On Optimal Signaling Over Gaussian MIMO Channels Under Interference Constraints

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***Abstract*—Gaussian MIMO channel under total transmit and interference power constraints (TPC and IPC) is considered. A closed-form solution for the optimal transmit covariance matrix is obtained using the KKT-based approach. While closed-from solutions for optimal dual variables are possible in special cases, an iterative bisection algorithm (IBA) is proposed to find the optimal dual variables in the general case and its convergence is proved. Numerical experiments illustrate its efficient performance. Bounds for the optimal dual variables are given, which facilitate numerical solutions. An interplay between the TPC and IPC is studied, including the transition from power-limited to interference-limited regimes as the total transmit power increases.**

I. Introduction

Cognitive radio (CR) has recently attracted significant attention as a powerful approach to exploit underutilized spectrum and hence possibly resolve the spectrum scarcity problem [1][2]. Allowing secondary systems to use resources allocated to primary systems call for a careful management of possible interference to the latter from the former. In this respect, multi-antenna (MIMO) systems have significant potential due to their significant signal processing capabilities, including interference cancellation and precoding [3], which can also be done in an adaptive and distributed manner [7]. A promising approach is to limit interference to primary receivers (PR) by properly designing secondary transmitters (Tx) while exploiting their multi-antenna capabilities.

The capacity and optimal signalling for the Gaussian MIMO channel under the total power constraints (TPC) is well-known: the optimal (capacity-achieving) signaling is Gaussian and, under the TPC, is on the eigenvectors of the channel with power allocation to the eigenmodes given by the water-filling (WF) [3][4]. Under per-antenna power constraints (PAC), in addition or instead of the TPC, Gaussian signalling is still optimal but not on the channel eigenvectors anymore so that the standard water-filling solution over the channel eigenmodes does not apply [5][6]. Much less is known under the added interference power consitraint (IPC), which limits the power of interference induced by the secondary transmitter to a primary receiver. A game-theoretic approach to this problem was proposed in [7], where a fixed-point equation was formulated from which the optimal covariance matrix can in principle be determined. Unfortunately, no closed-form solution is known for this equation. In addition, this approach is limited in the following respects: the channel to the primary receiver is required to be full-rank (hence excluding the important case of single-antenna devices communicating to a multi-antenna base station or, in general, the cases where the number of Rx antennas is less than the number of Tx antennas); the TPC is not included explicitly (rather, being ”absorbed” into the IPC), hence eliminating the important case of inactive IPC (since this is the only explicit constraint); consequently, no interplay between the TPC and the IPC can be studied.

In this paper, we obtain a closed-form expression for an optimal covariance matrix of the Gaussian MIMO channel under the TPC and the IPC using the KKT conditions. Both constraints are included explicitly and hence anyone is allowed to be inactive. This allows us to study the interplay between the power and interference constraints and, in particular, the transition from power-limited to interference limited regimes as the Tx power increases. As an added benefit, no limitations is placed on the rank of the channel to the PR, so that the number of PR antennas can be any. Under the added IPC, independent signaling is shown to be sub-optimal for parallel channels to the intended receiver (Rx), unless the PR channels are also parallel or if the IPC is inactive.

Optimal signaling for the Gaussian MIMO channel under the TPC and the IPC has been also considered in [8] using the dual problem approach. However, no closed-form solution was obtained for optimal dual variables. Hence, various sub-optimal solutions were proposed (e.g. partial channel projection). Our KKT-based approach includes explicit equations for the optimal dual variables, which can be solved efficiently. To this end, we propose an iterative bisection algorithm (IBA) and prove its convergence. Numerical experiments demonstrate its efficient performance. In some cases, our KKT-based approach allows the optimal dual variables to be determined in a closed-form analytically. Bounds to the optimal dual variables are derived, which facilitate numerical solutions. Properties of the optimal Tx covariance as a function of dual variables are explored: the total Tx power as well as interference power are shown to be decreasing functions of dual variables, which is an important part in the proof of the IBA convergence.

