On The Capacity of Gaussian MIMO Channels Under Interference Constraints

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Introduction

- Importance of spectrum sharing (SS)
  - significantly improves spectrum efficiency
  - cellular, cognitive radio (CR), HetNet, non-orthogonal multiple-access (NOMA)
  - promising technology for 5G & beyond
- Problem: multi-user interference
- Control via multi-antenna (MIMO) systems
- Optimal signaling? Capacity?

Standard Gaussian MIMO Channel (no interference)

- Tx Power Constraint (TPC)
- Capacity is well-known (log-det expression under TPC)[T'65][1][T'95][2]
- Capacity-achieving input: Gaussian
- Optimal signalling (covariance): on channel eigenvectors
- Power allocation: via water-filling (WF)

[1] B. S. Tsybakov, Capacity of Vector Gaussian Memoryless Channel, Problems of Information Transmission, v.1, n.1., 1965.

[2] I. E. Telatar, Capacity of Multi-Antenna Gaussian Channels, AT&T Bell Labs, Internal Tech. Memo, June 1995, (European Trans. Telecom., v.10, no. 6, Dec. 1999).

Cont.: Per-Antenna (PAC) and Joint Constraints

- Per-antenna power constraint (PAC)[Vu'11][3]
- Joint power constraint (TPC+PAC)[L'17][4]
  - Gaussian input is still optimal
  - optimal signaling: not on channel eigenvectors anymore
  - MISO: EGT or/and MRT
  - MIMO: open in general

[3] M. Vu, MISO Capacity With Per-Antenna Power Constraint, IEEE Trans. on Commun., vol. 59, no. 5, May 2011.

[4] S. Loyka, The Capacity of Gaussian MIMO Channels Under Total and Per-Antenna Power Constraints, IEEE Trans. Comm., v.65, n.3, Mar. 2017

MIMO Channel Under Interference Constraint

- Interference power constraint (IPC), in addition to the TPC
- Much less in known
- Capacity-achieving input: Gaussian
- Optimal signalling (Tx covariance): not known in general
- Numerical (algorithmic) solutions
  - game-theoretic approach (fixed point equation) [SP'10][5]
  - dual problem approach [Z'10][6]
  - many more

[5] G. Scurati, D.P. Palomar, MIMO Cognitive Radio: A Game Theoretical Approach, IEEE Trans. Signal Processing, v. 58, n. 2, Feb. 2010.

[6] R. Zhang et al, Dynamic Resource Allocation in Cognitive Radio Networks, IEEE Signal Procesing Magazine, v.27, n.3, May 2010.

This paper: closed-form solutions + properties

- Gaussian MIMO channel under TPC + IPCs
- Closed-form solutions for optimal signaling
  - the general case: two (or more) dual variables (via e.g. IBA)
  - explicit for full-rank and rank-1 cases
- Major differences to the standard WF
  - signaling on the channel eigenmodes is **not** optimal
  - e.g. independent signaling is **not** optimal for parallel channels
  - optimal covariance is **not** necessarily unique
  - TPC can be inactive

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- Interplay of TPC and IPCs: interference-limited and power-limited regimes
- Potential of spectrum sharing (SS)
  - via linear-algebraic structure of MIMO channels
  - favorable propagation via simple rank condition
- Optimality of "pre-whitening" filter
- Optimality of partial null forming

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Standard Gaussian MIMO (under TPC)

- Channel model:

$$\mathbf{y} = \mathbf{H}_1\mathbf{x} + \boldsymbol{\xi} \quad (1)$$

- Capacity:

$$C = \max_{\mathbf{R}\in S_R} \ln|\mathbf{I} + \mathbf{W}_1\mathbf{R}| \quad (2)$$

$\mathbf{W}_1 = \mathbf{H}_1^+\mathbf{H}_1$ = channel Gram matrix
$\mathbf{R} = \mathbb{E}\{\mathbf{x}\mathbf{x}^+\}$ = Tx covariance, $\mathbf{x} \sim \mathcal{CN}(0, \mathbf{R})$
$S_R = S_{TPC} \triangleq \{\mathbf{R} : \mathbf{R} \geq 0, tr(\mathbf{R}) \leq P_T\}$ = constraint set

- Optimal signaling (covariance) via WF:

$$\mathbf{R}^* = \mathbf{R}_{WF} \triangleq (\mu^{-1}\mathbf{I} - \mathbf{W}_1^{-1})_+ \quad (3)$$

