On Optimal Detection Ordering for Coded V-BLAST

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Abstract—**Optimum ordering strategies for the coded V-BLAST with capacity achieving temporal codes on each stream are analytically studied in this paper, including optimal power and/or rate allocations among data streams. A compact closed-form solution is obtained for the case of two transmit antennas and necessary optimality conditions are found for the general case. The optimal rate allocation is shown to have a major impact (stronger streams are detected last) while the optimal power allocation does not alter the original Foschini ordering (stronger streams are detected first). A sub-optimal ordering is proposed based on the necessary optimality conditions, which performs very close to the optimal one but has much smaller computational complexity. An SNR gain of ordering is introduced and studied. All the results also apply to a multiple-access channel under successive interference cancelation.**

I. INTRODUCTION

The multiple-input multiple-output (MIMO) communication architecture has been widely adopted by the academia and industry due to its high spectral efficiency unattainable by conventional techniques [1]. To reduce its processing complexity, Vertical Bell Labs Layered Space-Time (V-BLAST) was proposed by Foschini [2] as a low-complexity architecture that is able to achieve a substantial portion of the total MIMO channel capacity given that the multipath environment is rich enough and capacity-approaching temporal codes (e.g. LDPC, turbo or polar codes) are used for each data stream. Its key steps are interference cancelation from already detected symbols (i.e. successive interference cancelation (SIC)), interference nulling from yet-to-be-detected symbols (either zero-forcing or MMSE), and an optimal ordering of the detection sequence (to optimize the performance). Note that the V-BLAST architecture requires less complexity at the transmitter and also less feedback as compared to the SVD-based transmission, and the complexity burden is essentially shifted towards the receiver[1].

While its analysis becomes feasible without the optimal ordering procedure [3], the latter posses significant problem for the analysis and only the two Tx antennas case has been fully settled [4][5]. Thus, unordered V-BLAST became popular, also because its smaller complexity. Since its performance may be not satisfactory in some cases, various optimization techniques have been proposed (e.g. optimal power and/or rate allocation among data streams) [7]-[10], which can be considered as an alternative to a computationally-demanding optimal ordering procedure. Indeed, the optimal ordering requires $m!$ orderings to be compared in the general case, where m is the number of Tx antennas, which can be prohibitively complex for large m and real-time implementation. Thus, various sub-optimal orderings have been proposed [5][8].

While Foschini et al [2] has found the optimal ordering for the essentially uncoded system and uniform power/rate allocation (stronger streams are detected first), no analytical solution is known to date for coded V-BLAST (also including optimal power/rate allocation) and the only remaining option is a brute force approach (comparing all $m!$ orderings numerically). In the present paper, we partially settle this issue by providing an analytical solution for the (most practical) $m = 2$ case of coded V-BLAST, including optimal power and/or rate allocations. In the general case of $m > 2$, we provide compact necessary optimality conditions, which depend on the channel matrix only (SNR and other system parameters-independent) and can be used to rule out most of the possible $m!$ combinations so that the brute-force approach can be applied to a much-smaller set and thus becomes practically-feasible. These conditions provide a number of insights into the optimal ordering procedure and its properties which cannot be obtained numerically. Based on the necessary optimality conditions, we propose a sub-optimal ordering which performs very close to the optimal one and yet has much smaller computational complexity. The suboptimal and optimal orderings coincide for $m = 2$. To quantify the impact of optimal ordering, an SNR gain of ordering is introduced and studied.

The major insight from this study is that the optimal rate allocation among data streams has a much more pronounced impact on the optimal ordering (stronger streams are detected last) as opposed to the optimal power allocation, which does not alter the original Foschini ordering (stronger streams are detected first), regardless of whether temporal coding is used or not.

Finally, we mention that the V-BLAST system architecture naturally represents the multiple-access channel (MAC) (i.e. an uplink of a cellular system) under successive interference cancelation so that all our results (including optimal user detection order) also apply to such setting, where multiple Tx antennas represent different users.