Notations: bold capitals denote matrice while bold lower-case letters denote column vectors; $\boldsymbol{R}^+$ is the Hermitian conjugation of $\boldsymbol{R}$; $\boldsymbol{R} \geq 0$ means that $\boldsymbol{R}$ is positive semi-definite, $|\boldsymbol{R}|$ denotes determinant while $\lambda_i(\boldsymbol{R})$ is i-th eigenvalue of $\boldsymbol{R}$; unless indicated otherwise, eigenvalues are in decreasing order, $\lambda_1 \geq \lambda_2 \geq ..$; $\lceil \cdot \rceil$ denotes ceiling, while $(x)_+ = \max[0, x]$ is the positive part of x.

II. Channel Model and Capacity

Let us consider the standard discrete-time model of the Gaussian MIMO channel:

$$\boldsymbol{y}_1 = \boldsymbol{H}_1 \boldsymbol{x} + \boldsymbol{\xi}_1 \tag{1}$$

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where $\boldsymbol{y}_1, \boldsymbol{x}, \boldsymbol{\xi}_1$ and $\boldsymbol{H}_1$ are the received and transmitted signals, noise and channel matrix respectively. The noise is assumed to be Gaussian with zero mean and unit variance, so that the SNR equals to the signal power. Complex-valued channel model is assumed throughout the paper, with full channel state information available both at the transmitter and the receiver. Gaussian signaling is known to be optimal in this setting [3][4] so that finding the channel capacity C amounts to finding an optimal transmit covariance matrix $\boldsymbol{R}$:

$$C = \max_{\boldsymbol{R} \in S_R} C(\boldsymbol{R}) \tag{2}$$

where $C(\boldsymbol{R}) = \ln |\boldsymbol{I} + \boldsymbol{W}_1 \boldsymbol{R}|$, $\boldsymbol{W}_1 = \boldsymbol{H}_1^+ \boldsymbol{H}_1$, $\boldsymbol{R}$ is the Tx covariance and S_R is the constraint set. In the case of the total power constraint (TPC) only, it takes the form

$$S_R = \{\boldsymbol{R} : \boldsymbol{R} \geq 0, tr\boldsymbol{R} \leq P_T\}, \tag{3}$$

where P_T is the maximum total Tx power. The solution to this problem is well-known: optimal signaling is on the eigenmodes of $\boldsymbol{W}_1$, so that they are also the eigenmodes of optimal covariance $\boldsymbol{R}^*$, and the optimal power allocation is via the water-filling (WF). This solution can be compactly expressed as follows:

$$\boldsymbol{R}^* = \boldsymbol{R}_{WF} \triangleq (\mu^{-1}\boldsymbol{I} - \boldsymbol{W}_1^{-1})_+ \tag{4}$$

where μ is the "water" level found from the total power constraint $tr\boldsymbol{R}^* = P_T$ and $(\boldsymbol{R})_+$ denotes positive eigenmodes of Hermitian matrix $\boldsymbol{R}$:

$$(\boldsymbol{R})_+ = \sum_{i:\lambda_i > 0} \lambda_i \boldsymbol{u}_i \boldsymbol{u}_i^+ \tag{5}$$

where $\lambda_i,\ \boldsymbol{u}_i$ are i-th eigenvalue and eigenvector of $\boldsymbol{R}$.

In the case of cognitive radio system, there is a 2nd channel from the Tx to the primary receiver (PR), $\boldsymbol{y}_2 = \boldsymbol{H}_2 \boldsymbol{x} + \boldsymbol{\xi}_2$, and there is a limit of how much interference the Tx can induce (via $\boldsymbol{x}$) to the PR:

$$E\{\boldsymbol{x}^+ \boldsymbol{H}_2^+ \boldsymbol{H}_2 \boldsymbol{x}\} = tr\boldsymbol{H}_2^+ \boldsymbol{R} \boldsymbol{H}_2 \leq P_I \tag{6}$$

where P_I is the maximum acceptable interference power and the left-hand side is the actual interference power at the PR. In this setting, the constraint set becomes