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MIMO Channel Under Interference Constraint

- Channel model: Tx-Rx link is the same, $\mathbf{y} = \mathbf{H}_1\mathbf{x} + \boldsymbol{\xi}$
- Tx-PR(U) links: $\mathbf{y}_{2k} = \mathbf{H}_{2k}\mathbf{x} + \boldsymbol{\xi}_{2k}$, $k = 1...K$

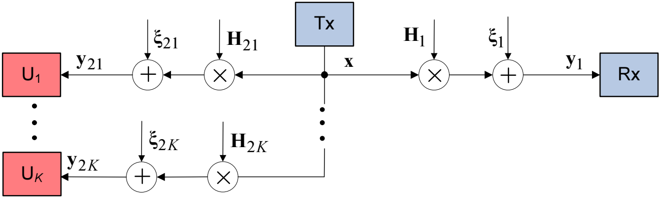

Figure: Multi-user MIMO channel under IPCs.

- Interference constraint (IPC): $tr(\mathbf{H}_{2k}\mathbf{R}\mathbf{H}_{2k}^{+}) \leq P_{Ik}\ \forall k$
- Constraint set: $S_R = \{\mathbf{R} : \mathbf{R} \geq 0, tr(\mathbf{R}) \leq P_T,\ tr(\mathbf{W}_{2k}\mathbf{R}) \leq P_{Ik}\ \forall k\}$

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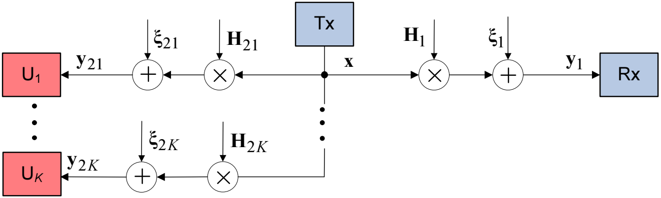

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MIMO Channel Under Interference Constraint

- The capacity:

$$C = \max_{\mathbf{R} \in S_R} \ln |\mathbf{I} + \mathbf{W}_1 \mathbf{R}| \tag{4}$$

- Optimal signaling?
- **Caution**:
  - constraint set S_R is not less important than the objective
  - unitary invariance is lost (due to IPCs): S_R is not isotropic
  - standard tricks (e.g. Hadamard inequality) do not apply

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Optimal Signaling Under Interference Constraint

Theorem (General case)

The optimal Tx covariance matrix is

$$\mathbf{R}^* = \mathbf{W}_\mu^\dagger (\mathbf{I} - \mathbf{W}_\mu \mathbf{W}_1^{-1} \mathbf{W}_\mu)_+ \mathbf{W}_\mu^\dagger \quad (5)$$

$\mathbf{W}_\mu = (\mu_1 \mathbf{I} + \sum_k \mu_{2k} \mathbf{W}_{2k})^{\frac{1}{2}}$;
$\mu_1, \mu_{2k} \geq 0$ *are Lagrange multipliers (dual variables),*

$$\mu_1(tr\mathbf{R}^* - P_T) = 0,\ \mu_{2k}(tr(\mathbf{W}_{2k}\mathbf{R}^*) - P_{Ik}) = 0 \quad (6)$$

s.t. $tr(\mathbf{R}^*) \leq P_T$, $tr(\mathbf{W}_{2k}\mathbf{R}^*) \leq P_{Ik}\ \forall k$. *The capacity is*

$$C = \sum_{i:\lambda_i > 1} \log \lambda_i (\mathbf{W}_\mu^\dagger \mathbf{W}_1 \mathbf{W}_\mu^\dagger) \quad (7)$$

Optimal Signaling: cont.

$$\mathbf{R}^* = \mathbf{W}_\mu^\dagger(\mathbf{I} - \mathbf{W}_\mu \mathbf{W}_1^{-1} \mathbf{W}_\mu)_+ \mathbf{W}_\mu^\dagger \quad (8)$$

- $\mathbf{W}_\mu^\dagger$ = "pre-whitening" filter; no IPCs: $\mathbf{W}_\mu^\dagger = \frac{1}{\sqrt{\mu_1}}\mathbf{I}$
- Closed-form solution up to μ_1, $\{\mu_{2k}\}$
- Explicit in some cases (full-rank, rank-1)
- In general: numerically, via an iterative bisection algorithm (IBA)[14]
- Alternative: $P_T(\mu_1, \{\mu_{2k}\})$, $P_{Ik}(\mu_1, \{\mu_{2k}\})$ parametrized by $\mu_1, \{\mu_{2k}\}$