II. SYSTEM MODEL

The standard discrete-time MIMO channel model is

$$\mathbf{r} = \mathbf{H}\Lambda\mathbf{q} + \xi = \sum\nolimits_{i=1}^{m} \mathbf{h}_i \sqrt{\alpha_i} q_i + \xi_i \tag{1}$$

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where $\mathbf{q} = [q_1, q_2, ..., q_m]^T$ and $\mathbf{r} = [r_1, r_2, ..., r_n]^T$ are the transmitted and received signal vectors respectively, $\mathbf{H} = [\mathbf{h}_1..\mathbf{h}_m]$ is the $n \times m$ channel matrix (n Rx and m Tx antennas, $n \geq m$) representing the complex channel gains from each transmit to each receive antenna, and $\mathbf{h}_i$ is its i-th column. $\mathbf{H}$ is assumed to be constant for a sufficiently long period of time so that the standard infinite-horizon information theory assumption holds; ξ is the circularly symmetric additive white Gaussian noise vector with i.i.d. entries i.e. $\xi \sim CN(0, \sigma_0^2\mathbf{I})$. Λ is a diagonal matrix whose entries are $\sqrt{\alpha_i}$ and α_i represents the normalized power allocation to i-th stream.

After the interference cancellation and nulling steps, for a given ordering, the equivalent scalar channel of the i-th stream is [9][10],

$$r_i = |\mathbf{h}_{i\perp}| \sqrt{\alpha_i} q_i + \xi_i \quad (2)$$

where r_i is the i-th component of $\mathbf{r}$, $\mathbf{h}_{i\perp}$ is the projection of $\mathbf{h}_i$ onto the sub-space orthogonal to that spanned by yet-to-be-detected streams , i.e. $\mathbf{h}_{i\perp} \perp \{\mathbf{h}_{i+1}, ..., \mathbf{h}_m\}$, $|\mathbf{h}|$ is the Euclidean norm (length) of vector $\mathbf{h}$. Assuming that each stream employs a capacity-achieving temporal code (this models well practical codes operating very close to the capacity, e.g. LDPC, turbo or polar codes [11]), this stream can support a target rate R up to its instantaneous capacity given by

$$C_i = \ln(1 + |\mathbf{h}_{i\perp}|^2 \alpha_i \gamma_0) \text{ [nat/s/Hz]} \quad (3)$$

where $\gamma_0 = 1/\sigma_0^2$ is the average SNR at each Rx antenna. The total system capacity C (this includes the channel as well as the transmission and reception strategy) depends on the power and rate allocation strategy [9][10].

For simplicity of exposition , we consider first the uniform power allocation, $\alpha_i = 1$. When the uniform rate/power allocation (URA) is used, i.e. all streams transmit at the same target rate, the system capacity is limited by the weakest stream so that

$$C_{URA} = m \min_i C_i = \ln\left(1 + \min_i |\mathbf{h}_{i\perp}|^2 \gamma_0\right) \quad (4)$$

When the optimal instantaneous rate allocation (IRA) is used, i.e. the rate of each stream is adjusted to match its capacity C_i, the system capacity is

$$C_{IRA} = \sum_{i=1}^{m} C_i \quad (5)$$

This two strategies can be further combined with the instantaneous power allocation to maximize the system capacity [10].

We note that this system model also applies to a multiple-access channel (MAC), where different streams represent difference users (i.e. an uplink of a cellular system).

To further improve the system performance, the stream detection order can be optimized to maximize the system capacity. Let $\pi = \{k_1, k_2, ..., k_m\}$ represents the detection order where stream k_1 is detected first etc. All the capacities above then become the functions of the detection order. Changing the detection order is equivalent to swapping the columns of the channel matrix $\mathbf{H}$ so that the re-ordered matrix is $\mathbf{H}_\pi = [\mathbf{h}_{k_1}..\mathbf{h}_{k_m}]$

Below, we consider an optimal ordering strategy for each of the power/rate allocation strategies. It turns out that it is the rate allocation strategy that affects the optimal detection ordering most. To make the analysis tractable, we consider first the case of 2 Tx antennas, and generalize the results later to the $m > 2$ case.

