Performance Analysis of Coded V-BLAST with Optimum Power and Rate Allocation

Victoria Kostina, Sergey Loyka

***Abstract*—Several optimization strategies for instantaneous rate and/or power allocation in the coded V-BLAST are studied analytically. Outage probabilities and system capacities of these strategies in a spatial multiplexing system are compared under generic settings. Since the conventional waterfilling algorithm is suboptimal for the coded V-BLAST, a recently-proposed ”fractional waterfilling” algorithm is studied, which simultaneously maximizes the system capacity and minimizes the outage probability. A comparative, closed-form performance analysis of this and other algorithms is presented, including bounds on the outage probability and its low-outage approximations. The fractional waterfilling algorithm attains the full MIMO channel diversity and outperforms the other algorithms by a wide margin.**

I. Introduction

V-BLAST algorithm has received a significant attention as a reasonable-complexity way to approach high spectral efficiencies promised by information-theoretic studies of MIMO channels [1]. Since it suffers from the error propagation effect, a number of efforts have been reported to improve the performance of the uncoded V-BLAST using adaptive power and/or rate allocation techniques [2]-[5]. Uncoded systems, however, are rare and most practical systems are coded. This motivates the study of coded V-BLAST. While the error rate analysis of coded systems is a formidable task hardly possible in a closed-from (except for some special cases), the analysis becomes feasible when powerful capacity-approaching codes are used (e.g. LDPC, turbo-codes or polar codes) [6][7]. Following this approach, we assume here that capacity-achieving temporal codes are used for each stream in the V-BLAST, so that the per-stream rates are set equal to the corresponding capacities and there are no errors when streams are not in outage, and also no error propagation in-between the streams.

This approach has been used by Zhang and Cioffi [9], who considered an instantaneous optimization of power, rate and antenna mapping for the coded zero-forcing (ZF) V-BLAST to minimize the total transmit power for given data rate under zero outage constraint, assuming capacity-achieving temporal codes or realistic ones via an SNR gap to capacity. The optimization in [9] is carried out under zero outage constraint, which requires unlimited power investment into particularly bad channel realizations (to support the target rate) and is not feasible when the peak power is constrained (i.e. by an RF power amplifier). Unlike [9], we allow non-zero outage probability and minimize it by proper power/rate allocation, under the total instantaneous power constraint, which automatically constraints the peak power as well. While [9] makes use of the conventional waterfilling (WF) algorithm to assign powers and rates, this algorithm does not achieve the maximum V-BLAST system capacity[1] (because of the successive interference cancelation) so that we consider here a recently-proposed ”fractional waterfilling” (FWF) algorithm that does maximize the system capacity [8]. While the complexity of the new FWF algorithm is higher compared to the conventional WF, its incremental complexity is small when the number of transmitters is not too large, as in realistic MIMO systems. Since [8] did not study the performance of the FWF, the present paper concentrates on its closed-form performance analysis, also in comparison to other known algorithms. To the best of our knowledge, it is the first time when the outage performance of waterfilling-type algorithms is presented in a compact, closed form. In particular, this paper shows that the FWF is superior to the conventional WF in terms of the outage probability when applied to the coded ZF V-BLAST; additionally, the FWF significantly outperforms the WF at low SNR for particularly bad channel realizations. We also consider instantaneous (per-stream) rate allocation (IRA) and joint instantaneous power/rate allocation (IPRA) to minimize the outage probability of the coded ZF V-BLAST under the constrained total transmit power and a given target rate. Since the total transmit power is limited, the peak power of RF amplifiers is limited as well.

Section II introduces the system model. In Section III, we briefly summarize the relevant prior results that are used in the following analysis, including a description of the fractional water-filling algorithm. A closed-form comparative performance analysis of the IRA, the WF and the FWF is presented in section IV. In the low outage regime, the WF achieves the same diversity gain as the IRA, while the FWF brings in an additional diversity gain, achieving the full MIMO channel diversity (Corollary 2) and attaining simultaneously the minimum possible outage probability and the maximum system capacity under the total power constraint. In the low rate (also known as wideband) regime, the outage probabilities of all the instantaneous optimization strategies discussed in the paper are found in explicit, closed form (Theorem 3). Section V gives an example to compare the average and instantaneous optimization, to demonstrate the superiority of the FWF and to

V. Kostina is with the Department of Electrical Engineering, Princeton University, NJ, 08544, USA, e-mail: vkostina@princeton.edu.

S. Loyka is with the School of Information Technology and Engineering, University of Ottawa, Ontario, Canada, K1N 6N5, e-mail: sergey.loyka@ieee.org.