$$S_R = \{\boldsymbol{R} : \boldsymbol{R} \geq 0,\ tr\boldsymbol{R} \leq P_T,\ tr\boldsymbol{W}_2 \boldsymbol{R} \leq P_I\}, \tag{7}$$

where $\boldsymbol{W}_2 = \boldsymbol{H}_2^+ \boldsymbol{H}_2$. The Gaussian signalling is still optimal and the capacity subject to the TPC and IPC can still be expressed as in (2) but the optimal covariance is not $\boldsymbol{R}_{WF}$ anymore, as discussed in the next section.

III. Optimal Signalling Under Interference Constraint

The following Theorem gives a closed-form solution for the optimal Tx covariance matrix under the TPC and the IPC in (7) in the general case.

Theorem 1. *The optimal Tx covariance matrix to achieve the capacity of the Gaussian MIMO channel in* (2) *under the joint TPC and IPC in* (7) *can be expressed as follows:*

$$\boldsymbol{R}^* = \boldsymbol{W}_\mu^{-\frac{1}{2}} (\boldsymbol{I} - \boldsymbol{W}_\mu^{\frac{1}{2}} \boldsymbol{W}_1^{-1} \boldsymbol{W}_\mu^{\frac{1}{2}})_+ \boldsymbol{W}_\mu^{-\frac{1}{2}} \tag{8}$$

where $\boldsymbol{W}_\mu = \mu_1 \boldsymbol{I} + \mu_2 \boldsymbol{W}_2$*;* $\mu_1, \mu_2 \geq 0$ *are Lagrange multipliers (dual variables) responsible for the total Tx and interference power constraints found as solutions of the following non-linear equations:*

$$\mu_1(tr\boldsymbol{R}^* - P_T) = 0,\ \mu_2(tr\boldsymbol{W}_2\boldsymbol{R}^* - P_I) = 0 \tag{9}$$

subject to $tr\boldsymbol{R}^* \leq P_T$*,* $tr\boldsymbol{W}_2\boldsymbol{R}^* \leq P_I$*. The capacity can be expressed as follows:*

$$C = \sum_{i:\lambda_{ai} > 1} \log \lambda_{ai} \tag{10}$$

where $\lambda_{ai} = \lambda_i(\boldsymbol{W}_\mu^{-1} \boldsymbol{W}_1)$.

Proof. Since the problem is convex and Slater's condition holds (e.g. take $\boldsymbol{R} = a\boldsymbol{I} > 0$, $a = \frac{1}{2m}\min\{P_T, P_I/\lambda_1(\boldsymbol{W}_2)\}$, which is strictly feasible), the KKT conditions are both sufficient and necessary for optimality [9]. They take the following form:

$$-(\boldsymbol{I} + \boldsymbol{W}_1\boldsymbol{R})^{-1}\boldsymbol{W}_1 - \boldsymbol{M} + \mu_1\boldsymbol{I} + \mu_2\boldsymbol{W}_2 = 0 \tag{11}$$

$$\boldsymbol{M}\boldsymbol{R} = 0,\ \mu_1(tr\boldsymbol{R} - P_T) = 0,\ \mu_2(tr\boldsymbol{W}_2\boldsymbol{R} - P_I) = 0, \tag{12}$$

$$\boldsymbol{M} \geq 0,\ \mu_1 \geq 0,\ \mu_2 \geq 0 \tag{13}$$

$$tr\boldsymbol{R}^* \leq P_T,\ tr\boldsymbol{W}_2\boldsymbol{R}^* \leq P_I,\ \boldsymbol{R} \geq 0 \tag{14}$$

