Problem 2.61

1) No. The input $\Pi(t)$ has a spectrum with zeros at frequencies $f = k$, $(k \neq 0, k \in \mathcal{Z})$ and the information about the spectrum of the system at those frequencies will not be present at the output. The spectrum of the signal $\cos(2\pi t)$ consists of two impulses at $f = \pm 1$ but we do not know the response of the system at these frequencies.

2)

$$\begin{aligned}
h_1(t) \star \Pi(t) &= \Pi(t) \star \Pi(t) = \Lambda(t) \\
h_2(t) \star \Pi(t) &= (\Pi(t) + \cos(2\pi t)) \star \Pi(t) \\
&= \Lambda(t) + \frac{1}{2}\mathcal{F}^{-1}\left[\delta(f-1)\text{sinc}^2(f) + \delta(f+1)\text{sinc}^2(f)\right] \\
&= \Lambda(t) + \frac{1}{2}\mathcal{F}^{-1}\left[\delta(f-1)\text{sinc}^2(1) + \delta(f+1)\text{sinc}^2(-1)\right] \\
&= \Lambda(t)
\end{aligned}$$

Thus both signals are candidates for the impulse response of the system.

3) $\mathcal{F}[u_{-1}(t)] = \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$. Thus the system has a nonzero spectrum for every f and all the frequencies of the system will be excited by this input. $\mathcal{F}[e^{-at}u_{-1}(t)] = \frac{1}{a+j2\pi f}$. Again the spectrum is nonzero for all f and the response to this signal uniquely determines the system. In general the spectrum of the input must not vanish at any frequency. In this case the influence of the system will be present at the output for every frequency.

Problem 2.62

$$\begin{aligned}
\mathcal{F}[A\widehat{\sin(2\pi f_0 t + \theta)}] &= -j\text{sgn}(f)A\left[-\frac{1}{2j}\delta(f+f_0)e^{j2\pi f\frac{\theta}{2f_0}} + \frac{1}{2j}\delta(f-f_0)e^{-j2\pi f\frac{\theta}{2f_0}}\right] \\
&= \frac{A}{2}\left[\text{sgn}(-f_0)\delta(f+f_0)e^{j2\pi f\frac{\theta}{2f_0}} - \text{sgn}(-f_0)\delta(f-f_0)e^{-j2\pi f\frac{\theta}{2f_0}}\right] \\
&= -\frac{A}{2}\left[\delta(f+f_0)e^{j2\pi f\frac{\theta}{2f_0}} + \delta(f-f_0)e^{-j2\pi f\frac{\theta}{2f_0}}\right] \\
&= -A\mathcal{F}[\cos(2\pi f_0 t + \theta)]
\end{aligned}$$

Thus, $A\widehat{\sin(2\pi f_0 t + \theta)} = -A\cos(2\pi f_0 t + \theta)$

Problem 2.63

Taking the Fourier transform of $\widehat{e^{j2\pi f_0 t}}$ we obtain

$$\mathcal{F}[\widehat{e^{j2\pi f_0 t}}] = -j\text{sgn}(f)\delta(f - f_0) = -j\text{sgn}(f_0)\delta(f - f_0)$$

Thus,

$$\widehat{e^{j2\pi f_0 t}} = \mathcal{F}^{-1}[-j\text{sgn}(f_0)\delta(f - f_0)] = -j\text{sgn}(f_0)e^{j2\pi f_0 t}$$

To find the power content of the modulated signal we write $u^2(t)$ as

$$\begin{aligned} u^2(t) &= A^2\cos^2(2\pi(4\times 10^3+\frac{200}{\pi})t)+A^2\cos^2(2\pi(4\times 10^3-\frac{200}{\pi})t) \\ &\quad +4A^2\sin^2(2\pi(4\times 10^3+\frac{250}{\pi})t+\frac{\pi}{3})+4A^2\sin^2(2\pi(4\times 10^3-\frac{250}{\pi})t-\frac{\pi}{3}) \\ &\quad +\text{terms of cosine and sine functions in the first power} \end{aligned}$$

Hence,

$$P=\lim_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}}u^2(t)dt=\frac{A^2}{2}+\frac{A^2}{2}+\frac{4A^2}{2}+\frac{4A^2}{2}=5A^2$$

