Summer 2007 ELG3170 Final Exam

- Three hours
- Two-page double sided aid-sheet allowed
- Calculator allowed
- 52 points total

Question 1 (2 Points)

One of the most fundamental results in digital communications is the the channel coding theorem, stating that any given channel has a capacity, and that at any rate below the capacity, reliable communication is possible. Give the name of the person who discovered this result.

Question 2 (8 Points)

Suppose that signal $m(t) = sinc(8000t)$ is intended to be transmitted.

1. (5 points) Determine the Fourier transform $M(f)$ of $m(t)$.
2. (7 points) Suppose that the only available channel is the frequency band from 900KHz to 905KHz. If amplitude modulation is used, is it possible to transmit the signal $m(t)$ through the channel? If yes, describe a modulation scheme. If not, explain why.

Question 3 (10 Points)

Suppose that an error control coding scheme that encodes every 4-bit message vector $(m_1, m_2, \ldots, m_4)$ to a 8-bit codeword $(c_1, c_2, \ldots, c_8)$ as follows.

1. First encode $(m_1, m_2, \ldots, m_4)$ to $(c_1, c_2, \ldots, c_7)$ using a $(7, 4)$ Hamming code.
2. Then append bit c_8 to vector $(c_1, c_2, \ldots, c_7)$ such that the resulting vector $(c_1, c_2, \ldots, c_8)$ has even weight.

Answer the following questions.

1. (5 points) Is the code linear? Justify your answer.
2. (5 points) Find the minimum distance of the code.
3. (3 points) Comment on the number of errors the code can correct and can detect respectively.

Question 4 (8 Points)

The set of signals $\{s_1(t), s_2(t), s_3(t), s_4(t)\}$ is defined as follows.

$$
\begin{aligned}
s_1(t) &= \begin{cases} cos(2\pi t/T), & t \in [0, T] \\ 0, & \text{otherwise} \end{cases} \\
s_2(t) &= \begin{cases} cos(2\pi t/T + \pi/4), & t \in [0, T] \\ 0, & \text{otherwise} \end{cases} \\
s_3(t) &= \begin{cases} 2\sin(2\pi t/T), & t \in [0, T] \\ 0, & \text{otherwise} \end{cases} \\
s_4(t) &= \begin{cases} cos(2\pi t/T - \pi/4), & t \in [0, T] \\ 0, & \text{otherwise} \end{cases}
\end{aligned}
$$

Find an orthonormal basis of the set of signals, and determine their signal constellation under this choice of basis.

Question 5 (8 Points)

Suppose that discrete-time signal $X[k]$ is to be quantized using a 8-bit uniform quantizer, and it is known that at every k, $X[k]$ is uniformly distributed over interval $[0, 1000]$ and samples of $X[k]$ are independent. Determine the SQNR of this quantization scheme.

Question 6 (8 Points)

In a voice communication system, voice signals are sampled at 8KHz. Suppose that each sample is quantized by an 8-bit quantizer, the bit sequence is then coded using an error correction code that is a $(3000, 1000)$ binary linear block code, and the coded bit sequence is then modulated by a binary PAM scheme. Determine the symbol rate. What is the minimal required bandwidth if the transmission needs to be ISI free?

Question 7 (8 Points)

Verify whether the following statements are correct. Justify your answer.

1. DSB-TC AM is less power-efficient than DSB-SC AM.
2. If $s(t)$ is a Nyquist pulse, so is $s^2(t)$.
3. Every binary linear block code contains all-zero vector as a codeword.
4. For any binary block code, the syndrome of the received vector is independent of transmitted codeword.