Rateless Coding for Wireless Relay Channels

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***Abstract*— We propose a novel coding framework for wireless relay channels based on rateless codes, which allows for a natural extension to multiple antenna and multiple relay settings. The relaying protocol is half-duplex, and relays independently choose when to collaborate, if at all. With a simulated fountain code based implementation of this framework, we show that the use of rateless codes is both robust and efficient when the channel state information is not available at the transmitters.**

***Index Terms*— Channel coding, cooperative diversity, rateless coding, relay networks.**

I. Introduction

RELAY-BASED strategies provide a new dimension to the design space of wireless networks in which coverage and throughput may be significantly enhanced. Among recent literature approaching the problem from different directions (see, e.g., [1]–[6]), the setting and results of Mitran *et al.* [6] form the main motivation for this research.

In [6], a two-phase communication scheme is proposed for wireless devices in a network that cannot transmit and receive simultaneously. The two phases are the *listening* phase, in which the source node broadcasts and other nodes listen, and the *collaboration* phase, in which multiple nodes cooperate to transmit to the destination. It is assumed that the channel state information is not available at the transmitters but is available at the receivers. The information-theoretic results of [6] suggest that such a collaborative communication scheme can lead to significant diversity enhancement.

Approaching the setup of [6] from a coding-theoretic perspective, we are interested in designing a practical coding framework that effectively implements such a collaboration strategy. However, it appears that no fixed-rate code (traditional code) is capable of driving the outage probability to zero without channel state information at the source. Also, unless operating at a very low efficiency (i.e. low rate), no fixed-rate code is robust to the severe variation of channel statistics typical in a wireless scenario. Furthermore, fixed-rate codes require that the relay devices be committed to assisting the communication in order to be efficient.

Comparing with fixed-rate codes, a rateless code, pioneered by fountain codes in [7] and [8], can operate "universally" over classes of channels, adapt its rate to the channel realization, and requires no knowledge of channel state information or even channel statistics at the transmitter. This leads us to propose a rateless coding framework for space-time collaboration over relay channels in the setting of [6].

The proposed framework in this paper naturally accounts for communication efficiency and system robustness simultaneously. We present a concrete implementation of this framework based on fountain codes to demonstrate these advantages. We also modestly extend the theoretical results of [6] and present an achievable rate result (for fixed rate codes), which serves as a performance criterion to compare with our rateless code implementation.

Other coding strategies, such as those in [9] and [10], have also been proposed for collaborative communication over relay channels. Additionally, Hybrid-ARQ approaches, which are conceptually similar to rateless coding, have been investigated [11], [12]. However, to the best of our knowledge, this work is the first coding framework that implements collaborative communication for the setting of [6], and the first to simultaneously take both robustness and efficiency into consideration.

II. From Fixed-Rate Codes to Rateless Codes

Consider the system shown in Fig. 1 with three wireless devices. The source s wishes to communicate with the destination d, possibly with the help of the third device, relay r. We consider the scenario where the relay does not receive from the source and transmit to the destination at the same time. We assume that each transmitted symbol from a source antenna or from a relay antenna has the same average energy E_s. Let $X[i]$ and $U[i]$ be respectively the symbol vectors at time i transmitted from the source antennas and the relay antennas. Following [6], we consider the quasi-static Rayleigh fading model and use H_r, H_s and H_c to denote the channel gain matrices for the source-to-relay channel, source-to-destination channel, and the compound channel from the combined antennas of source and relay to the antennas of the destination. Let $Y[i]$ and $Z[i]$ be the received signals at time instant i at the relay and destination respectively. During each codeword transmission,

$$\begin{aligned} Y[i] &= H_r X[i] + N_Y[i], \\ Z[i] &= H_c [X[i]^T \; U[i]^T]^T + N_Z[i], \end{aligned}$$

where N_Y and N_Z are zero-mean (vector-valued) white complex Gaussian noise processes received at the relay and the destination respectively. We note that H_s is a submatrix of H_c and if the relay does not assist the source at time instant i, the received signal $Z[i]$ reduces to

$$Z[i] = H_s X[i] + N_Z[i].$$

We assume that the entries of H_r and H_c are drawn i.i.d. respectively from complex Gaussian distributions with variances G and 1. This restricts the source-to-destination channel and