Problem 3.2

$$u(t)=m(t)c(t)=A(\text{sinc}(t)+\text{sinc}^2(t))\cos(2\pi f_c t)$$

Taking the Fourier transform of both sides, we obtain

$$\begin{aligned} U(f) &= \frac{A}{2}[\Pi(f)+\Lambda(f)]\star(\delta(f-f_c)+\delta(f+f_c)) \\ &= \frac{A}{2}[\Pi(f-f_c)+\Lambda(f-f_c)+\Pi(f+f_c)+\Lambda(f+f_c)] \end{aligned}$$

$\Pi(f-f_c)\neq 0$ for $|f-f_c|<\frac{1}{2}$, whereas $\Lambda(f-f_c)\neq 0$ for $|f-f_c|<1$. Hence, the bandwidth of the bandpass filter is 2.

Problem 3.3

The following figure shows the modulated signals for $A=1$ and $f_0=10$. As it is observed both signals have the same envelope but there is a phase reversal at $t=1$ for the second signal $Am_2(t)\cos(2\pi f_0 t)$ (right plot). This discontinuity is shown clearly in the next figure where we plotted $Am_2(t)\cos(2\pi f_0 t)$ with $f_0=3$.

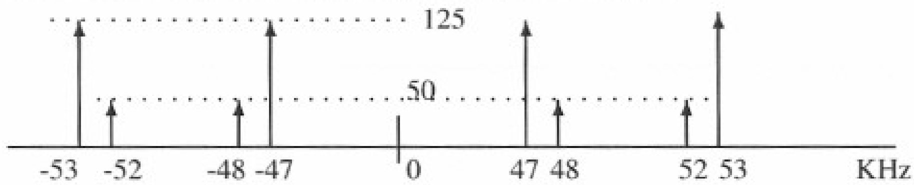
Problem 3.5

$$
\begin{aligned}
u(t) &= m(t) \cdot c(t) \\
&= 100(2\cos(2\pi 2000t) + 5\cos(2\pi 3000t))\cos(2\pi f_c t)
\end{aligned}
$$

Thus,

$$
\begin{aligned}
U(f) &= \frac{100}{2}\left[\delta(f-2000) + \delta(f+2000) + \frac{5}{2}(\delta(f-3000) + \delta(f+3000))\right] \\
&\quad \star [\delta(f-50000) + \delta(f+50000)] \\
&= 50\left[\delta(f-52000) + \delta(f-48000) + \frac{5}{2}\delta(f-53000) + \frac{5}{2}\delta(f-47000)\right. \\
&\quad \left. + \delta(f+52000) + \delta(f+48000) + \frac{5}{2}\delta(f+53000) + \frac{5}{2}\delta(f+47000)\right]
\end{aligned}
$$

A plot of the spectrum of the modulated signal is given in the next figure



Problem 3.6

The mixed signal $y(t)$ is given by

$$
\begin{aligned}
y(t) &= u(t) \cdot x_L(t) = Am(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \theta) \\
&= \frac{A}{2} m(t)\left[\cos(2\pi 2 f_c t + \theta) + \cos(\theta)\right]
\end{aligned}
$$

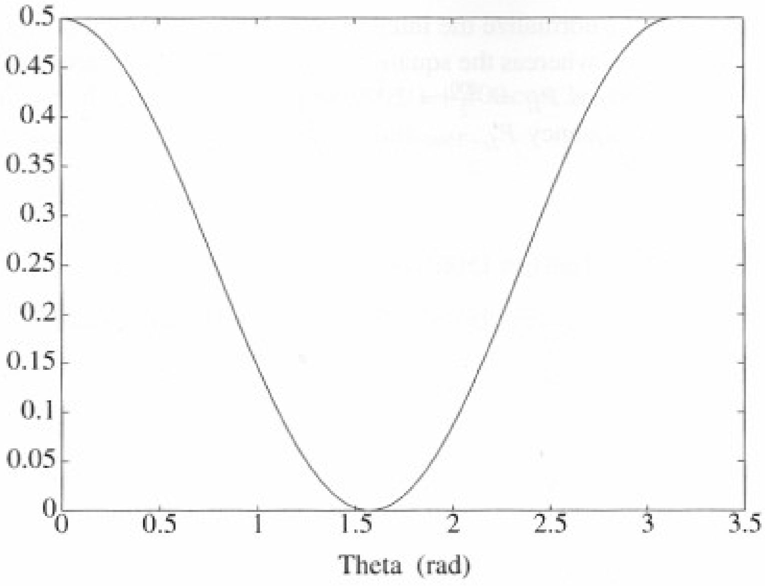
The lowpass filter will cut-off the frequencies above W, where W is the bandwidth of the message signal $m(t)$. Thus, the output of the lowpass filter is