[1] The system capacity is the capacity of an extended channel, which includes the channel itself and also the V-BLAST transmission/processing architecture.

validate the analytical results and conclusions via simulations.

Due to similar processing strategies, most of these results also apply to multiuser detection and inter-symbols equalization systems that use successive interference cancelation.

II. System Model

Motivated by lower complexity and to make the analysis feasible, we consider unordered ZF V-BLAST. The standard baseband discrete-time MIMO system model is used [5],

$$\mathbf{r} = \mathbf{H\Lambda s} + \boldsymbol{\xi} = \sum\nolimits_{i=1}^{m} \mathbf{h}_i \sqrt{\alpha_i} s_i + \boldsymbol{\xi} \tag{1}$$

where $\mathbf{s} = [s_1, s_2, ...s_m]^T$ and $\mathbf{r} = [r_1, r_2, ...r_n]^T$ are the vectors representing the Tx and Rx symbols respectively, "T" denotes transposition, $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, ...\mathbf{h}_m]$ is the $n \times m$ matrix of the complex channel gains between each Tx and each Rx antenna, where $\mathbf{h}_i$ denotes i-th column of $\mathbf{H}$, n and m are the numbers of Rx and Tx antennas respectively, $n \geq m$, $\boldsymbol{\xi}$ is the vector of circularly-symmetric additive white Gaussian noise (AWGN), which is independent and identically distributed (i.i.d.) in each receiver, $\boldsymbol{\Lambda} = diag\left(\sqrt{\alpha_1}, \ldots, \sqrt{\alpha_m}\right)$, where α_i is the power allocated to the i-th transmitter (stream). For the unoptimized V-BLAST, the total power is distributed uniformly among the transmitters, $\alpha_1 = \alpha_2 = ... = \alpha_m = 1$. The channel will be assumed to be either ergodic, in which case the key performance measure is the ergodic system capacity, or non-ergodic (quasi-static block-fading fading), in which case the key performance measures are the outage probability and the outage capacity and also the instantaneous system capacity (for given channel realization) [7]. Details of a mathematical model of the uncoded V-BLAST, on which our model of the coded V-BLAST is based, and its analysis can be found in [2][5][10] and are omitted here.

III. Prior Results

Below, we briefly summarize the relevant prior results from [8] to set the stage for further analysis in Section IV.

A. Instantaneous vs. Average Optimization

Let us consider a generic spatial multiplexing system (not only V-BLAST) operating in a fading channel of generic statistics (not only i.i.d. Rayleigh), which is quasi-static (non-ergodic or "slow block fading"). The main performance indicator in this setting is the system outage probability, i.e. the probability that the system cannot support a target total rate mR [6][7],

$$P_{out} = \Pr\{C < mR\} \tag{2}$$

where C is the instantaneous (i.e. for given channel realization) system capacity (i.e. the sum of per-stream capacities), and an optimization strategy should target this measure. On the other hand, when the channel is ergodic, the mean (ergodic) capacity $\overline{C}$ is an appropriate performance measure and its optimization is of interest. In both scenarios, the optimization can be instantaneous (i.e. for each channel realization) or average (i.e. based on the channel statistics), and may include per-stream power, rate or joint power/rate allocation.

Let us compare the performance of four different optimization strategies: average power and/or rate allocation $\overline{\boldsymbol{\alpha}}_C$ to maximize the mean capacity, average power/rate allocation $\overline{\boldsymbol{\alpha}}_{out}$ to minimize the outage probability, instantaneous power/rate allocation $\boldsymbol{\alpha}_C$ to maximize the instantaneous system capacity, instantaneous power/rate allocation $\boldsymbol{\alpha}_{out}$ to minimize the outage probability, all subject to the total power $\sum_{i=1}^{m} \alpha_i = m$ constraint,

$$\overline{\boldsymbol{\alpha}}_C = \arg\max\nolimits_{\boldsymbol{\alpha}(\gamma_0)} \overline{C}(\boldsymbol{\alpha}), \tag{3}$$

$$\overline{\boldsymbol{\alpha}}_{out} = \arg\min\nolimits_{\boldsymbol{\alpha}(\gamma_0)} P_{out}(\boldsymbol{\alpha}), \tag{4}$$

$$\boldsymbol{\alpha}_C = \arg\max\nolimits_{\boldsymbol{\alpha}(\gamma_0, \mathbf{H})} C(\boldsymbol{\alpha}), \tag{5}$$

$$\boldsymbol{\alpha}_{out} = \arg\min\nolimits_{\boldsymbol{\alpha}(\gamma_0, \mathbf{H})} P_{out}(\boldsymbol{\alpha}), \tag{6}$$

where C, $\overline{C}$ and P_{out} are considered as functions of the power and/or rate allocation $\boldsymbol{\alpha}$ [2], and γ_0 is the average SNR. These optimization strategies can be ordered as follows [8].