III. Optimum Ordering Under the IRA

Under the IRA, the per-stream rates are adjusted to match the per-stream capacities with uniform power allocation. The optimum detection ordering maximizes the instantaneous sum capacity of the system,

$$\pi^* = \arg\max_\pi C_{IRA}(\pi) = \arg\max_\pi \sum\nolimits_{i=1}^{m} C_i(\pi) \quad (6)$$

where $C(\pi)$ and $C_i(\pi) = \ln(1 + |\mathbf{h}_{k_i\perp}|^2 \gamma_0)$ are the total system capacity and the per-stream capacity as functions of the detection ordering π, and $\mathbf{h}_{k_i\perp}$ is the projection of $\mathbf{h}_{k_i}$ orthogonal to $\{\mathbf{h}_{k_{i+1}}..\mathbf{h}_{k_m}\}$.

For $m = 2$, the optimum detection order is as follows.

Proposition 1: The optimum detection ordering for the coded V-BLAST with two Tx and n Rx antennas under the IRA is to detect the strongest stream (with highest unprojected channel gain) last,

$$\pi^* = \arg\max_\pi \sum\nolimits_{i=1}^{2} C_i(\pi) = \{1, 2\} \text{ iff } |\mathbf{h}_1| \leq |\mathbf{h}_2| \quad (7)$$

The "only if" part in (7) holds when $\mathbf{h}_1$, $\mathbf{h}_2$ are not orthogonal, $\phi \neq \pi/2$, where ϕ is the angle between them. When $\phi = \pi/2$ and/or $|\mathbf{h}_1| = |\mathbf{h}_2|$, any ordering delivers the same system capacity.

Proof: First observe that when $\mathbf{h}_1$, $\mathbf{h}_2$ are orthogonal or/and of equal length, any ordering delivers the same system capacity. Therefore, we have to consider only the case when $\phi \neq \pi/2$ and $|\mathbf{h}_1| \neq |\mathbf{h}_2|$. Assuming $|\mathbf{h}_1| < |\mathbf{h}_2|$, it is straightforward to see, after some manipulations, that $\pi^* = \{1, 2\}$ by comparing it with $\pi = \{2, 1\}$. The "only if" part is proved in the same way. ■

Note that this ordering is opposite of that of the uncoded V-BLAST [2][4], which detects the strongest stream first. It is also SNR and other system parameters-independent, since it is based on the channel matrix only. Unfortunately, as numerical observations indicate, this independence does not hold anymore for larger systems ($m > 2$), where, in general, the optimal ordering is SNR-dependent.

However, using the same reasoning as in Proposition 1, a necessary optimality condition can be formulated for any m.

Proposition 2: Given that $\mathbf{h}_{k_{i-1}\perp}$ and $\mathbf{h}_{k_i\perp}$ are non-orthogonal to each other, an optimum channel ordering $\pi^* = \{k_1, k_2, ..., k_m\}$ must satisfy the following necessary conditions:

$$|\mathbf{h}_{k_{i-1}\perp}| \leq |\mathbf{h}_{k_i\perp}| \;\; \forall\; 2 \leq i \leq m \quad (8)$$

where $\mathbf{h}_{k_{i-1}\perp}$ and $\mathbf{h}_{k_i\perp}$ are the projections of vectors $\mathbf{h}_{k_{i-1}}$ and $\mathbf{h}_{k_i}$ orthogonal to the sub-space spanned by $\{\mathbf{h}_{k_{i+1}}, ...\mathbf{h}_{k_m}\}$. If some $\mathbf{h}_{k_{i-1}\perp}$ and $\mathbf{h}_{k_i\perp}$ are of equal length and/or orthogonal to each other, any ordering among them is optimum. ■

Three important conclusions follow from the necessary optimality conditions:

* Given that all $\mathbf{h}_{k_i\perp}$ are of different length and non-orthogonal to each other, swapping two consecutive columns for a given order that meets the necessary optimality conditions results in a lower system capacity.