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General Properties: Unbounded Growth

- When does $C(P_T) \to \infty$ as $P_T \to \infty$?
- i.e. arbitrary large spectral efficiency of spectrum sharing (given enough power budget)
- Trivial under TPC alone, but not for TPC + IPCs

Proposition

Let $0 \leq P_{Ik} < \infty$ be fixed for all k. Then, the capacity grows unbounded as P_T increases, i.e. $C(P_T) \to \infty$ as $P_T \to \infty$, if and only if

$$\bigcap_k \mathcal{N}(\mathbf{W}_{2k}) \notin \mathcal{N}(\mathbf{W}_1) \quad (9)$$

or, equivalently,

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Unbounded Growth: Some Observations

- "iff": exhaustive characterization of all cases of unbounded growth
- holds if $r(\sum_k \mathbf{W}_{2k}) < r(\mathbf{W}_1)$
- does not hold if $\mathcal{N}(\sum_k \mathbf{W}_{2k}) \in \mathcal{N}(\mathbf{W}_1)$, e.g. if $\sum_k \mathbf{W}_{2k} > 0$

General Properties: Zero Capacity

- When is $C = 0$?
- i.e. no spectrum sharing is possible
- Trivial under the TPC alone, but not for TPC + IPC

Proposition

Consider the Gaussian MIMO channel under the TPC and IPCs and let $P_T > 0$, $\mathbf{W}_1 \neq 0$. Its capacity is zero iff $P_{Ik} = 0$ for some k and

$$\mathcal{N}\Big(\sum_{k\in\mathcal{K}_0} \mathbf{W}_{2k}\Big) \in \mathcal{N}(\mathbf{W}_1). \quad (11)$$

where $\mathcal{K}_0 = \{k : P_{Ik} = 0\}$.

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where $\mathcal{K}_0 = \{k : P_{Ik} = 0\}$.

Zero Capacity: Some Observations

- $P_{Ik} = 0$: equivalent to ZF, , i.e. $C = 0$ only if ZF is required for at least one user
- $C > 0$ if $r(\mathbf{W}_1) > r(\sum_k \mathbf{W}_{2k})$

Spectrum Sharing: favorable propagation

Corollary

If $r(\mathbf{W}_1) > r(\sum_k \mathbf{W}_{2k})$, *then*
1. $C \neq 0 \; \forall \; P_{Ik} \geq 0$ *and* $P_T > 0$.
2. $C(P_T) \rightarrow \infty$ *as* $P_T \rightarrow \infty \; \forall \; P_{Ik} \geq 0$

- $r(\mathbf{W}_1) > r(\sum_k \mathbf{W}_{2k})$: represents **favorable propagation** scenarios
- spectrum sharing is possible for any P_{Ik}
- arbitrary large spectral efficiency given enough Tx power budget
- holds for massive MIMO ?

Rank Bound

- regular MIMO: $r(\mathbf{R}^*) \leq r(\mathbf{W}_1)$
- is it still true under IPCs ?

Proposition

If the TPC is active or/and active $\sum_k \mathbf{W}_{2k}$ is full-rank, then

$$r(\mathbf{R}^*) \leq r(\mathbf{W}_1) \tag{12}$$

Otherwise: (i) $\mathbf{R}^*$ may be not unique; (ii) there exists $\mathbf{R}^*$ for which (12) *holds.*

- Note that $\mathbf{R}^*$ is not necessarily unique - a stark difference to the TPC alone case (the standard WF).