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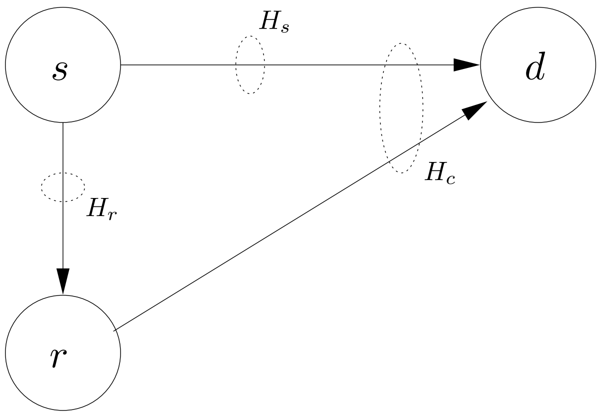

Fig. 1. Three node wireless relay network.

relay-to-destination channel to have the same fading statistics. Such a restriction plays no essential role in the applicability of this work, and merely serves as a simplification assumption to make our results comparable with those of [6].

As in [6], H_r and H_c are assumed to be *known* at the respective receivers but *unknown* at each transmitter. The variance of N_Y and N_Z are both $N_0/2$, also known at the receivers. It is also assumed that the relay is capable of synchronizing with the source at the symbol level.

For such a setup, an information-theoretic analysis of block coding schemes is presented in [6], which may be summarized as follows. The source selects a rate R to transmit a block of k-bits of information using a block code of length $n := k/R$. The relay, aware of the channel state H_r, decides a number $f \in (0, 1]$ such that after listening for $n_1 := fk/R$ symbols, the relay is able to decode the information block and collaboratively transmits to the destination.

Under this scheme, Theorem 1 of [6] shows that given a channel realization (H_r, H_c), and given a choice of f, any rate R satisfying

$$R < fC(H_s, \gamma) + (1 - f)C(H_c, \gamma) \tag{1}$$

and

$$R < fC(H_r, \gamma) \tag{2}$$

simultaneously, or satisfying

$$R < C(H_s, \gamma) \tag{3}$$

is achievable, where $C(H, \gamma) := \log_2 \det(I + \gamma HH^T)$ is the MIMO capacity formula under equal power allocation across antennas, and γ in our setup equals E_s/N_0. Since H_r and H_c are both random variables, the authors of [6] argue that for any fixed R and any f, the outage probability may be defined as the probability that R is outside the interval prescribed by (1), (2) and (3), under the channel law for H_r and H_c. Furthermore, they show that the optimal choice $\hat{f}$ of f — in the sense of *minimizing the outage probability for a fixed rate* R — is

$$\hat{f} = \min\left\{1, \frac{R}{C(H_r, \gamma)}\right\}, \tag{4}$$

The minimal outage probability as a function of R can then be re-expressed as

$$P_{\text{out}} = P\left[R > \hat{f}C(H_s, \gamma) + (1 - \hat{f})C(H_c, \gamma)\right]. \tag{5}$$

The work of [6] presents great insights into the potential of diversity enhancement by utilizing relays. However, we argue that the fixed-rate strategy presented in [6] is challenged by practical issues. As long as the source selects a transmission rate R, there is no opportunity for the relay to improve the rate, even when the channel supports much higher rates; the only benefit that the relay offers is a decreased outage probability. It is also remarkable that no matter what rate is chosen, the outage probability is bounded above zero for quasi-static or slow fading channels. In addition, the optimal choice of rate (subject to an outage constraint) requires that the source be fully aware of the existence of the relay as well as full channel knowledge. Furthermore, such a choice is fragile to the variation of channel conditions and relay availability.

To handle efficiency and robustness simultaneously, we propose an alternative strategy based on rateless codes — a concept first brought to attention by fountain codes of [7] and [8] and recently generalized in [13].