Theorem 1. *The outage probabilities of the optimization strategies in* (3)-(6) *are ordered as follows,*

$$\begin{aligned} \Pr\{C(\overline{\boldsymbol{\alpha}}_C) < mR\} &\geq \Pr\{C(\overline{\boldsymbol{\alpha}}_{out}) < mR\} && (7)\\ &\geq \Pr\{C(\boldsymbol{\alpha}_{out}) < mR\} = P_{out}^{\star} && (8)\\ &= \Pr\{C(\boldsymbol{\alpha}_C) < mR\} && (9) \end{aligned}$$

i.e. the instantaneous optimizations of the capacity and outage probability achieve the same lowest outage probability $P_{out}^{\star}$, the average optimization of the outage probability gives an intermediate result, and the average optimization of the ergodic capacity is the worst.

We further remark that under an average (or fixed) rate allocation, the inequality in (8) becomes equality, i.e. instantaneous power allocation is not better than the average one. The importance of (9) in Theorem 1 is due to the fact that while the problem in (6) is non-convex, has multiple solutions (see [8] for examples) and difficult to deal with in general, either numerically or analytically, the problem in (5) has a well-known solution via the waterfilling (when no successive interference cancelation is used at the receiver) or via the fractional waterfilling for the coded V-BLAST (see Section III-C).

B. Instantaneous Rate Allocation (IRA)

Let us consider the optimum instantaneous rate allocation (IRA) with the uniform power allocation, $\alpha_i = 1$, across all streams of the coded V-BLAST. With the uniform rate allocation, i.e. when the per-stream target rate is R, the system outage takes place if any of the streams is not able to support this rate, i.e. if the capacity of at least one stream is lower than the target rate,

$$P_{out}^{u} = 1 - \prod_{i=1}^{m} (1 - \Pr\{C_i < R\}); \tag{10}$$

[2] to simplify the notations, rate allocation is also incuded in $\boldsymbol{\alpha}$ here.

where $C_i = \ln(1 + g_i\gamma_0)$ is the instantaneous capacity of i-th stream in [nat/s/Hz], $g_i = |\mathbf{h}_{i\perp}|^2$ is i-th stream power gain, $\mathbf{h}_{i\perp}$ is i-th column of the channel matrix projected onto the subspace orthogonal to $\mathtt{span}\{\mathbf{h}_{i+1}, \dots, \mathbf{h}_m\}$; we also used the fact that in the i.i.d. Rayleigh fading channels different g_i are independent of each other [2][10].

When the IRA is employed and capacity-achieving codes are used for each stream, the per-stream rates are set equal to the corresponding per-stream instantaneous capacities C_i. The system outage probability is then given by

$$P_{out}^{IRA} = \Pr\left\{\sum_i C_i < mR\right\}, \tag{11}$$

i.e. the outage takes place only when the sum capacity is below the target rate mR.

C. Joint Instantaneous Power/Rate Allocation (IPRA)

The IPRA solves the problem in (5), which also solves (6). The key observation here is that, contrary to what one would expect [9], the conventional waterfilling algorithm (WF) does not provide an optimal solution to (5). Indeed, an implicit assumption behind the conventional WF is that the channel gains do not depend on the allocated power. This is not so for the V-BLAST because the SIC forces the equivalent channel gains $g_i = |\mathbf{h}_{i\perp}|^2$ to depend on the allocated powers, albeit in a binary way: if some transmitters are not active ($\alpha_i = 0$), there is no need to project out interference from those streams. Thus, turning off $i-$th stream affects the gains of lower-level streams $g_1 ... g_{i-1}$. This results in (5) being a non-convex problem for the V-BLAST, for which the conventional WF is in general not a solution. However, the problem can be split into 2^{m-1} convex sub-problems, one per each set of inactive transmitters, and each of the sub-problems can be solved via the conventional WF algorithm. The following theorem [[8], Theorem 5] makes this idea precise.

Theorem 2 (FWF)**.** *The joint optimum allocation of instantaneous power/rate for the coded V-BLAST (i.e.* (5)*) is given by the Fractional Waterfilling Algorithm (FWF) below:*

- *A. Split the problem:* `for` $l = 1, ..., 2^{m-1}$

Select a set of participating transmitters: if i-th bit in m-digit binary representation of $2l - 1$ (l is an index of the set) is $l^{(i)} = 1$, then transmitter i participates in l-th set (1st transmitter always participates).