* A channel matrix under the optimum detection ordering will never contain the column with minimum norm at the last position.

* A channel matrix under the optimum detection ordering will never contain the column with maximum norm at the second last position.

IV. Optimum Ordering Under the URA

The coded V-BLAST with uniform power and rate allocation among the data streams may be used to simplify the system design. Since its system capacity is dominated by the weakest stream [10], the optimum ordering is

$$\pi^* = \arg\max_{\pi} \min_{i} C_i(\pi) = \arg\max_{\pi} \min_{i} |\mathbf{h}_{k_i\perp}(\pi)|^2 \quad (9)$$

i.e. maximizes the weakest after-projection stream.

In the case of $m = 2$, this can be evaluated explicitly.

Proposition 3: The optimum detection ordering for the coded V-BLAST with two Tx and n Rx antennas under the URA is to detect the strongest (before-projection) stream first,

$$\begin{aligned} \pi^* &= \arg\max_{\pi} \min_{i} C_i(\pi) \\ &= \{1,2\} \text{ iff } |\mathbf{h}_1| \geq |\mathbf{h}_2| \text{ for } \forall\phi \neq 0 \end{aligned} \quad (10)$$

The “only if” part in (10) holds if $\phi \neq \pi/2$. If $\phi = \pi/2$ and/or $|\mathbf{h}_1| = |\mathbf{h}_2|$, any ordering delivers the same capacity. If $\phi = 0$, the system capacity is zero. ■

Note that this is in fact the Foschini ordering. Hence, unlike the IRA, the power allocation strategy has no impact on the optimal ordering, even when coding is used.

V. Optimum ordering under the IPA

Let us now consider the optimal instantaneous power allocation (IPA) under the uniform rate allocation (e.g. different streams make use of the same code/modulation format). From [10], the system capacity under the IPA for a given ordering π is given by

$$C_{IPA}(\pi) = m\ln(1 + \overline{g}(\pi)\gamma_0) \text{ if } |\mathbf{h}_{i\perp}(\pi)| > 0 \ \forall i \quad (11)$$

and 0 otherwise, where $\overline{g}(\pi)$ is the harmonic mean per-stream power gain for a given ordering,

$$\overline{g}(\pi) = \left(\frac{1}{m}\sum_i |\mathbf{h}_{i\perp}(\pi)|^{-2}\right)^{-1} \quad (12)$$

so that the optimum ordering is to maximize the harmonic mean gain,

$$\pi^* = \arg\max_{\pi} \overline{g}(\pi) \quad (13)$$

Note that this holds for any m and is SNR-independent, as opposed to the case of the IRA. For $m = 2$, one obtains:

Proposition 4: The optimum detection ordering for the coded V-BLAST with two Tx and n Rx antennas under the IPA (and uniform rate allocation) is to detect the strongest stream first (i.e. the Foschini ordering),

$$\pi^* = \arg\max_{\pi} \overline{g}(\pi) = \{1,2\} \text{ iff } |\mathbf{h}_1| \geq |\mathbf{h}_2| \ \forall\phi \neq 0 \quad (14)$$

The “only if” part in (14) holds when $\phi \neq \pi/2$. If $\phi = \pi/2$ and/or $|\mathbf{h}_1| = |\mathbf{h}_2|$, any ordering delivers the same capacity.

Proof: follows by comparing the harmonic mean gains for the two orderings. ■

Observe that this ordering is the same as for the URA under the uniform power allocation. Thus, we conclude that, for $m = 2$, power allocation does not affect the ordering, only the rate allocation does.