A rateless code is parametrized by a single number k, the length of the information block. The codebook of a rateless code with parameter k consists of a set of 2^k codewords of infinite length, where the codeword symbols may in general take on values from any alphabet. When using a rateless coding scheme, the transmitter encodes each k-bit information block into a codeword and sequentially transmits the codeword symbols. The receiver tries to decode the information block as it receives output symbols from the channel. When it can successfully decode, it sends back an ACK to inform the transmitter so that transmitter can terminate transmission and transmit the next information block if it needs. As is indicated by its name, a rateless code does not have a fixed rate, but rather the rate is determined "on the fly" by the time at which the receiver decodes the message.

Under the same channel model as described above, we propose the following rateless coding strategy, noting it is straightforward to generalize rateless coding to multiple antenna systems. The source encodes a block of $k-$bit information using a rateless code with parameter k and broadcasts the codeword to both the destination and the relay. The relay attempts to decode the information until it succeeds. At this time, if the destination has not decoded the information, the relay then collaborates with the source by transmitting to the destination using another rateless code. Starting from time instant 1, the destination has also attempted to decode the source information, and whenever it can decode, it sends an ACK back to the source and the relay in order to terminate the current transmission. Here we note that the one-bit feedback for acknowledgement entails little implementational difficulty as long as the feedback channels exist. For example, a TDD system may provide many fast feedback slots for such a purpose.

In this proposed strategy, the source does not need to be aware that a relay exists. The rateless nature of the coding scheme allows the source to communicate with the destination at a rate adapted to the channel conditions, to the availability of the relay, and to the collaborating strategy of the relay. Furthermore, although we are dealing with fading channels, the outage probability can be made arbitrarily small. This is because the successful decoding almost surely occurs at a time corresponding to a rate supported by the channel.

A fundamental question then arises: what rates are achievable with the proposed scheme? We do not have a closed-form answer to this question. However, to serve as a guideline, we present an achievable rate result for block coded schemes, built on the work of [6].

Let n be the time needed for the destination to *declare* a message, where by using the term "declare" rather than "decode", we include the possibility that the declared message may be incorrect. Similarly, we denote by n_1 the time needed for the relay to *declare* a message. We define the *realized rate R of a transmission* by $R := k/n$ bits/channel use. Notice that both n_1 and n are random variables depending on channel realization (H_r, H_c) and noise realizations. Therefore the realized rate R is also a random variable.

Corollary 1 (of Mitran et al [6] Theorem 1): [1]
Let

$$\tilde{f} := \begin{cases} \frac{C(H_c,\gamma)}{C(H_c,\gamma)+C(H_r,\gamma)-C(H_s,\gamma)} & \text{if } C(H_r,\gamma)>C(H_s,\gamma) \\ 1 & \text{otherwise} \end{cases} \tag{6}$$

and let

$$\tilde{R} := \tilde{f}C(H_s,\gamma) + (1-\tilde{f})C(H_c,\gamma). \tag{7}$$

Then for any $\delta > 0$, there exists a block coding scheme at rate $\tilde{R}-\delta$ such that with increasing block length, the decoding error probability is driven arbitrarily close to 0.

This result is implied by Theorem 1 of [6], but the fact that it is not mentioned therein should come as no surprise. It is critical to notice that this result only states that there *exists* such a code, it never implies that such a code can actually be chosen by the source. Since the source must know the channel to choose the optimal rate parameter, this result has no significance for block codes. However, this achievable rate is meaningful for a rateless coding scheme, since a rateless code can adapt to the channel, providing the possibility for this rate to be achieved. We now give a proof of this result.

Proof: First consider the case when $C(H_r,\gamma) \leq C(H_s,\gamma)$. Then $\tilde{f} = 1$ and $\tilde{R} = C(H_s)$. By (3) or a standard MIMO capacity argument, rate $\tilde{R} - \delta$ is achievable with arbitrarily small decoding error for sufficiently large block-length. Now consider the case when $C(H_r,\gamma) > C(H_s,\gamma)$. Referring to Fig. 2, by noting that $C(H_c,\gamma) > C(H_s,\gamma)$, it can be verified that the supremum $\tilde{R}$ of R is defined by the intersection of boundary of constraint (1) and that of constraint (2). Therefore, we can solve for the intersecting point by solving the two linear equations. This gives $\tilde{f}$ and $\tilde{R}$ in the theorem. With the standard information theoretic argument, for any rate $\tilde{R} - \delta$ and the corresponding $\tilde{f}$, the decoding error probability at the destination can be made arbitrarily small for sufficiently large block length. □

[1] We note that Theorem 1 of [6] requires some extra mild conditions. Since this result is a consequence of that theorem, the same conditions apply. For the ease of reading and simplicity, we however choose not to include the conditions when stating this result. The reader is referred to [6] for details.

