**Lectures of July 7th, 2006**
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# 1 Some Important RV's

Uniform, Bernoulli, Binomial, Geometric, Poisson, Exponential, Gaussian

- *Poisson*: $p_x(k) = \frac{\alpha^k}{k!}e^{-\alpha}$
- *Exponential*: $f_X(x) = \lambda e^{-\lambda x}u(x)$

  $\lambda$ is arrival rate: number calls/time unit

  $T$ is time window.

  The number of calls arriving during a time window of duration $T$ is a poisson RV with $\alpha = \lambda T$
- *Gaussian*

  Q-function
- *Memoryless property, as a consequence of independent arrival*
- See poisson RV and exponential RV from poisson process perspective.

  $N(t)$ is poisson process

$$p[N(t) = k] = \frac{(\lambda t)^k}{k!}e^{-\lambda t}$$

**Example 1** ***Final 2005, Q8***

*$\lambda$=5*

*Suppose during [0,2], 5 emails have arrived*

1. *P[20 emails during [0,5]]*
   *=P[15 emails during [2,5]] (poisson PMF with $\alpha = 5 \times 3$, at k=15)*
   $=\frac{(3\times 5)^{15}}{15!}e^{(-3\times 5)}$
2. *Find probability that the 6th email arrives at time $\geq 5$*
   *Let $Y$ be the time we need to wait from time 2 until the 6th email arrive.*
   *$Y$ is an exponential RV and the probability of interest is:*

$$P[Y \geq 3] = \int_3^\infty 5e^{-5y}dy$$

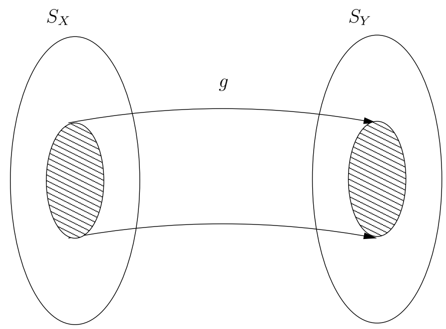


Figure 1: Diagram of fig 1.

## 2 Function of RV's

Given RV X and function g, find PDF of RV := g(x)

1. First principle approach, as shown in Figure 1.
2. If X is continuous and g is smooth, as shown in Figure 2

$$f_Y(y) = \sum_k \frac{f_X(x)}{|\frac{dy}{dx}|X = x_k}$$

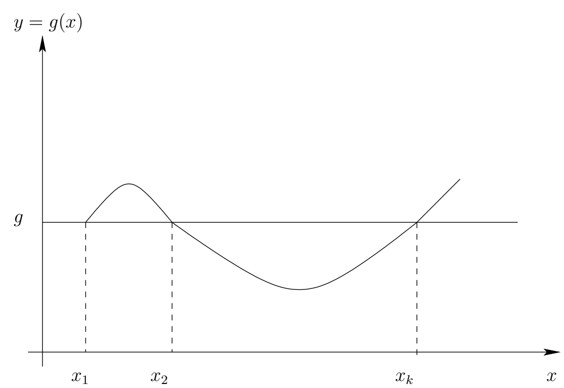


Figure 2: Diagram of fig 2.

**Example 2** *Mock Final 2006, Q1 part 3*
$g(x) = u(x-2) + 2u(x-4.5)$
*The figure is illustrated in Figure 3*
*Find PDF Y:=g(X) conditioned on* $X > 2$

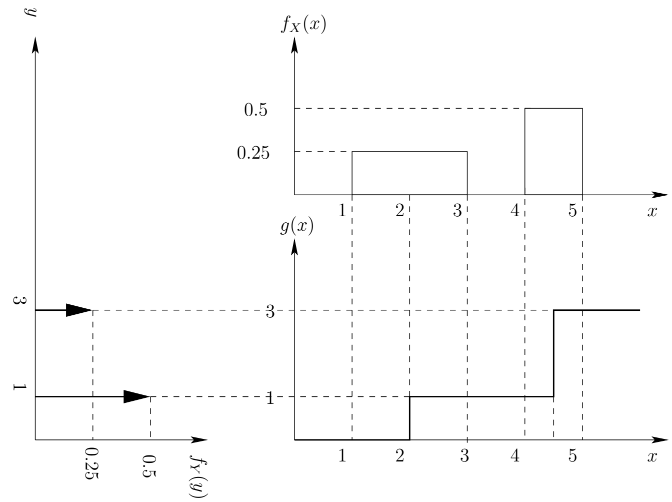


Figure 3: Diagram of fig 3.

*The condition $X > 2$ is equivalent to the condition $Y \geq 1$*

$$f_{Y|X>2}(y) = f_{Y|Y\geq 1}(y)$$

*Ths figure is shown in Figure 4*

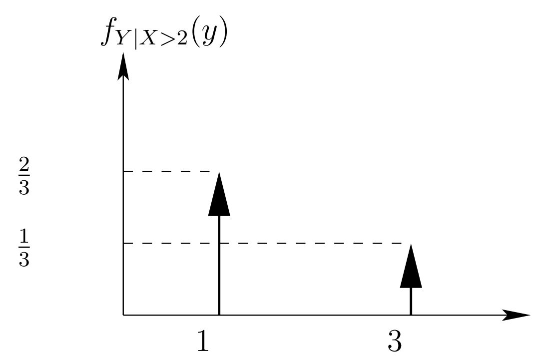


Figure 4: Diagram of fig 4.

**Example 3** ***Final 2005, Q2***

$X = g(Y)$

$$X := \begin{cases} 1, & Y < 0.5, \\ Y^2, & Y \geq 3.5. \end{cases}$$

*Find PDF of x, as shown in Figure 5*

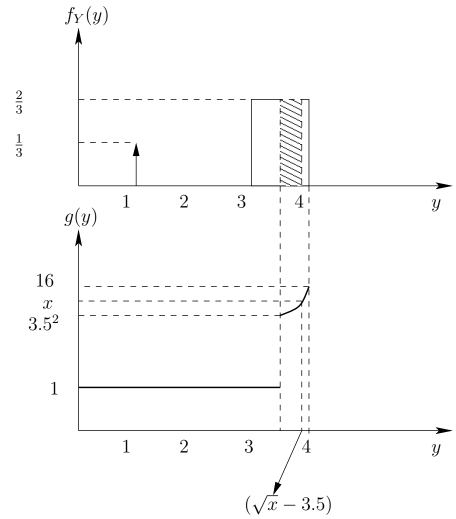


Figure 5: Diagram of fig 5.

***Solution***:

*CDF first*

*If* $x < 1$, $F_x(x) = 0$

*If* $1 \leq x < 3.5^2$, $F_X(x) = P[X \leq x] = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

*If* $3.5^2 < x \leq 1\ 6$, $F_X(x) = \frac{2}{3} + (\sqrt{x} - 3.5)\frac{2}{3}$

*If* $x > 16$ $F_X(x) = 1$, *as shown in Figure 6*

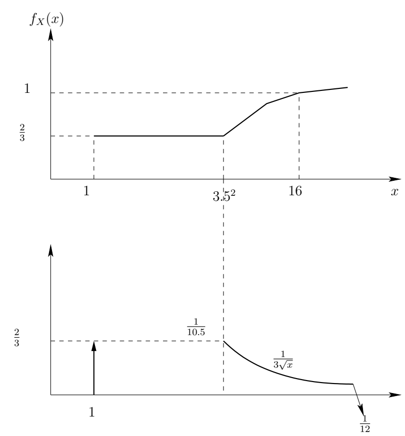


Figure 6: Diagram of fig 6.