VI. Optimum Ordering Under the IPRA

It was demonstrated in [10] that, for a given ordering, the well-known water-filling (WF) algorithm does not maximize (in general) the system capacity of the coded V-BLAST via optimum power/rate allocation (IPRA) (due to the successive interference cancellation) and a new algorithm was proposed, the fractional water-filling (FWF), which does so. Extensive numerical simulations show that both algorithms dictate the same optimal ordering. Since the WF is more amendable to the analysis, we proceed with it in this section. The optimal ordering can be formulated as follows:

$$\pi^* = \arg\max_{\pi} \sum_i \ln(1 + \alpha_i^*(\pi)|\mathbf{h}_{k_i\perp}|^2\gamma_0) \quad (15)$$

where the optimum power allocation $\alpha_i^*(\pi)$ is given by the WF algorithm,

$$\alpha_i^*(\pi) = \left[\mu(\pi) - \frac{1}{\gamma_0|\mathbf{h}_{k_i\perp}|^2}\right]_+ \quad (16)$$

where $[x]_+ = \max\{x, 0\}$, $\mu(\pi)$ is the water level for a given order π and is calculated from the total power constraint. In the general case (any m), the problem is difficult due to the fact that different ordering may result in different number of active streams. However, if $m = 2$, either one or two streams are active and the analysis becomes feasible. According to the number of active streams, we consider three different SNR regimes, exploiting well-known property of the WF algorithm: while all streams are active at high SNR, only one is active at low SNR.

- *Low SNR regime:* Both orderings have one active stream,

$$\gamma_0 \leq \frac{1}{2}\left|\frac{1}{g_1} - \frac{1}{g_2\beta}\right| \quad (17)$$

where $g_i = |\mathbf{h}_i|^2$ and $\sin^2\phi = \beta$, and we assume, without loss of generality, that $g_1 \leq g_2$.

- *High SNR regime:* Both orderings have two active streams.

$$\gamma_0 > \frac{1}{2}\left(\frac{1}{g_1\beta} - \frac{1}{g_2}\right) \quad (18)$$

- *Intermediate SNR regime:* The optimum ordering has one active stream and the suboptimum has two active streams when the SNR is between the bounds in (17) and (18).

The case of one active stream for the sub-optimal two active streams for the optimal ordering never takes place. Regardless of the SNR regime, the optimal ordering can be characterized as follows.

Proposition 5: The optimum detection ordering for the coded V-BLAST with two Tx and n Rx antennas under the IPRA via the WF is to detect the strongest stream last,

$$\pi^* = \arg\max_{\pi} \sum_{i=1}^{2} \ln(1 + \alpha_i^*(\pi)|\mathbf{h}_{k_i\perp}|^2\gamma_0)$$
$$= \{1, 2\} \text{ iff } |\mathbf{h}_1| \le |\mathbf{h}_2| \quad (19)$$

The "only if" part in (19) holds when $\phi \neq \pi/2$. If $\phi = \pi/2$ and/or $|\mathbf{h}_1| = |\mathbf{h}_2|$, any ordering delivers the same system capacity.

Proof: Comparison of the system capacities in the three different SNR regimes above under the two ordering results, after some lengthy manipulations, in this ordering. ■

It is a remarkable fact that, whether uniform or optimal power allocation is used, optimal rate allocation always results in the inverse ordering as in (19). This re-enforces our earlier conclusion that it is the rate allocation that is critical for optimal ordering, with power allocation playing no significant role. This conclusion is especially important for the MAC channel, where different users are likely to have different rates.

VII. SNR Gain of Ordering

To quantify the impact of optimal ordering, we introduce an SNR gain of ordering, which compares the optimally-ordered and unordered systems. We focus on the analytically-tractable case of two Tx antennas and the IPRA via the WF.

The SNR gain G of ordering is defined as the difference in SNR required by the unordered V-BLAST to achieve the same capacity as the optimally ordered i.e.

$$C_{\pi*}(\gamma_0) = C(G\gamma_0) \quad (20)$$

where $C_{\pi*}(\gamma_0)$ and $C(G\gamma_0)$ are the system capacities with and without optimal ordering.

As in previous sections, the analysis for two Tx antennas will be divided into three SNR regimes: low, intermediate and high. Without loss of generality and following (19), we assume that $g_1 \le g_2$.