$$
z(t) = \frac{A}{2} m(t)\cos(\theta)
$$

If the power of $m(t)$ is P_M, then the power of the output signal $z(t)$ is $P_{\text{out}} = P_M \frac{A^2}{4}\cos^2(\theta)$. The power of the modulated signal $u(t) = Am(t)\cos(2\pi f_c t)$ is $P_U = \frac{A^2}{2} P_M$. Hence,

$$
\frac{P_{\text{out}}}{P_U} = \frac{1}{2}\cos^2(\theta)
$$

A plot of $\frac{P_{\text{out}}}{P_U}$ for $0 \leq \theta \leq \pi$ is given in the next figure.

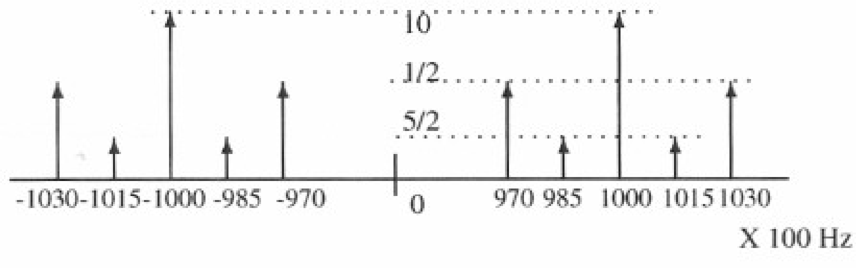


Problem 3.7

1) The spectrum of $u(t)$ is

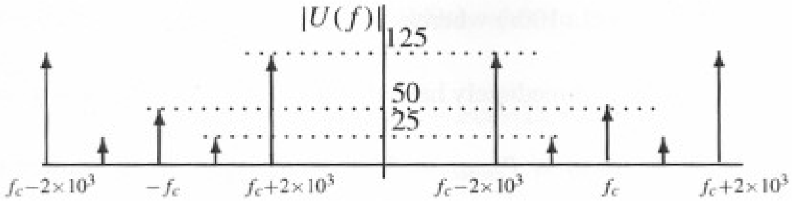
$$\begin{aligned} U(f) &= \frac{20}{2}[\delta(f-f_c)+\delta(f+f_c)] \\ &+\frac{2}{4}[\delta(f-f_c-1500)+\delta(f-f_c+1500) \\ &+\delta(f+f_c-1500)+\delta(f+f_c+1500)] \\ &+\frac{10}{4}[\delta(f-f_c-3000)+\delta(f-f_c+3000) \\ &+\delta(f+f_c-3000)+\delta(f+f_c+3000)] \end{aligned}$$

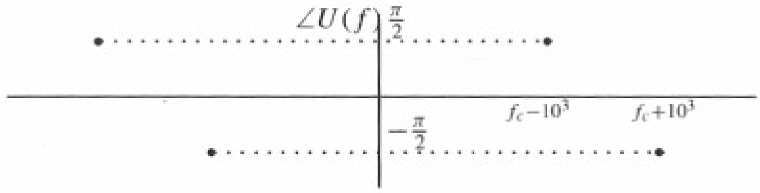
The next figure depicts the spectrum of $u(t)$.



2) The square of the modulated signal is

$$\begin{aligned} u^2(t) &= 400\cos^2(2\pi f_c t)+\cos^2(2\pi(f_c-1500)t)+\cos^2(2\pi(f_c+1500)t) \\ &+25\cos^2(2\pi(f_c-3000)t)+25\cos^2(2\pi(f_c+3000)t) \\ &+\text{ terms that are multiples of cosines} \end{aligned}$$





2) The average power in the carrier is

$$P_{\text{carrier}} = \frac{A_c^2}{2} = \frac{100^2}{2} = 5000$$

The power in the sidebands is

$$P_{\text{sidebands}} = \frac{50^2}{2} + \frac{50^2}{2} + \frac{250^2}{2} + \frac{250^2}{2} = 65000$$

3) The message signal can be written as

$$\begin{aligned} m(t) &= \sin(2\pi 10^3 t) + 5\cos(2\pi 2\times 10^3 t) \\ &= -10\sin(2\pi 10^3 t) + \sin(2\pi 10^3 t) + 5 \end{aligned}$$

As it is seen the minimum value of $m(t)$ is -6 and is achieved for $\sin(2\pi 10^3 t) = -1$ or $t = \frac{3}{4\times 10^3} + \frac{1}{10^3}k$, with $k \in Z$. Hence, the modulation index is $\alpha = 6$.