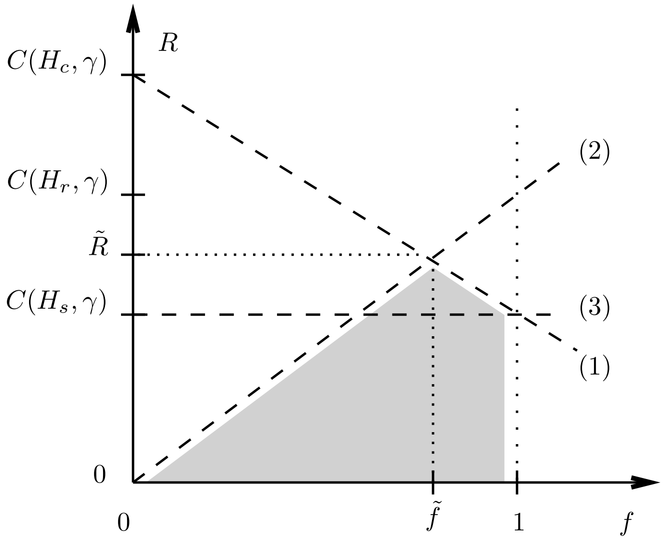

Fig. 2. Sketch of the relationship between (1), (2) and (3) for $C(H_r,\gamma) > C(H_s,\gamma)$.

Although this achievable rate is only shown to be valid for block coding schemes and for the case in which the channel is *known* at the transmitters, it is interesting to investigate whether there exists a rateless code that can also achieve this rate *without* channel knowledge at the transmitter. In fact, as will be shown in our fountain code implementation, this rate is actually closely approached in the low SNR regime.

Some insights may be obtained by distinguishing $\tilde{f}$ with $\hat{f}$. Given a rate R, $\hat{f}$ minimizes outage probability resulting from channel uncertainty, and such minima are strictly bounded above zero. However, knowing channel realizations at the source, $\tilde{f}$ maximizes the achievable rate dictated by (1), (2) and (3) and allows virtually zero outage probability.

III. Rateless Code Implementation

The proposed rateless coding scheme may be implemented with fountain codes. More specifically, we propose to use a Raptor Code as an outer code concatenated with a space-time inner code to form a rateless code. The source and relay may possibly use different such rateless codes.

This proposed architecture is tested over a simplified channel model, where the source, relay and destination each have only one antenna, and are time-synchronized at the symbol level. A Raptor code with information block length $k = 9,500$ is used as the inner code both at the source and the relay. The LDPC component code of the Raptor code has rate 0.95. We use the degree distributions as in [8] for the inner (LDPC) and outer (LT) components of the Raptor code. The performance of Raptor codes with these parameters has been reported in [14] for several lossy channel models including the AWGN channel. Consecutive output symbols from the Raptor code are then QPSK modulated. The choice of modulation scheme here is for implementation simplicity.

The rateless code used by the relay is the same Raptor code also with QPSK modulation. When in the collaboration phase, the relay aligns its output symbols temporally to correspond to the source's output symbols.