where $\boldsymbol{M}$ is Lagrange multiplier responsible for the positive semi-definite constraint $\boldsymbol{R} \geq 0$. Denoting $\boldsymbol{W}_\mu = \mu_1\boldsymbol{I} + \mu_2\boldsymbol{W}_2$ and introducing new variables $\tilde{\boldsymbol{R}} = \boldsymbol{W}_\mu^{\frac{1}{2}}\boldsymbol{R}\boldsymbol{W}_\mu^{\frac{1}{2}}$, $\tilde{\boldsymbol{W}}_1 = \boldsymbol{W}_\mu^{-\frac{1}{2}}\boldsymbol{W}_1\boldsymbol{W}_\mu^{-\frac{1}{2}}$, $\tilde{\boldsymbol{M}} = \boldsymbol{W}_\mu^{-\frac{1}{2}}\boldsymbol{M}\boldsymbol{W}_\mu^{-\frac{1}{2}}$, it follows that $\tilde{\boldsymbol{M}}\tilde{\boldsymbol{R}} = 0$ so that (11) can be transformed to

$$(\boldsymbol{I} + \tilde{\boldsymbol{W}}_1\tilde{\boldsymbol{R}})^{-1}\tilde{\boldsymbol{W}}_1 + \tilde{\boldsymbol{M}} = \boldsymbol{I} \tag{15}$$

for which the solution is

$$\tilde{\boldsymbol{R}} = (\boldsymbol{I} - \tilde{M})^{-1} - \tilde{\boldsymbol{W}}_1^{-1} = (\boldsymbol{I} - \tilde{\boldsymbol{W}}_1^{-1})_+ \tag{16}$$

Transforming back to the original variables results in (8). (9) are complementary slackness conditions in (12); (10) follows, after some manipulations, by using $\boldsymbol{R}^*$ of (8) in $C(\boldsymbol{R})$. $\square$

For simplicity of exposition, we implicitly assumed above that $\boldsymbol{W}_\mu$ is full-rank. If this is not the case, pseudo-inverse should be used instead.

Note that (9) allow anyone of the dual variables to be inactive (i.e. $\mu_1 = 0$ or $\mu_2 = 0$, but not simultaneously), unlike the standard WF solution, where the TPC is always active. While it is not feasible to find dual variables μ_1, μ_2 in a closed form in general (since (9) is a system of coupled non-linear equations), they can be found in such form in some special cases. The next section develops an iterative bisection algorithm (IBA) to find the optimal dual variables in the general case with any desired accuracy and proves its convergence.

IV. Iterative Bisection Algorithm

Let $f(x)$ be a function with the following property: $f(x) \geq 0$ for any $x < x_0$ and $f(x) \leq 0$ for any $x > x_0$, where x_0 is a solution of $f(x) = 0$. Then, the following bisection algorithm (BA) can be used to solve $f(x) = 0$, where x_l, x_u are the upper and lower bounds to x_0: $x_l \leq x_0 \leq x_u$, and $\epsilon > 0$ is any desired accuracy. In fact, it is straightforward to show that this algorithm will converge in a finite number N of steps such that

$$N \leq \left\lceil \log_2 \left(\frac{x_u - x_l}{\epsilon} \right) \right\rceil \tag{17}$$

where $\lceil \cdot \rceil$ denotes ceiling, so that the convergence is exponentially fast and hence the algorithm is very efficient [9].

Algorithm 1 Bisection algorithm (BA)

Require: $f(x),\ x_l,\ x_u,\ \epsilon$
 repeat
  1. Set $x = \frac{1}{2}(x_l + x_u)$.
  2. If $f(x) < 0$, set $x_u = x$. Otherwise, set $x_l = x$. Terminate if $f(x) = 0$.
 until $|x_u - x_l| \leq \epsilon$.

An alternative stopping criteria for this algorithm is $|f(x)| \leq \epsilon$ and the two criteria are equivalent when $f(x)$ is continuous. The BA can be used to solve for $\mu_1,\ \mu_2$ in (9) in an iterative way, as we show below. To this end, we need to establish lower and upper bounds to the solutions $\mu_1^*,\ \mu_2^*$ required by the BA.