Proposition 6: The SNR gain of the optimum ordering procedure in the low SNR regime as in (17) is given by:

$$G = \min\left[\frac{1}{\beta}, \frac{g_2}{g_1}\right], \quad (21)$$

at high SNR as in (18) by

$$G = 1 + \frac{(1-\beta)(g_2 - g_1)}{2g_1 g_2 \beta \gamma_0} \quad (22)$$

and at intermediate SNR by

$$G = \frac{1}{\gamma_0}\left(\sqrt{\frac{1 + 2g_2\gamma_0}{g_1 g_2 \beta}} - \frac{g_2\beta + g_1}{2g_1 g_2 \beta}\right) \quad (23)$$

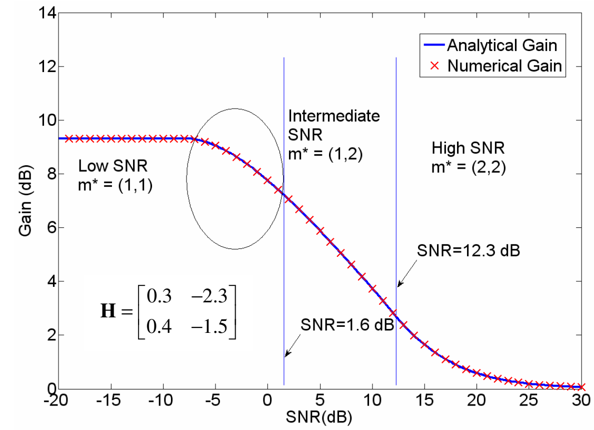

Fig. 1. The SNR gain of ordering vs. SNR (numerical and analytical) for the 2×2 system and given $\mathbf{H}$; m^* is the number of active streams for both orderings. The low and intermediate SNR regimes are the largest beneficiaries.

Proof: Follows from the definition (20) after some manipulations and using the optimal ordering in (19). ■

The SNR gain of ordering is illustrated in Fig. 1. Some conclusions follow from Proposition 6:

$*$ If $g_1 = g_2$ (the per-stream SNRs are equal) and/or $\beta = 1$ ($\mathbf{h}_1$ and $\mathbf{h}_2$ are orthogonal), there is no gain (both orderings offer the same capacity) at any SNR.

$*$ In the low SNR regime, the gain is SNR-independent, and it is an increasing function of g_2/g_1 and decreasing in β.

$*$ In the high SNR regime and for fixed g_1, g_2 and β, G is decreasing in SNR. For fixed g_1, g_2 and γ_0, it is decreasing in β. For fixed g_2, β and γ_0, it is decreasing in g_1.

Based on the gain behavior in the low and high SNR regimes above, we arrive at the following conjecture.

Conjecture: At any SNR, G is bounded as follows,

$$1 \le G \le \min\left[\frac{1}{\beta}, \frac{g_2}{g_1}\right] \quad (24)$$

Note: This conjecture is suggested by the low and high SNR regimes. However, since $G(\gamma)$ may exhibit a non-monotonic behavior in the intermediate SNR regime, we do not have a complete proof of this result at the moment.

Fig. 1 shows the SNR gain of ordering (numerical and analytical) vs. SNR for the 2×2 coded V-BLAST system under the IPRA (via WF) for a fixed channel realization $\mathbf{H}$. Note that the SNR gain is a decreasing function of the SNR, so that there is no much advantage from the optimal ordering at high SNR. In the low SNR regime, the SNR gain is highest and is SNR-independent when both ordering employ only 1 active stream. In the intermediate SNR regime, the gain decreases with the SNR but is still considerable, while it becomes low at high SNR. Thus, we conclude that the major advantage of the optimal ordering is at low SNR, i.e. precisely when it is needed. It can be further proved that $G \to 1$ as $\gamma_0 \to \infty$ for any m.

VIII. An Inverse Ordering

Since optimal ordering can be computationally demanding for $m > 2$, we consider a suboptimal ordering that perform very close to the optimum one (either under IRA or IPRA) in this section.

