

ELG 3121 Summer 2006 Mock Final Exam

July 4, 2006

- 45 points in total
- One page single-sided aid-sheet allowed.
- 3 hours

Question 1 (13 points) *The PDF of random variable X is given in Figure 1.*

1. (4 points) Find the mean and variance of X .
2. (4 points) Find the CDF of X .
3. (5 points) Let function $g(\cdot)$ be defined by

$$g(x) = u(x - 2) + 2u(x - 4.5),$$

where $u(x)$ is the unit-step function. Define random variable $Y := g(X)$. Find the PDF of Y conditioned on event $X > 2$.

Question 2 (5 points) *A gambler has a fake quarter coin which looks identical to a regular quarter. It is known that when tossing a regular quarter, HEAD and TAIL occur with the same probability, and when tossing the fake quarter, HEAD occurs with probability $4/5$ and TAIL occurs with probability $1/5$. One day the gambler accidentally mixed up the fake quarter with 9 regular quarters. Not being able to tell which one was fake, he picked a quarter from the 10 quarters at random and tossed it 5 times. The outcome was HEAD, HEAD, HEAD, HEAD, TAIL. What is the probability that the quarter is the fake one?*

Question 3 (9 points) *Verify whether the following statements are correct. Prove the correct ones or give counter-examples to disprove the incorrect ones.*

1. If $E[XY] = E[X]E[Y]$, then X and Y are independent.
2. If $COV[X, Y] = 0$ and both X and Y have zero mean, then X and Y are independent.
3. Suppose that X and Y are jointly Gaussian variables. Then if X and Y are uncorrelated, they are independent.

Question 4 (5 points) *Service requests arriving at a web server are modelled as a Poisson process with average arrival rate 3 requests per minutes.*

1. What is the probability that there are more than 2 requests arriving during time window $[0, 3]$ (time unit is minute)?
2. Suppose that during the time window $[0, 2]$, no request has arrived. What is the probability that there are more than 2 requests arriving during time window $[0, 3]$?

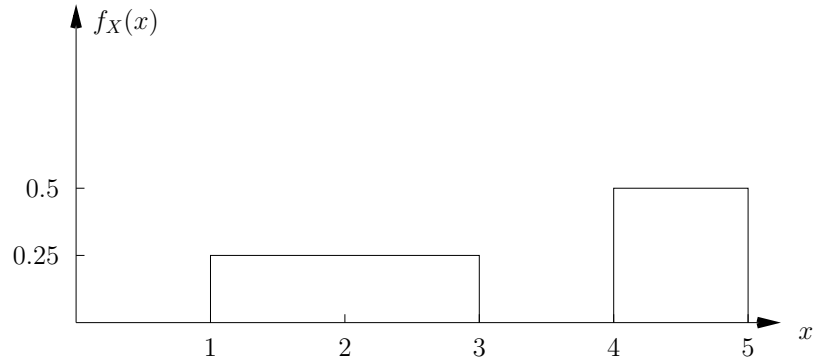


Figure 1: A probability density function

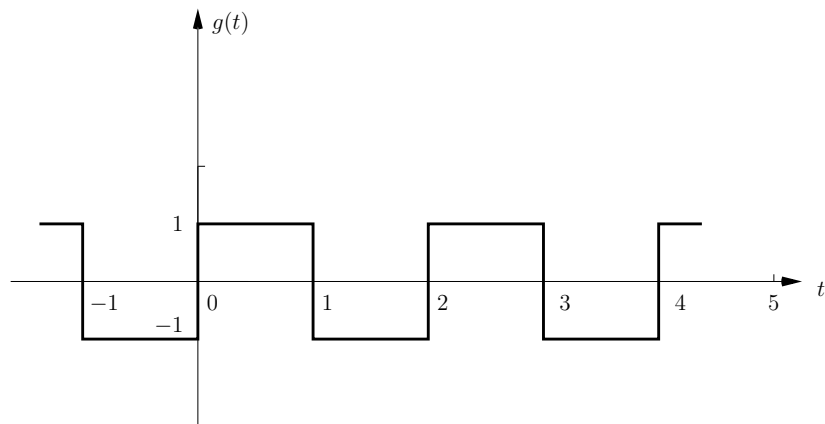


Figure 2: Function $g(t)$

Question 5 (7 points) Function $g(t)$ is defined as in Figure 2. Suppose that a random process $X(t)$ is defined by

$$X(t) = g(t - T),$$

where T is a uniform random variable over interval $[0, 2)$.

1. (5 points) Is $X(t)$ wide-sense stationary?
2. (2 points) Is $X(t)$ Markov?

Question 6 (6 points) Random variable Z is generated as follows. First toss a fair die, namely, one whose outcome is equally likely to be 1, 2, 3, 4, 5, 6. Denote the outcome by K ; generate K iid Gaussian random variables X_1, X_2, \dots, X_K each having mean 1 and variance 1; random variable Z is defined as the sum $X_1 + X_2 + \dots + X_K$.

1. Find the PDF of Z .
2. Determine the probability that $Z > 6$ in terms of Q-function.