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Rank-1 Solution (beamforming)

Corollary

If $r(\mathbf{W}_1) = 1$*, then* $r(\mathbf{R}^*) = 1$*, i.e. beamforming is optimal.*

- mimics the respective property for the standard WF
- however, signalling on the (only) active eigenvector of $\mathbf{W}_1$ is not optimal under TPC + IPC (unlike TPC alone/WF)

Rank-1 Solution, $K = 1$

- $\mathbf{W}_1 = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^+$ i.e. rank-1,
- $K = 1$ (one PR),
- $\gamma_I = P_I / P_T =$ "interference-to-signal" ratio (ISR),
- $\gamma_{1,2}$ are ISR thresholds:

$$\gamma_1 = \frac{\mathbf{u}_1^+ \mathbf{W}_2^\dagger \mathbf{u}_1}{\mathbf{u}_1^+ (\mathbf{W}_2^\dagger)^2 \mathbf{u}_1} \leq \gamma_2 = \mathbf{u}_1^+ \mathbf{W}_2 \mathbf{u}_1 \tag{13}$$

where $\gamma_1 = \gamma_2 = 0$ if $\mathbf{u}_1 \in \mathcal{N}(\mathbf{W}_2)$.

Rank-1 Solution, $K = 1$

Proposition (low ISR)

1. If $\gamma_I < \gamma_1$ (low ISR regime), then the TPC is redundant and

$$\mathbf{R}^* = P_I \frac{\mathbf{W}_2^\dagger \mathbf{u}_1 \mathbf{u}_1^+ \mathbf{W}_2^\dagger}{\mathbf{u}_1^+ \mathbf{W}_2^\dagger \mathbf{u}_1}, \;\; C = \log(1 + \lambda_1 \alpha P_T) \tag{14}$$

where $\alpha =$ "SNR loss" $= \gamma_I \mathbf{u}_1^+ \mathbf{W}_2^\dagger \mathbf{u}_1 < 1$.

- beamforming on $\mathbf{W}_2^\dagger \mathbf{u}_1$, not on $\mathbf{u}_1$!
- $\mathbf{W}_2^\dagger =$ "pre-whitening" filter ($= \mathbf{I}$ without IPCs)

Rank-1 Solution, $K = 1$

Proposition (moderate ISR)

2. If $\gamma_1 \leq \gamma_I < \gamma_2$ (moderate ISR), both constraints are active and

$$\mathbf{R}^* = P_T \frac{\mathbf{W}_{2\mu}^{-1}\mathbf{u}_1\mathbf{u}_1^{+}\mathbf{W}_{2\mu}^{-1}}{\mathbf{u}_1^{+}\mathbf{W}_{2\mu}^{-2}\mathbf{u}_1}, \ \mathbf{W}_{2\mu} = \mathbf{I} + \mu_2\mathbf{W}_2 \tag{15}$$

- beamforming on $\mathbf{W}_{2\mu}^{-1}\mathbf{u}_1$, not on $\mathbf{u}_1$!
- $\mathbf{W}_{2\mu}^{-1}$ = "pre-whitening" filter

Rank-1 Solution $K = 1$

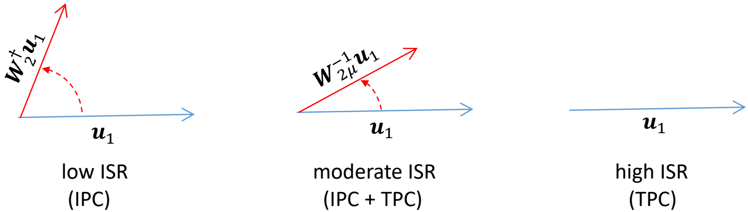
Proposition (high ISR)

3. If $\mathbf{u}_1 \in \mathcal{N}(\mathbf{W}_2)$ or $\gamma_I \geq \gamma_2$ (high ISR regime), then the IPC is redundant and the standard beamforming solution applies: $\mathbf{R}^* = P_T \mathbf{u}_1 \mathbf{u}_1^+$; $\alpha = 1$ (no SNR loss).

- beamforming on $\mathbf{u}_1$ (standard)
- no "pre-whitening" filter

Rank-1 Solution, $K = 1$

- 3 regimes (ISR) $\Rightarrow$ 3 different solutions
- different from the standard beamforming on $\mathbf{u}_1$, unless IPC is redundant or $\mathbf{u}_1$ is an eigenvector of $\mathbf{W}_2$



Full-Rank Solutions, $K = 1$

- Let $\mathbf{W}_1, \mathbf{W}_2 > 0$ and P_I be bounded as follows:

$$P_I > m\lambda_1(\mathbf{W}_2\mathbf{W}_1^{-1}) - tr(\mathbf{W}_2\mathbf{W}_1^{-1}) \quad (16)$$

$$P_I \leq \frac{m}{tr(\mathbf{W}_2^{-1})}(P_T + tr(\mathbf{W}_1^{-1})) - tr(\mathbf{W}_2\mathbf{W}_1^{-1})$$

then

Proposition (large SNR & INR)

