Summer ELG3121 Final Exam

Total 52 + 4 (bonus) points; 3 hours; two-page aid sheet; calculators allowed

Your answer to each question should only contain what is necessary. Including unnecessary, irrelevant and wrong statements in your answers will be penalized in grading.

Question 1 (8 Points)

Verify whether the following statements are correct. Justify your answer.

1. The Q-function $Q(x)$ is an increasing function of x.
2. If random variables X and Y are jointly Gaussian and if their covariance is 0, then X and Y are independent.
3. If $VAR[X+Y] = VAR[X] + VAR[Y]$, then X and Y are uncorrelated.
4. If a null hypothesis that a set of data are drawn from a given distribution is rejected at significance level of 0.02, then the null hypothesis will also be rejected at significance level of 0.05.

Question 2 (8 Points)

Suppose that the probability that a senior woman has Cancer X is 0.1. Now there is new test for Cancer X. If a senior woman has Cancer X, the test will show "positive" with probability 0.9; and if the woman does not have Cancer X, the test will show "positive" with probability 0.3.

The test result for Jennifer, a senior woman, is positive. What is the probability that she has Cancer X?

Question 3 (8 Points)

Consider the random variable obtained in the following two-step experiment.

- Step 1: Toss a biased coin where TAIL occurs with probability 0.4.
- Step 2: If HEAD occurs in Step 1, then toss a fair die and record the number; if TAIL occurs in Step 1, then draw a random number uniformly distributed over interval $[0, 1]$, and record the number.

Determine the cdf of the resulting random variable.

Question 4 (8 Points)

Draw a random number X uniformly distributed over interval $[0, 10]$. Then draw a random number Y uniformly distributed over interval $[0, X]$. Determine the pdf of Y.

Question 5 (8 Points)

The number of phone calls calling John in any time window of 1 hour is a Poisson random variable with mean 1.4. Under this setting, let random variable X be the number of phone calls calling John tomorrow between 1pm and 3pm. Find the variance of X.

Question 6 (8 Points)

Considering the following game which charges M dollars for a player to play: The dealer will toss a loaded die where number "1" occurs with probability 1/2 and the other outcomes are equally likely; if the outcome of the die is number a and $a \neq 6$, the player will receive a^2 dollars; if the outcome of the die is 6, the player will receive 1 dollar. As the dealer, how should you set up the price M so that when you offer this game for a large number of times, with probability approaching 1, you will make profit? (You will need to give the necessary and sufficient condition on M under which you will make profit with probability close to 1 when offering the game for a sufficient number of times.)

Question 7 (8 Points)

There is a subroutine named `uniGen()` written in certain programming language. When the subroutine is called, it will return a random real number uniformly distributed over the interval $[0, 1]$. Now you need to write a computer program (using pseudo-code) by calling `uniGen()` so that your program returns a random number X distributed according to pdf $f_X(x)$ given as follows.

$$f_X(x) = g(x) + 0.75\delta(x - 10),$$

where

$$g(x) = \begin{cases} x + 1/2, & x \in [-1/2, 0) \\ -x + 1/2, & x \in [0, 1/2) \\ 0, & \text{otherwise} \end{cases}$$

Note the subroutine `uniGen()` is the only source of randomness which you are allowed to use.