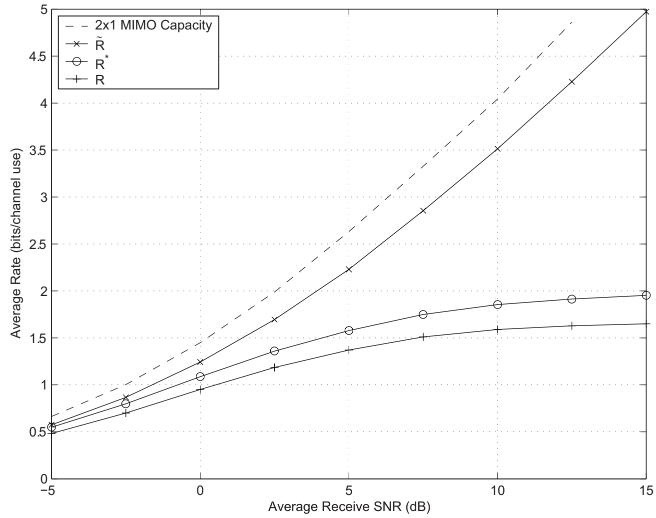

Fig. 3. Average system rates vs. average receive SNR, $G = 15$dB.

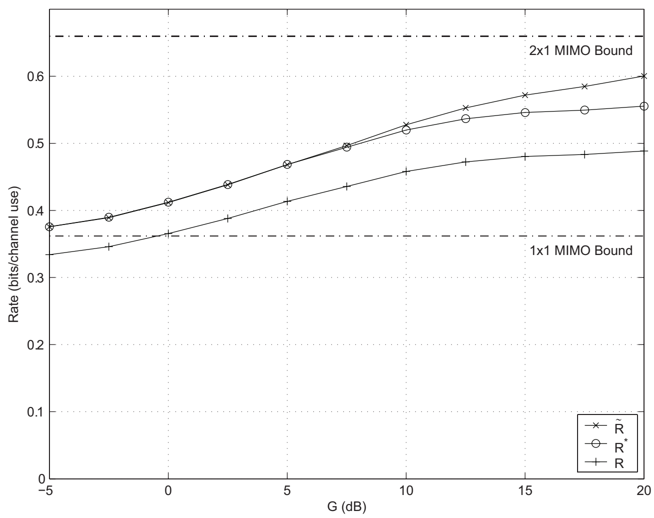

Fig. 4. Average system rates vs. G, $\gamma = -5$dB.

The output symbols of the source and relay are input to the distributed space-time inner code. The space-time inner code uses the Alamouti scheme [15] and works as follows. During both the listening and collaboration phases, the source simply passes through the input symbols. The relay, once in the collaboration phase, acts as the secondary antenna in which consecutive pairs of input symbols are transformed according to the Alamouti scheme.

This distributed space-time code is received and decoded at the destination. During the listening phase the destination receives symbols only from the source, and during the collaboration phase the destination performs standard Alamouti decoding.

For each transmitted codeword, belief propagation decoding is attempted periodically at intervals of 100 channel uses. For each attempt, the initial messages for belief propagation are calculated based on the received signal and the receiver's (perfect) knowledge of channel gains and noise variance. During each iteration of decoding, the decoder examines whether hard decisions on the messages form a codeword. When this is the case, a k-bit information vector is declared and the transmission of this codeword is terminated. If the hard decisions do not form a codeword within 100 iterations, the current decoding attempt is stopped and the decoder waits for the next decoding attempt. For the purposes of our simulation we assume that the receiver knows whether it decodes correctly. In practice, this may correspond to the case where CRC bits are embedded within the k-bit message, and we note that the entailed rate loss is negligible.

IV. Simulation Results

We perform a Monte-Carlo simulation of the proposed rateless code implementation over a range of SNR from -5dB to 15dB, and we vary G, the variance of H_r, from -5dB to 20dB. For the purpose of analyzing our results, we denote for each codeword transmission the declared decoding times at the relay and destination by n_1 and n respectively. The realized fraction of time that the relay was listening is defined as $f := \min\{1, n_1/n\}$. The maximum achievable realized rate for a given realized f is defined as

$$R^* := fC(H_s, \gamma) + (1 - f)C(H_c, \gamma), \tag{8}$$

which is prescribed by (1), (2) and (3). Then the gap between $\tilde{R}$ and R^* indicates the suboptimality of the *realized* f in comparison to the optimal $\tilde{f}$ for a given channel realization. To an extent, this gap also reflects the spectral efficiency limitation associated with the modulation scheme. The realized rate of a single simulated codeword transmission is denoted by $R := k/n$ and if decoding fails after 200,000 channel uses or an incorrect message is declared we regard $R = 0$. The gap between R^* and R indicates the coding loss due to the suboptimality of the Raptor code.

