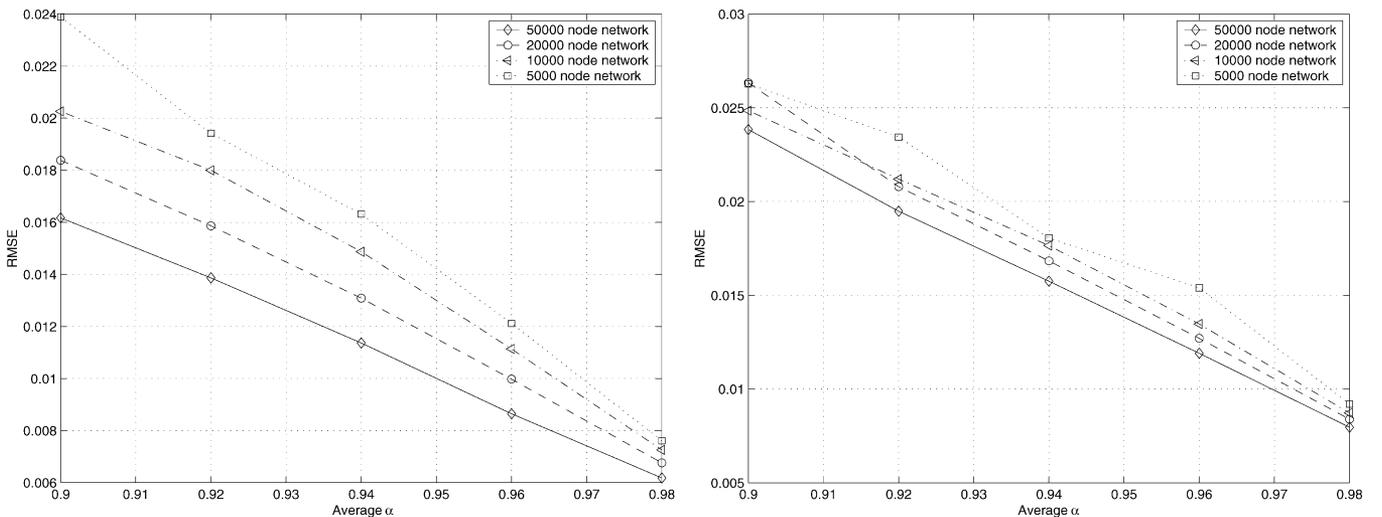


Fig. 6. RMSE with  $n = 2000$  (left) and with  $n = 500$  (right), as a function of  $\bar{\alpha}$ .

as  $\alpha_e - \hat{\alpha}_e$ . For each simulated network, the root mean square error (RMSE) is computed as

$$\text{RMSE} = \left( \sum_{e \in E} \frac{|\alpha_e - \hat{\alpha}_e|^2}{|E|} \right)^{1/2}.$$

### B. Simulation Results

Figs. 5 and 6 show the RMSE as a function of the number  $n$  of batches, and as a function of the  $\bar{\alpha}$ , respectively, for each simulated network. These results exhibit the general performance trend of this algorithm, which is also observed in other parameter settings of our simulations.

First, the estimation error decreases as the number of batches increases. However, it is worth noting that error will have a lower bound of  $1/2L$ , the quantization error of  $\alpha_e$ . If we can infinitely quantize  $\alpha_e$ , as the number of packet batches  $n$  approaches infinity, we expect  $\hat{\alpha}_e$  approaches the actual  $\alpha_e$ .

Second, the algorithm favors large networks. That is, in general, the larger the network, the better the algorithm performs.

This is because, as the network grows, the factor graph corresponding to the network becomes more sparsely connected; to a certain extent, Table I indicates the sparsity of the factor graphs. This result is consistent with the observation of the sum-product algorithm in other applications where the factor graph contains cycles (see, for example, [11]), since the independence assumption [2] of the incoming messages in that case is a closer approximation.

Third, without the need of increasing the number of batches, the same or better estimation accuracy can be achieved for large networks. For practical purposes, our simulations suggest that 500–2000 packet batches are sufficient for inferring the link loss rates in large networks.

Furthermore, the estimation error decreases as  $\bar{\alpha}$  increases. This implies that more batches are required for the same estimation error, as the average loss rate increases. In another experiment, we observe that in the 50 000-node network, based on the observations of 2000 batches of packets, the RMSE increases from 0.03 to 0.11, as  $\bar{\alpha}$  decreases from 0.8 to 0.7. That is, significantly more batches are needed for an accurate inference of

the latter loss rates. This is consistent with the discussion in [3] and [15]. Fortunately, in reality, action should have been taken before the network experiences such heavy link losses, and there is unlikely a need for inferring the link losses for these cases.

In summary, the simulation results suggest that the loss-rate estimates obtained from the proposed algorithm are sufficiently accurate for any practical purpose.