Proposition 1. *Let $\mu_1^*,\ \mu_2^*$ be solutions of* (9)*, i.e. the optimal dual variables. They can be bounded as follows:*

$$0 \leq \mu_1^* \leq \mu_{1u} = m(P_T + \lambda_1^{-1}(\boldsymbol{W}_1))^{-1} \tag{18}$$

$$0 \leq \mu_2^* \leq \mu_{2u} = (P_I/r_2 + \lambda_m(\boldsymbol{W}_2)/\lambda_1(\boldsymbol{W}_1))^{-1} \tag{19}$$

where r_2 is the rank of $\boldsymbol{W}_2$ and m is the number of Tx antennas.

Proof. From the KKT conditions in (11),

$$(\boldsymbol{I} + \boldsymbol{W}_1\boldsymbol{R})^{-1}\boldsymbol{W}_1\boldsymbol{R} = \mu_1\boldsymbol{R} + \mu_2\boldsymbol{W}_2\boldsymbol{R} \tag{20}$$

so that

$$\mu_1 P_T + \mu_2 P_I = tr(\boldsymbol{I} + \boldsymbol{W}_1\boldsymbol{R})^{-1}\boldsymbol{W}_1\boldsymbol{R} \tag{21}$$

Let $\lambda_{11} = \lambda_1(\boldsymbol{W}_1)$. Since

$$tr(\boldsymbol{I} + \boldsymbol{W}_1\boldsymbol{R})^{-1}\boldsymbol{W}_1\boldsymbol{R} \leq mP_T(\lambda_{11}^{-1} + P_T)^{-1} \tag{22}$$

2nd inequality in (18) follows from (21). Let $\lambda_{2m} = \lambda_m(\boldsymbol{W}_2)$. Using (8),

$$\begin{aligned} P_I = tr\boldsymbol{W}_2\boldsymbol{R}^* &\leq tr\boldsymbol{W}_2\boldsymbol{W}_\mu^{-1}(1 - \lambda_{11}^{-1}(\mu_1 + \mu_2\lambda_{2m})) \\ &\leq r_2(\mu_2^{-1} - \lambda_{2m}\lambda_{11}^{-1}) \end{aligned}$$

from which 2nd inequality in (19) follows. □

To proceed further, let

$$x_\epsilon = L[f(x), x_l, x_u, \epsilon] \tag{23}$$

formally denote an ϵ-accurate solution of $f(x) = 0$ given by the BA and let

$$f_1(\mu_1, \mu_2) = \mu_1(tr\boldsymbol{R}^*(\mu_1, \mu_2) - P_T) \tag{24}$$

$$f_2(\mu_1, \mu_2) = \mu_2(tr\boldsymbol{W}_2\boldsymbol{R}^*(\mu_1, \mu_2) - P_I) \tag{25}$$

where $\boldsymbol{R}^*(\mu_1, \mu_2)$ denotes $\boldsymbol{R}^*$ in (8) for *given* $\mu_1,\ \mu_2$. Then, the optimal dual variables μ_1^*, μ_2^* satisfy $f_1(\mu_1^*, \mu_2^*) = 0$ and $f_2(\mu_1^*, \mu_2^*) = 0$. For a given μ_2^*, one could use the BA to formally express μ_1^* as

$$\mu_1^* = L[f(x) = f_1(x, \mu_2^*), \mu_l, \mu_{1u}, 0] \tag{26}$$

where, from (18), $\mu_l = 0$, and likewise for μ_2^* (since the convergence of the BA is exponentially fast, the inaccuracy ϵ can be set to be arbitrary small in practice so that we disregard here this small inaccuracy by setting $\epsilon = 0$ to simplify the analysis; numerical experiments support this approach). The following proposition shows that $f_1(x, \mu_2),\ f_2(\mu_1, x)$ have the property needed for the convergence of the BA as stated above. To this end, let $P_1(\mu_1, \mu_2) = tr\boldsymbol{R}^*(\mu_1, \mu_2)$, $P_2(\mu_1, \mu_2) = tr\boldsymbol{W}_2\boldsymbol{R}^*(\mu_1, \mu_2)$, i.e. the transmit and interference powers for given μ_1, μ_2.