A key idea for a sub-optimal ordering is to satisfy the necessary optimality conditions in Proposition 2 by following the same principle as in the Foschini ordering in [2] but in the inverse direction, i.e. strongest streams are detected last (while in the Foschini ordering, strongest streams are detected first), which we term "inverse ordering". The algorithm is as follows:

1) Select the largest $|\mathbf{h}_i|$; the corresponding stream is detected last: $k_m = \arg\max_i |\mathbf{h}_i|$.
2) Select the second largest $|\mathbf{h}_{i\perp k_m}|$; the corresponding stream is detected second last: $k_{m-1} = \arg\max_i |\mathbf{h}_{i\perp k_m}|$.
3) Repeat step 2 until finish (always projecting orthogonally to already selected streams). The sub-optimal ordering is $\pi = \{k_1...k_m\}$.

Note that this ordering always satisfies the necessary optimality condition in Proposition 2 (and hence "sub-optimal"). The inverse ordering is a SNR-independent strategy (since it is based on $\mathbf{H}$ only) and thus cannot be optimal in general, since, from numerical experiments, the optimum detection ordering is SNR-dependent for $m \geq 3$. However, as illustrated in Fig.2-3, this ordering performs very close to the optimum one either under the IRA or the IPRA. Furthermore, its computational complexity is greatly reduced compared to the optimal ordering: while the latter compares all $m!$ possible orderings, the former compares only $m(m+1)/2-1$ orderings, most of which are in sub-spaces of reduced dimension ($< m$), i.e. a significant advantage for large m.

Fig. 2 compares various ordering strategies in terms of the outage probability in i.i.d. Rayleigh-fading channel. The optimality of Foschini ordering under the URA is clearly observed. It can also be seen that the inverse ordering under the IRA is almost optimum. Finally, the better performance of the ordered systems as compared to the unordered detection is evident.

The performance of the inverse and Foschini orderings are evaluated under the WF and the IPA respectively in Fig. 3. It can be seen that both orderings are almost optimum for each respective case.

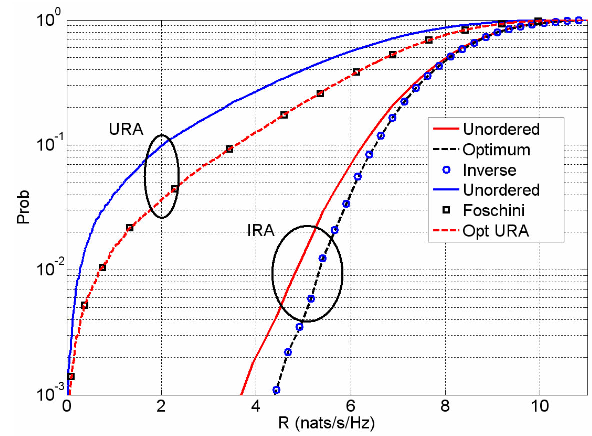

Fig. 2. Empirical outage probability of the 3×3 unoptimized (URA) system and under the IRA, with optimal and sub-optimal orderings; SNR=10dB; 10^4 channel realizations.

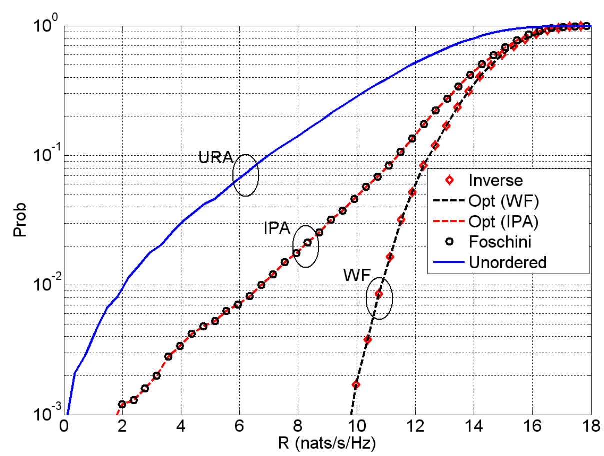

Fig. 3. Empirical outage probability of the 3×3 unoptimized (URA), unordered system compared to the IPA under Foschini and optimal orderings, and the WF under the inverse and optimum orderings; SNR=20dB; 10^4 channel realizations.

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