### C. Implementation and Complexity

The complexity of this factor-graph based algorithm is linear in the number  $n$  of batches and in the number  $|E|$  of links. This is in the same order as the existing algorithms (e.g., [3] and others). However, as we will explain next, when the algorithm is implemented as a constantly running monitoring daemon, its complexity for inferring loss rates *at any time instant* is in fact independent of the number of packet batches  $n$ . This makes the proposed algorithm much more suited for long-term monitoring purpose, as compared with other algorithms. We describe this implementation next.

As one may notice, a salient feature of this algorithm is its independent processing of the path states obtained by observing each batch of packets. That is, in practice, we may implement the link-loss monitoring daemon based on the path states obtained by observing a moving window of the most recent  $n$  batches of packets, and little computation is in fact needed to infer the current link loss rates. In detail, we may fix a choice of  $n$ , then as the monitoring daemon is initiated, upon observing the arrival or loss of the packets in the  $i^{\text{th}}$  batch, we start computing  $\mu_e^{(i)}$  (for all  $e \in E$ ), for  $i = 1, 2, \dots, n$ , sequentially. This only involves Step 1 of the algorithm. After obtaining the arrival or loss of the packets in the  $n^{\text{th}}$  batch, we finish computing  $\mu_E^{(n)}$ . Then, we move to Step 2 of the algorithm and for each  $e \in E$ , compute the posterior

$$P[\alpha_e | X_W^{(1,n)}] \propto \prod_{i=1 \dots n} (\mu_e^{(i)} \alpha_e + (1 - \mu_e^{(i)})(1 - \alpha_e)) \quad (10)$$

and find the maximizing  $\hat{\alpha}_e$ . After this initial computation, at any later time instant  $t_k$ ,  $k > n$ , the posterior of the link-loss rate for any link  $e$  can be simply updated recursively by

$$P[\alpha_e | X_W^{(k-n+1,k)}] = P[\alpha_e | X_W^{(k-n,k-1)}] \times \frac{\mu_e^{(k)} \alpha_e + (1 - \mu_e^{(k)})(1 - \alpha_e)}{\mu_e^{(k-n)} \alpha_e + (1 - \mu_e^{(k-n)})(1 - \alpha_e)}.$$

That is, at any time instant  $t_k$ , we only need to compute  $\mu_e^{(k)}$  according to Step 1, and reuse the previously computed posterior of  $\alpha_e$ . This significantly decreases the required computation. In this implementation, the computational complexity for inferring link loss rates at any time instant becomes independent of  $n$ . Comparing with other existing algorithms as mentioned earlier (the complexity of which all increases with  $n$ ), this translates to an appealing computational saving of hundreds or thousands of folds, if the algorithm is to be implemented as a monitoring daemon.

It is worth noting that with this “moving window” implementation, one should expect a tradeoff between the estimation accuracy and the estimation sensitivity to the change of link loss rates. That is, for a small window size  $n$ , the estimate is less accurate, but can better track the change of  $\alpha_e$  with time; whereas for a large window size  $n$ , the estimate is more accurate (provided the link loss rates stay static), but is less sensitive to the change of link loss rates.

### D. Extension to Other Models

Until this point, we have assumed that packets are constantly sent from the terminals to the center in a synchronous manner. In fact, the proposed approach can be adapted to other network scenarios where packets are sent by the terminals asynchronously, or when the terminals do not send packets continuously. However, in those cases, there will be a need of a reliable signaling mechanism from the terminals to the center, or a delicate protocol that allows the center to be aware of a packet sent from a terminal. For example, an out-of-band signaling mechanism may be employed, whereby each terminal node informs the center node directly or via multiple hops when it transmits a packet to the center. Then, the center node will expect the arrival of the packets sent from the terminals; upon receiving (or not receiving) the packets, the center node can perform link loss inference using the method presented in this paper. Alternatively, similar mechanisms may be made possible via higher level protocols. For example, with transmission control protocol (TCP), when a short series of out-of-order packets are received, congestion is assumed to have occurred and source transmission rates are reduced using window-based flow control. Built on TCP, by checking the order of the received packets, a node (intermediate node or center node) can decide whether an expected packet has arrived—it is safe to assume that with high probability an expected packet is lost, if a number of out-of-order packets are received. With this strategy, the details of such a protocol will depend on the data-fusion mechanism employed in the sensor network.

