

ELG 3121 Mid-Term Examination

June 9th, 2005; Total 24 points

Question 1 (4 points) Pick a number X at random from interval $(0, 2)$ where each point in the interval is equally likely to be picked. Then independent of the value of X , pick a number Y at random from interval $(0, 2)$ again where each point in the interval is also equally likely to be picked. Note the ordered pair (X, Y) .

1. (1 point) What is the sample space of this experiment. Show it graphically.
2. (3 points) What is the probability that $X + Y < 1$?

Question 2 (6 points) In a random experiment, you draw four cards without replacement from a regular deck of playing cards (i.e., 52 cards consisting of 4 suits and each suit ranging from “A” to “K”).

1. (3 points) What is the probability that the drawn four cards are all Hearts?
2. (3 points) What is the probability that the drawn four cards are from the same suit?

Question 3 (7 points) Let S be the sample space of a random experiment, and A , B and C be three events (i.e., subsets of S). It is known that $P[A] = 0.3$, $P[B] = 0.4$ and $P[C] = 0.5$.

1. (5 points) Suppose that $P[A \cap B] = P[A]P[B]$. Then based on the axioms of probability theory, can you conclude whether \overline{A} and \overline{B} are independent? Justify your answer.
2. (2 points) Independent of part 1 of this question, suppose that $P[A \cap B \cap C] = P[A]P[B]P[C]$. Then based on the axioms of probability theory, can you conclude whether \overline{A} , \overline{B} and \overline{C} are independent? Justify your answer.

Question 4 (7 points) It is well-known that the English letters “M” and “N” sound similar, particularly over the phone. Suppose that over the phone, when letter “M” is pronounced, with probability 0.2 it is heard as “N”, and with probability 0.8 it is heard correctly — as “M”. Similarly, suppose that over the phone, when letter “N” is pronounced, with probability 0.2 it is heard as “M”, and with probability 0.8 it is heard correctly — as “N”.

Now suppose that your friend John is calling you to get your password to unlock a computer. Below is the phone conversation based on what you hear and what you say.

You hear John saying: “Can you give me the password?”

You say: “The password is MM, both letters capitalized.”

You hear John saying: “MM, OK, I got it.”

Suppose that the only possibility of misunderstanding in this conversation is the possible confusion between the pronunciation of “M” and that of “N”.

Notice that this dialogue consists of three sentences. In the first sentence, John asks you for the password; in the second sentence, you answer John; and in the third sentence, John confirms. You must realize that you will never be sure whether John hears your answer correctly, nor can you be sure whether you hear John’s confirmation correctly.

1. (3 points) After you answer John and before you hear John’s confirmation, what is the probability that John has got the correct password? — The correct password is obviously “MM”.

2. (4 points) After you hear John's confirmation, what is the probability that John has got the correct password?

Reality is merely an illusion, albeit a very persistent one.

— *Albert Einstein*