Following [6], and since the entries of H_c have unit variance, the average receive SNR at the destination is

$$\text{SNR} = (2 - E[f])\gamma, \tag{9}$$

and similarly, the average optimal SNR at the destination associated with the optimal choice $\tilde{f}$ is

$$\widetilde{\text{SNR}} = (2 - E[\tilde{f}])\gamma. \tag{10}$$

Fig. 3 presents the curves of R versus SNR, R^* versus SNR, and $\tilde{R}$ versus $\widetilde{\text{SNR}}$ averaged over the ensemble of channel realizations, as well as the standard 2×1 MIMO capacity bound as a function of receive SNR, for $G = 15$dB. Notice from the figure that R^* converges with $\tilde{R}$ in the low SNR regime, which indicates that the realized f is nearly optimal. As SNR increases, the R^* curve reaches the asymptote of 2 bits/channel use, governed by the QPSK modulation scheme. There is an approximately constant fraction of rate loss across all SNR when comparing R with R^*. This is better visualized in Fig. 4, where the same performance metrics are presented along the G dimension at $\gamma = -5$dB. Combining these two figures one can conclude that there is about 10%-15% rate loss due to code suboptimality. This is in fact consistent with the results of [14] for Gaussian channels. Clearly, at high SNR, both R^* and R can be increased by using higher-order

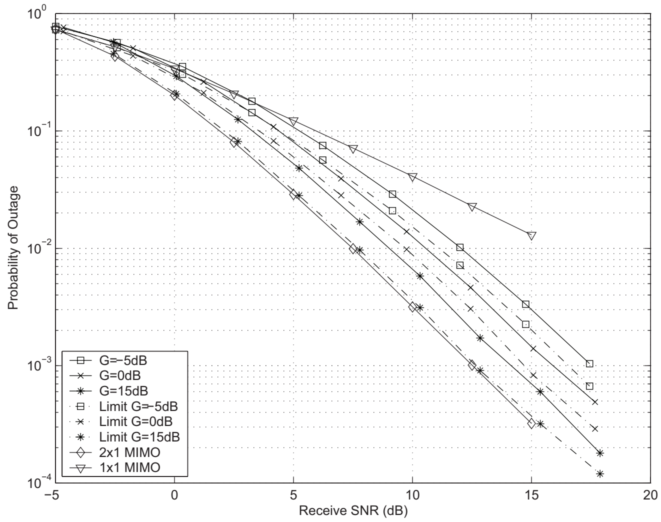

Fig. 5. Probability of outage vs. average receive SNR, $R = 0.5$.

modulations. The 2×1 and 1×1 MIMO capacity values are also given in Fig. 4, and bound $\tilde{R}$.

To make a comparison with the theoretical limits presented in [6] for fixed-rate codes (see also (5)), we present a plot of outage probability vs. SNR for the proposed implementation. Here the natural definition of outage probability for the rateless code at a given rate α is the relative frequency at which the decoder can *not* decode the message before time k/α. Given rate α, we are essentially treating the rateless code as a fixed-rate code of codeword length k/α. Fig. 5 contains outage probability curves for our rateless code implementation, the theoretical limits of [6], and the standard theoretical limits for a 2×1 and 1×1 MIMO system for $R = 0.5$ and some values of G. From these plots, it can be seen that our results are only about 1 dB away from the fixed-rate code outage limits of [6], achieve the same order of diversity based on the slopes of the curves, and improve upon a 1×1 MIMO system, particularly at high SNR.

V. Conclusions

We have presented a framework for collaboration over wireless relay channels using a two-phase collaboration scheme based on rateless codes that is simultaneously robust against outage and efficient in rate. In particular, we have shown that this system can be implemented using fountain codes and is capable of performing at rates approaching theoretical limits across a wide variety of channel configurations. We have also modestly extended the work of [6] which provides achievable rates to validate our performance.

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