The TPC is redundant, $\mathbf{R}^*$ is of full-rank and is given by:

$$\mathbf{R}^* = \mu_2^{-1}\mathbf{W}_2^{-1} - \mathbf{W}_1^{-1}, \ \mu_2^{-1} = \frac{1}{m}(P_I + tr(\mathbf{W}_2\mathbf{W}_1^{-1})) \quad (17)$$

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- Let $\mathbf{W}_1$ be of full rank, $\mathbf{W}_2 = \lambda_2 \mathbf{u}_2 \mathbf{u}_2^+$ be of rank-1.

Proposition (large SNR & INR)

1. The IPC is redundant, the optimal covariance is of full rank and is given by the standard WF solution, $\mathbf{R}^* = \mathbf{R}^*_{WF}$, if

$$
\begin{aligned}
P_I &\geq P_{I,th} = m^{-1}\lambda_2(P_T + tr(\mathbf{W}_1^{-1})) - \lambda_2 \mathbf{u}_2^+ \mathbf{W}_1^{-1} \mathbf{u}_2 \\
P_T &> m\lambda_1(\mathbf{W}_1^{-1}) - tr(\mathbf{W}_1^{-1})
\end{aligned}
\tag{18}
$$

Full-Rank Solutions, $K = 1$

Proposition (large SNR, moderate INR)

2. The TPC and IPC are active if

$$\lambda_2 \lambda_1(\mathbf{W}_1^{-1}) - \lambda_2 \mathbf{u}_2^+ \mathbf{W}_1^{-1} \mathbf{u}_2 < P_I < P_{I,th}, \quad (19)$$

$$P_T > m\lambda_2^{-1} P_I + m\mathbf{u}_2^+ \mathbf{W}_1^{-1} \mathbf{u}_2 - tr(\mathbf{W}_1^{-1}) \quad (20)$$

and the optimal covariance is of full rank and is given by

$$\mathbf{R}^* = \mu_1^{-1} \mathbf{I} - \mathbf{W}_1^{-1} - \alpha \mathbf{u}_2 \mathbf{u}_2^+ \quad (21)$$

where $\alpha, \mu_1 > 0$ are given in the paper.

- 1st 2 terms of (21) = the standard WF
- last term = correction due to IPC

Full-Rank Solutions, $K = 1$

Proposition (large SNR, moderate INR)

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Full-Rank Solutions, $K = 1$

$$\mathbf{R}^* = \mu_1^{-1}\mathbf{I} - \mathbf{W}_1^{-1} - \alpha \mathbf{u}_2 \mathbf{u}_2^+ \tag{22}$$

Link to the adaptive antenna array literature:

- 1st 2 terms of (22) = the standard WF
- last term = correction due to IPC
- well-known in the adaptive antenna array literature as partial null forming[7]
- hence, partial null forming is also optimal from the information-theoretic perspective (for spectrum sharing)

[7] H.L. Van Trees, Optimum Array Processing, Wiley, New York, 2002.

Conclusion

- Gaussian MIMO channel under interference constraints
  - CR, spectrum sharing, HetNet, NOMA (5G)
- Optimal signaling/covariance?
- General properties
  - arbitrary large SE (unbounded growth)
  - zero SE
  - qualitative behaviour via the natural linear-algebraic structure
  - favorable propagation via the simple rank condition
- Explicit closed-form solutions (rank-1, full rank)
- Optimality of pre-whitening filter
- Optimality of partial null forming
- Independent signaling is **not** optimal for parallel channel

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  - favorable propagation via the simple rank condition
- Explicit closed-form solutions (rank-1, full rank)
- Optimality of pre-whitening filter
- Optimality of partial null forming
- Independent signaling is **not** optimal for parallel channel

Conclusion

- Gaussian MIMO channel under interference constraints
  - CR, spectrum sharing, HetNet, NOMA (5G)
- Optimal signaling/covariance?
- General properties
  - arbitrary large SE (unbounded growth)
  - zero SE
  - qualitative behaviour via the natural linear-algebraic structure
  - favorable propagation via the simple rank condition
- Explicit closed-form solutions (rank-1, full rank)
- Optimality of pre-whitening filter
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- Independent signaling is **not** optimal for parallel channel