Proposition 2. *Let μ_{10} be a solution of $f_1(x, \mu_2) = 0$ for a given μ_2 and subject to $P_1(x, \mu_2) \leq P_T$. Then, $f_1(\mu, \mu_2) \geq 0$ for any $\mu < \mu_{10}$ and $f_1(\mu_1, \mu_2) \leq 0$ for any $\mu_1 > \mu_{10}$. Likewise, if μ_{20} is a solution of $f_2(\mu_1, x) = 0$ for a given μ_1 and subject to $P_2(\mu_1, x) \leq P_I$, then $f_2(\mu_1, \mu_2) \geq 0$ for any $\mu_2 < \mu_{20}$ and $f_2(\mu_1, \mu_2) \leq 0$ for any $\mu_2 > \mu_{20}$.*

Proof. see the full version of this paper [10]. □

Thus, this proposition shows that the BA can be used to solve $f_1(x, \mu_2) = 0$ for a given μ_2 and likewise for $f_2(\mu_1, x) = 0$. Unfortunately, neither of the optimal dual variables is known in advance. Hence, we propose the following iterative bisection algorithm (IBA) which finds optimal dual variables without such advance knowledge.

Algorithm 2 Iterative Bisection Algorithm (IBA)

Require: $f_1(\mu_1, \mu_2),\ f_2(\mu_1, \mu_2),\ \mu_{1u},\ \mu_{2u},\ \delta$
 1. Set $\mu_{20} = 0$, $k = 1$.
 repeat
  2. Set $\mu_{1k} = L[f_1(x, \mu_{2(k-1)}), 0, \mu_{1u}, \delta]$.
  3. Set $\mu_{2k} = L[f_2(\mu_{1k}, x), 0, \mu_{2u}, \delta]$.
  4. $k := k + 1$.
 until stopping criterion is met.

Note that the BA used in steps 2 and 3 will converge, as follows from Proposition 2. A possible stopping criteria for this algorithm is $|f_{1(2)}(\mu_{1k}, \mu_{2k})| \leq \epsilon$ or when a number of steps exceeds maximum k_{max}. The following proposition shows that the IBA generates converging sequences of dual variables $\{\mu_{1k}\}$, $\{\mu_{2k}\}$ under a mild technical condition; see [10] for a proof.

Proposition 3. *The sequences $\{\mu_{1k}\}_{k=1}^\infty$, $\{\mu_{2k}\}_{k=1}^\infty$ generated by the IBA above converge if $\delta = 0$ and $P_{1(2)}(\mu_1, \mu_2)$ are decreasing functions of μ_1, μ_2. In particular, this holds in any of the following cases:*

1. The IPC is inactive, in which case the IBA converges in 1 iteration.

2. $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ have the same eigenvectors.

3. $\boldsymbol{R}^*(\mu_1, \mu_2)$ is full-rank.

The following proposition shows that any stationary (and hence convergence) point of the IBA solves the dual optimality conditions in (9).

Proposition 4. *Any stationary point of the IBA is a solution of* (9) *if* $\delta = 0$*. Hence, the IBA converges to a solution of* (9) *under the conditions of Proposition 3.*

Proof. Let $\mu_{1s},\ \mu_{2s}$ be a stationary point of the IBA, so that

$$\begin{aligned} \mu_{1s} &= L[f_1(x, \mu_{2s}), 0, \mu_{1u}, 0] \\ \mu_{2s} &= L[f_2(\mu_{1s}, x), 0, \mu_{2u}, 0] \end{aligned} \tag{27}$$

It follows from 1st equality that $f_1(\mu_{1s}, \mu_{2s}) = 0$ and $f_2(\mu_{1s}, \mu_{2s}) = 0$ from 2nd one. Thus, $\mu_{1s},\ \mu_{2s}$ solves (9). Since a convergence point is stationary, it follows that the IBA converges to a solution of (9). □

