ELG3121 Summer 2006 Midterm Examination

Closed-book; calculators allowed; two hours; 25 points in total

May 30th, 2006

Warning: Inclusion of unnecessary, irrelevant and incorrect statements in your solution will be penalized in marking

Question 1 (7 points) *Suppose that S is the sample space of a random experiment. Verify whether each of the following statements is correct. If you think a statement is correct, prove it; otherwise give a counter-example to show it is incorrect. No marks will be given to answers without correct justification.*

- (3 points) For any two events $A \subseteq S$ and $B \subseteq S$, if A and B are independent, then A^c and B^c are independent, where the notation $(\cdot)^c$ refers to set complement with respect to S.
- (4 points) For any two *non-empty* events $A \subseteq S$ and $B \subseteq S$, if $A \cap B = \emptyset$, then A^c and B^c are independent.

Question 2 (8 points) *Due to some problem in the manufacturing process of an electrical thermometer, there is 10% probability that a thermometer has defect. Let T denote the true temperature. It is known that if a thermometer has no defect, it outputs a temperature reading in set $\{T - 0.1, T, T + 0.1\}$, where the probability of outputting $T - 0.1, T$, and $T + 0.1$ are respectively $0.1, 0.8$, and 0.1. It is also known that when a thermometer has defect, it outputs a temperature reading in set $\{T, T + 0.1, T + 1\}$, where the probabilities of outputting $T, T + 0.1$, and $T + 1$ are respectively $0.1, 0.3$ and 0.6.*

Suppose that the true temperature T is 20, and I will pick a thermometer at random to measure temperature. Let X be the reading of the thermometer.

1. (4 points) If you are told that the temperature reading X is **not lower** than 20.1, find the PDF and CDF of X given this information.
2. (4 points) If the temperature reading X is precisely 20.1, what is the probability that the thermometer has defect?

Question 3 (10 points) *Let random experiment E be defined as follows.*

1. *Keep picking cards at random from a regular deck of 52 cards* **with replacement** *until the first red card is drawn, where it is assumed that each card in the deck is drawn with equal probability in each pick and that the sequence of card pickings are independent.*
2. *Record the sequence of cards drawn.*

Now two players A and B will perform experiment E independently. Let $(A_1, A_2, \ldots, A_m)$ and $(B_1, B_2, \ldots, B_n)$ be respectively the sequence of cards picked by A and the sequence of cards picked by B, where, of course, A_m and B_n are the last cards picked by A and by B respectively.

Note: the four parts of this question below are independent.

1. (2 points) What is the probability that $A_1 = B_1$?
2. (2 points) What is the probability that $A_m = B_n$?
3. (3 points) If you are told $m = 5$, what is the probability that the cards picked by A consist of precisely two Kings ("K") and three Queens ("Q")?
4. (3 points) Let X and Y be the respectively the number of red cards and the number of black cards picked by A. (Clearly, by definition, $X = 1$.) This question asks whether it is more likely for A to pick more red cards or to pick more black cards. Specifically, you need to determine which one of the following is true and justify your answer.
   (a) $P[X > Y] > P[X < Y]$.
   (b) $P[X > Y] < P[X < Y]$.
   (c) $P[X > Y] = P[X < Y]$.