Case 2: S is infinite and countable

We note that equal-likelyhood assignment is NOT valid.

Exampel 5: Keep tossing a coin until the first "H" is seen, and record the number of the tossing trivals. Find the sampling space S.

Solution:

The sampling sapce $S = \{1, 2, 3, ...\}$

If we assume the coin is unbiased, the following assignment is "reasonable:"

$P[\{1\}] = (\frac{1}{2})$

$P[\{2\}] = (\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$

$P[\{3\}] = (\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \frac{1}{8}$

$P[\{k\}] = (\frac{1}{2})^k$, $k \in S$

Verify the sum of all k: $\sum_{k=1}^{\infty} P[\{k\}] = (\frac{1}{2}) + (\frac{1}{2})^2 + ... = 1$

Note: $1 + a + a^2 + a^3 + ... + a^n = \frac{1-a^n+1}{1-a}$

If we assume the coins biased such that with probability $\frac{2}{3}$ a "H" occurs, then the following assignment is "reasonable":

$P[\{1\}] = (\frac{2}{3})$

$P[\{2\}] = (\frac{1}{3})(\frac{2}{3}) = \frac{2}{3^2}$

$P[\{3\}] = (\frac{1}{3})(\frac{1}{3})(\frac{2}{3}) = \frac{2}{3^3}$

$P[\{k\}] = (\frac{2}{3})^k$, $k \in S$

Verify the sum of all k: $\sum_{k=1}^{\infty} P[\{k\}] = (\frac{1}{2}) + (\frac{1}{2})^2 + ... = 1$

Case 3: S is infinite and Uncountable

[Aside: If S is finite or countably infinite, we sometime say S is discrete; if S is uncountably infinite, we say S is continuous]

Typical options: uniform assignment and non-uniform assignment

Example 6: Measured the temparature if this rooom using an infinite precission thermometer.

Suppose the measurement real number in interval [10,20]

Under the uniform assignment of probability for any $x \in [10, 20]$

We assined probability to event [10, x] as follows:

$p[10, x] = \frac{(x-10)}{(20-10)}$

Exercise:

Based on this assignment,calculate the probability of the interval [14,18]

$P[14, 18] = \frac{4}{10}$

Solution: $[10, 18] = [10, 14] \bigcup$[14,18]

$P[(10, 18)] = P[(10, 14)] + P[(14, 18)]$

$\Rightarrow \frac{(18-10)}{(20-10)} = \frac{(14-10)}{(20-10)} + p[(14, 18)]$

$\Rightarrow P[(14, 18)] = 0.8 - 0.4 = 0.4$

Ex: Suppose the present time is time 0

Record the time which nest phome call arrive

S = $[o, +\infty]$

We immediately say that uniform assignment is invalid

We will chose the following assignment

For every interval $[t, +\infty] t \in S$

$P[t, +\infty] = e^{-\alpha t}$, for some choice of positive value of α

Let $t = 100, \alpha = 1$

$P[100, +\infty] = e^{-100}$

$P[\frac{1}{1000}, +\infty] = e^{-\frac{1}{1000}} \approx 1$

Under this assignment probability that the next call arrive before time t is $1 - e^{-\alpha t}$.

Combinatories Basics

Preliminaties:

There are k sets, A1,........Ak

Pick on element from each set and form a vector (a1,a2....ak) there and $a_i \in A_i$

for i =1,2...k

How many distinct such vector are there in?

Example: Let $K = 2, A_1 = \{1, 2, 3\}.A_2 = \{a, b\}$

Clearly there are 2×3 in different pairs.

$Answer = A_{1\times}|A_2| \times .... \times |A_k|$.

Sampling with/without replacement and with/without ordering

(here, sampling means drawing, replacement means putting back)

Case1: Sampling with replacement and with ordering

setup: Chose one object from set A of m distinct object.

Do this K- times and record the K-tuples.

What is the size of the sample space? Howmany K-tuples are there?

eg: $n \times n............. = n^k$

Pick 5 cards at random from a deck of regular card witrh replacement. and suppose in every draw each card is picked with equal probabilitty.

From a 5-type, what is the probability that this 5-types starts with black A and end with a black A?

the number of 5-taples start with black A and end with black A.

$\clubsuit A_1 .... \clubsuit A_5$

$\spadesuit A_1 ..... \spadesuit A_5$

$2 \times 52 \times 52 \times 52 \times 2$

Therefore the probability of draw such a 5- taple is $= 2^2 \times \frac{52^3}{52^5} = (\frac{1}{26})^2$

Extension:

After we pick the first two cards, we stop , the probability of both cards are Black Aces is still the same:

$P = (\frac{2}{52})(\frac{2}{52})$

Case2: Sampling without replacement and with ordering.

Setup: Pick K objects from a set A of n object (without putting the drawn objects back) and record the K-type($K \leq n$)

How many distint K- type are there?

$52 \times 51 \times 50 \times 49 \times 48$

The general form is $n \times (n-1) \times$ ....$(n-k+1) = \frac{n!}{(n-k)!}$