ELG 3121 Mid-Term Solution

June 20, 2005

Question 1 1. *The sample space is* $(0,2) \times (0,2)$. *(Drawing on 2-D plane omitted).*

2. *The total area of the sample space is 4, in which the area corresponding to the event of interest is* $1/2$. *The probability of the event is then* $\frac{1/2}{4} = 1/8$.

Question 2 1. $\binom{13}{4}/\binom{52}{4}$ *or* $(13/52)(12/51)(11/50)(10/49)$

2. *Four times the above number.*

Question 3 1.

$$\begin{aligned} P[\overline{A} \cap \overline{B}] &= P[S] - P[A] - P[B] + P[A \cap B] \\ &= 1 - P[A] - P[B] + P[A]P[B] \\ &= (1 - P[A])(1 - P[B]) \\ &= P[\overline{A}]P[\overline{B}] \end{aligned}$$

Therefore $\overline{A}$ *and* $\overline{B}$ *are independent.*

2. *We can not conclude* $\overline{A}$, $\overline{B}$ *and* $\overline{C}$ *are independent. Notice that the sample space* S *can be partitioned into 8 disjoint regions, in terms of whether it is contained in* A, B *or* C. *We may consider the probability of each region as a variable.*

   We are given $P[A]$, $P[B]$, $P[C]$, *which means that* $P[\overline{A}], P[\overline{B}]$, *and* $P[\overline{C}]$ *are also given to us. In addition, we are also given* $P[A \cap B \cap C]$.

   Notice that each of these 7 *probabilities gives us a linear (summation) equations involving some of the 8 variables. If we were to conclude that* $\overline{A}$, $\overline{B}$, *and* $\overline{C}$ *are independent, by the definition of independence of three events, it is possible to see that we would essentially be able to solve for the 8 variables. But having only 7 equations, this is impossible.*

Question 4 1. *After you answer John and before your hear John's confirmation, the probability that John gets the first letter correct is* 0.8, *as given in the question; and similarly the probability that John gets the second letter correct is also* 0.8. *By the independence assumption between John mis-interpreting the first letter and he mis-interpreting the second letter, the probability that John gets both letters correct is* 0.8^2.

2. *Note that whatever John says is what he hears. Then we need to find the* $P[John\ hears\ MM|I\ hear\ MM]$ *(which I will write as* $P[JMM|IMM]$ *for simplicity). This equals*

$$\begin{aligned} &= \frac{P[JMM\ AND\ IMM]}{P[IMM]} \\ &= \frac{P[IMM|JMM]P[JMM]}{P[IMM|JMM]P[JMM] + P[IMM|JMN]P[JMN] + P[IMM|JNM]P[JNM] + P[IMM|JNN]P[JNN]} \\ &= \frac{0.8 \cdot 0.8 \cdot 0.8 \cdot 0.8}{0.8 \cdot 0.8 \cdot 0.8 \cdot 0.8 + 0.8 \cdot 0.2 \cdot 0.8 \cdot 0.2 + 0.2 \cdot 0.8 \cdot 0.2 \cdot 0.8 + 0.2 \cdot 0.2 \cdot 0.2 \cdot 0.2} \end{aligned}$$