ELG3121 Summer 2006 Midterm Solutions

1. (a) Correct, since

$$\begin{aligned} P[A^c \cap B^c] &= 1 - P[A] - P[B] - P[A \cap B] = 1 - P[A] - P[B] - P[A]P[B] \\ &= (1 - P[A])(1 - P[B]) = P[A^c]P[B^c]. \end{aligned}$$

(b) Incorrect. For example, $S := \{0, 1\}$, $A := \{0\}$, $B := \{1\}$ and $P[A] = P[B] = 1/2$. It is clear that

$$0 = P[A^c \cap B^c] \neq P[A^c]P[B^c] = 1/4$$

2. (a) When $T = 20$, X takes values from $\{19.9, 20, 20.1, 21\}$. Denote $E := \{20.1, 21\}$.

$$\begin{aligned} P[X = 20.1] &= P[defect]P[X = 20.1|defect] + P[no\ defect]P[X = 20.1|no\ defect] \\ &= 0.1 \times 0.3 + 0.9 \times 0.1 = 0.12 \\ P[X = 21] &= P[defect]P[X = 21|defect] = 0.1 \times 0.6 = 0.06 \end{aligned}$$

The PDF and CDF of X given information $X \in E$ are respectively

$$\begin{aligned} f_{X|X\in E}(x) &= \frac{0.12}{0.12 + 0.06}\delta(x - 20.1) + \frac{0.06}{0.12 + 0.06}\delta(x - 21) \\ &= \frac{2}{3}\delta(x - 20.1) + \frac{1}{3}\delta(x - 21). \\ F_{X|X\in E}(x) &:= \begin{cases} 0, & x \in (-\infty, 20.1) \\ 2/3, & x \in [20.1, 21) \\ 1, & x \in [21, \infty) \end{cases} \end{aligned}$$

(b)

$$P[defect|X = 20.1] = \frac{P[defect\ and\ X = 20.1]}{P[X = 20.1]} = \frac{0.1 \times 0.3}{0.12} = 0.25$$

3. (a) 1/52

(b) 1/26

(c) We consider only the space of sequences consisting of 5 cards that end with a red card.

$$\begin{aligned} P[3\ Ks\ and\ 2\ Qs\ ending\ with\ Q] &= (1/13)(2/52)^3(2/52) \times \binom{4}{1} \\ P[3\ Ks\ and\ 2\ Qs\ ending\ with\ K] &= (1/13)(2/52)^2(2/52)^2 \times \binom{4}{2} \end{aligned}$$

$$\begin{aligned} P[3\ Ks\ and\ 2Qs] &= P[3\ Ks\ and\ 2Qs\ ending\ with\ Q] + P[3\ Ks\ and\ 2Qs\ ending\ with\ K] \\ &= 10/(13 \times 26^4) \end{aligned}$$

(d) The answer is (a).

$$P[Y < X] = P[Y < 1] = P[Y = 0] = 1/2.$$

Since $1 = P[Y < X] + P[Y = X] + P[Y > X]$ and $P[Y = X]$ (or equivalently $P[Y = 1]$) is clearly non-zero, we have $P[Y > X] < 1/2$.