The independent identically distributed (i.i.d.) assumption of link losses in our model is perhaps in reality over simplified. In those cases, direct application of the algorithm presented in this paper may lead to less accurate estimates. Nevertheless, we note that the framework of factor graphs is a universal language for probabilistic modeling, and can be applied to models with arbitrary dependency structure. By using the simple i.i.d. assumption, this paper establishes a “proof of concept” for applying factor graph-based approaches to link loss inference in sensor networks. To demonstrate how to extend the presented method to more realistic (non-i.i.d.) loss models, we give a small contrived example: in the toy example shown in Fig. 2, we will allow loss rates of link  $a$  and link  $b$  to be dependent. Then, this dependency can be modeled by introducing a function  $\phi_{ab}(\alpha_a, \alpha_b)$  to the joint distribution  $P(\alpha_E, X_E^{(1,n)}, X_W^{(1,n)})$ , i.e.,

$$P(\alpha_E, X_E^{(1,n)}, X_W^{(1,n)}) \propto \phi_{ab}(\alpha_a, \alpha_b) \prod_{i=1}^n B_E(X_E^{(i)}, \alpha_E) P_{W|E}(X_W^{(i)}, X_E^{(i)})$$

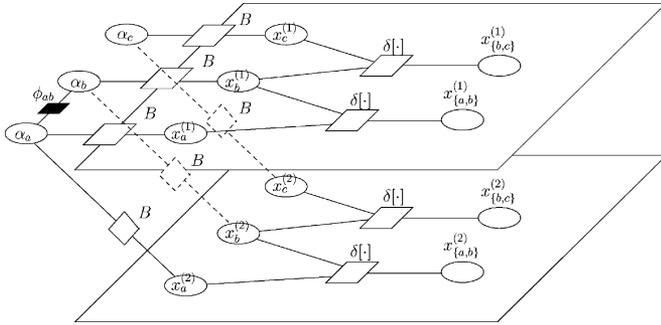


Fig. 7. Factor graph for the example in Fig. 2, where  $\alpha_a$  and  $\alpha_b$  are modeled as dependent. The function node  $\phi_{ab}$  is to model this dependency.

where function  $\phi_{ab}$  accounts for the dependency between  $\alpha_a$  and  $\alpha_b$ . By properly choosing function  $\phi_{ab}$ , one can obtain any desired dependence model between  $\alpha_a$  and  $\alpha_b$ . The factor graph representing the above factorization is shown in Fig. 7. For real networks, it is possible that the loss rates at a number of links have dependency. This would correspond to function node(s) in the factor graph connecting the variable nodes representing the loss rates at these links.

The sum-product algorithm can be derived similarly on such a graph for the inference of  $\alpha_E$ . Specifically, one may notice as in Fig. 7 that such a factor graph still contains “intralayer” connections and “interlayer” connections. However, it is possible in this case that the structure of “interlayer” connections contains cycles (Fig. 7 is *not* such an example; if we introduced two extra function nodes  $\phi_{bc}$  connecting  $\alpha_b$  and  $\alpha_c$  and  $\phi_{ac}$  connecting  $\alpha_a$  and  $\alpha_c$ , the resulting graph would be such an example). On such a graph, one may still carry out the two-step passing of messages for the sum-product algorithm. The “intralayer” message passing may remain the same as we presented earlier, whereas the “interlayer” message-passing may potentially need modification. Alternatively, one may consider different message-passing schedules. We expect the tradeoff between estimation accuracy and computational complexity to depend on the structure of the graph and the choice of message-passing schedules. In practice, more careful investigation of this tradeoff is likely to be necessary.

Finally we remark that the sum-product algorithm is not the only algorithm for factor-graph based inference. Other algorithms, such as the max-product algorithm, the EM algorithm and various variational methods<sup>5</sup> have also been developed in the framework of factor graphs (see, e.g., [2] and [29]). The performance and complexity of these algorithms for solving the problem of this paper certainly deserve further investigation.

## V. CONCLUSION

In this paper, we exploit the data-aggregation characteristic of wireless sensor networks in the implementation of a link loss monitoring daemon, where the network-wide link loss rates are

<sup>5</sup>In fact, it has been shown that in a variational formulation, the sum-product algorithm on factor graphs with cycles may be understood as iterative maximization of Bethe free energy [30]. Such a nature appears similar to the method of [31], where the true likelihood function is approximated by a “pseudolikelihood function” for computational tractability.

inferred upon observing whether packets sent from terminals have arrived. We present a factor-graph based algorithm for this purpose. We show that with very low complexity, this algorithm gives a good estimation of link loss rates, and the algorithm scales particularly well for large networks. We are one of the first to explore the design space toward efficient network inference algorithms in large-scale sensor networks with respect to link loss rates, and the first to use the factor-graph model and the sum-product algorithm to derive suboptimal but computationally lightweight inference mechanisms. We also take the communication paradigm of data aggregation into consideration in our design, which leads to the development of a network inference algorithm with virtually no costs of active probes.

On a side note, we would like to remark that the recent notion of factor graphs and other related graphical models are flexible modeling frameworks. Equipped with a set of well-developed efficient algorithms, such methodologies may be found useful in many networking scenarios, beyond this particular application in sensor networks.

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