While the analytical convergence results above are limited to $\delta = 0$, $\delta > 0$ is used in practice. Since the BA converges exponentially fast, very small δ can be selected in the IBA without significant increase in computational complexity of each step and hence the analysis serves as a reasonable approximation (due to the continuity of the problem and functions involved). Furthermore, numerous numerical experiments indicate that the IBA always converges, even when the conditions 1-3 of Proposition 3 are not met (we were not able to observe a single case where it did not). In the majority of the studied cases, a small to moderate number of IBA iterations (1...50) is needed to achieve a high accuracy of 10^{-5}, while up to 250 iterations are required in some exceptional cases with $\epsilon = 10^{-10}$ (which is hardly required in practice).

V. Numerical Experiments

In this section, we present some numerical results that illustrate the performance of the IBA. In 1st example, $P_I = 1$ and

$$\boldsymbol{W}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, \ \boldsymbol{W}_2 = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \tag{28}$$

Fig. 1 shows the number of iterations of the IBA required to solve (9) with the accuracy $\epsilon = 10^{-5}$ vs. P_T; the optimal dual variables $\mu_1^*,\ \mu_2^*$ as well as the actual Tx and interference powers ($P_1 = tr\boldsymbol{R}^*(\mu_1^*, \mu_2^*)$ and $P_2 = tr\boldsymbol{W}_2\boldsymbol{R}^*(\mu_1^*, \mu_2^*)$ respectively) are also shown. Note the transition from the the Tx power-limited regime (inactive IPC) to the interference-limited regime (inactive TPC) as P_T increases, which is visible when the respective dual variable sharply decreases to 0. In particular, the IPC is inactive when $P_T < 1.1$ and the TPC is inactive when $P_T > 1.8$, while both constraints are active otherwise. As P_T increases, the IPC becomes active at about $P_T \approx 1.1$, at which point the required number of iteration sharply increases from 1 to 36, gradually decreasing to a small number of 2...5. When the IPC is inactive, the number of iterations is 1, in agreement with Proposition 3. As this example demonstrates, anyone of the constraints can be inactive depending on the P_T, P_I and channel matrices. This

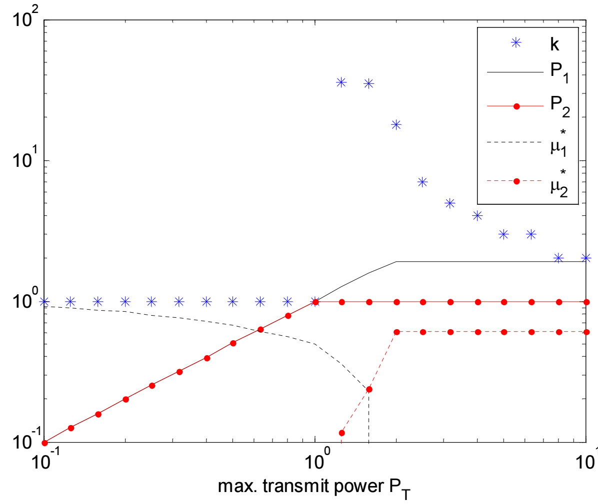

Fig. 1. Convergence of the IBA, i.e. the number k of iterations required to achieve $\epsilon = 10^{-5}$ vs. P_T; $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are as in (28), $P_I = 1$. $P_1, P_2, \mu_1^*, \mu_2^*$ are also shown.

changes if $\boldsymbol{W}_2$ is rank-deficient since the TPC is always active in that case.

It should also be noted that the optimal covariance $\boldsymbol{R}^*$ is not diagonal, even thoung $\boldsymbol{W}_1$ is, when the IPC is active - a sharp distinction to the TPC constraint only, where $\boldsymbol{R}^*$ and $\boldsymbol{W}_1$ have the same eigenvectors so that diagonal $\boldsymbol{W}_1$ implies diagonal $\boldsymbol{R}^*$. Hence, introducing the IPC makes independent signaling sub-optimal for independent channels in general (unless $\boldsymbol{W}_2$ is also diagonal or if the IPC is inactive).

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