4) The power delivered to the load is

$$P_{\text{load}} = \frac{|u(t)|^2}{50} = \frac{100^2(1+m(t))^2\cos^2(2\pi f_c t)}{50}$$

The maximum absolute value of $1+m(t)$ is 6.025 and is achieved for $\sin(2\pi 10^3 t) = \frac{1}{20}$ or $t = \frac{\arcsin(\frac{1}{20})}{2\pi 10^3} + \frac{k}{10^3}$. Since $2\times 10^3 \ll f_c$ the peak power delivered to the load is approximately equal to

$$\max(P_{\text{load}}) = \frac{(100\times 6.025)^2}{50} = 72.6012$$

Problem 3.14

1)

$$\begin{aligned} u(t) &= 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t) \\ &= 20(1 + \frac{1}{2}\cos(200\pi t))\cos(2000\pi t) \end{aligned}$$

The modulating signal is $m(t) = \cos(2\pi 100t)$ whereas the carrier signal is $c(t) = 20\cos(2\pi 1000t)$.

2) Since $-1 \leq \cos(2\pi 100t) \leq 1$, we immediately have that the modulation index is $\alpha = \frac{1}{2}$.

3) The power of the carrier component is $P_{\text{carrier}} = \frac{400}{2} = 200$, whereas the power in the sidebands is $P_{\text{sidebands}} = \frac{400\alpha^2}{2} = 50$. Hence,

$$\frac{P_{\text{sidebands}}}{P_{\text{carrier}}} = \frac{50}{200} = \frac{1}{4}$$

Problem 3.15

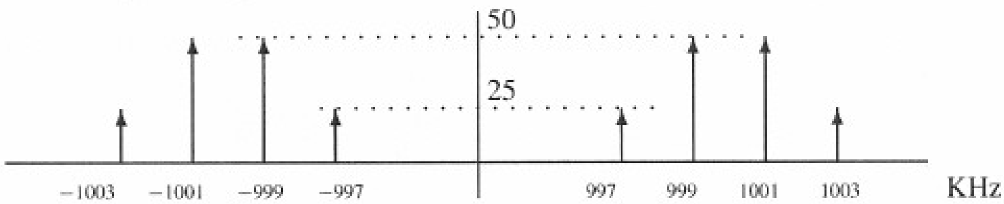
1) The modulated signal is written as

$$\begin{aligned} u(t) &= 100(2\cos(2\pi 10^3 t) + \cos(2\pi 3 \times 10^3 t))\cos(2\pi f_c t) \\ &= 200\cos(2\pi 10^3 t)\cos(2\pi f_c t) + 100\cos(2\pi 3 \times 10^3 t)\cos(2\pi f_c t) \\ &= 100\left[\cos(2\pi(f_c + 10^3)t) + \cos(2\pi(f_c - 10^3)t)\right] \\ &\quad +50\left[\cos(2\pi(f_c + 3 \times 10^3)t) + \cos(2\pi(f_c - 3 \times 10^3)t)\right] \end{aligned}$$

Taking the Fourier transform of the previous expression, we obtain

$$\begin{aligned} U(f) &= 50\left[\delta(f - (f_c + 10^3)) + \delta(f + f_c + 10^3)\right. \\ &+ \left.\delta(f - (f_c - 10^3)) + \delta(f + f_c - 10^3)\right] \\ &+ 25\left[\delta(f - (f_c + 3 \times 10^3)) + \delta(f + f_c + 3 \times 10^3)\right. \\ &+ \left.\delta(f - (f_c - 3 \times 10^3)) + \delta(f + f_c - 3 \times 10^3)\right] \end{aligned}$$

The spectrum of the signal is depicted in the next figure



2) The average power in the frequencies $f_c + 1000$ and $f_c - 1000$ is

$$P_{f_c+1000} = P_{f_c-1000} = \frac{100^2}{2} = 5000$$

The average power in the frequencies $f_c + 3000$ and $f_c - 3000$ is

$$P_{f_c+3000} = P_{f_c-3000} = \frac{50^2}{2} = 1250$$

Problem 3.16