Calculate the per-stream gains with interference from yet-to-be-detected symbols projected out, $g_i^l = |\mathbf{h}_{i\perp}^l|^2$, $\mathbf{h}_{i\perp}^l \perp \{\mathbf{h}_{i+1} l^{(i+1)}, \dots, \mathbf{h}_m l^{(m)}\}$, for $i = 1, \dots, m$.

- *B. Do the WF on the set of participating transmitters:*

Calculate the power allocation:

$$\alpha_i^l = l^{(i)}(\nu_l - 1/(\gamma_0 g_i^l))_+,$$

where $x_+ = x$ if $x > 0$ and 0 otherwise, and the water level ν_l is found from the total power constraint $\sum_{i=1}^m \alpha_i^l = m$. The per-stream and total capacities are:

$$C_i^l = \ln\left(1 + \gamma_0 \alpha_i^l g_i^l\right), \ C^l = \sum_{i=1}^m C_i^l.$$

- *C. Finalize:* `End for` *(l)*

The optimum power and rate allocations are given by $\alpha_i^{l^\star}$ and $C_i^{l^\star}$, where $l^\star = \arg\max_l C^l$.

While the FWF is more complex than the conventional WF, its incremental complexity is low for small m. Following Theorem 1, the FWF algorithm not only maximizes the instantaneous capacity C but also minimizes the system outage probability P_{out} and thus maximizes the outage capacity. As the further analysis shows, the FWF outperforms the WF by a wide margin.

IV. Performance Analysis

In this section, we present a comparative performance analysis of the unoptimized and optimized systems in different operating regimes.

A. Any SNR, any rate

Proposition 1. *For any channel realization, the system capacities of the coded V-BLAST with the FWF, the WF, the IRA and the uniform power/rate allocation are bounded as follows:*

$$\ln\left(1 + m\gamma_0 g_{max}\right) \le C_{FWF} \le m\ln\left(1 + \gamma_0 g_{max}\right) \tag{12}$$

$$\ln\left(1 + m\gamma_0 g_{max\perp}\right) \le C_{WF} \le m\ln\left(1 + \gamma_0 g_{max\perp}\right) \tag{13}$$

$$\ln\left(1 + \gamma_0 g_{max\perp}\right) \le C_{IRA} \le m\ln\left(1 + \gamma_0 g_{max\perp}\right) \tag{14}$$

$$C_u = m\ln\left(1 + \gamma_0 \min_i |\mathbf{h}_{i\perp}|^2\right) \tag{15}$$

where $g_{max} = \max_i |\mathbf{h}_i|^2$, $g_{max\perp} = \max_i |\mathbf{h}_{i\perp}|^2$ are the maximum unprojected and projected stream gains. The bounds are tight (i.e. there are channel realizations that achieve them). The relationship $C_u \le C_{IRA} \le C_{WF} \le C_{FWF}$ always holds. Moreover, $C_u = C_{IRA} = C_{WF} = C_{FWF} = m\ln\left(1 + \gamma_0|\mathbf{h}_1|^2\right)$ if and only if $\mathbf{H}^+\mathbf{H} = |\mathbf{h}_1|^2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, i.e the channel is orthogonal. No optimization is required in this case.

Proof: The left expression in (12) is the capacity in the regime with only one active transmitter, $\alpha_{i_{max}} = m$, where $i_{max} = \arg\max_i |\mathbf{h}_i|^2$ ($g_{i_{max}} = |\mathbf{h}_{i_{max}}|^2$ as there is no interference to project out when only one stream is active). Since the optimal capacity cannot be smaller, the lower bound in (12) holds. The upper bound in (12) is obtained by considering a hypothetical system where all per-stream gains are equal to $\max_i |\mathbf{h}_i|^2$. The bounds for conventional waterfilling (13) follow from the same reasoning. The difference between the two stems from the fact that the WF is oblivious to the possibility that some transmitters may be inactive and hence do not require projecting out their subspace, so that the maximum possible gain for the WF is $\max_i |\mathbf{h}_{i\perp}|^2$. The lower bound for IRA capacity in (14) is the largest term in the sum $C_{IRA} = \sum_i \ln\left(1 + \gamma_0|\mathbf{h}_{i\perp}|^2\right)$. The upper bound is obtained by upper bounding each term of the sum. For the regular V-BLAST (uniform power/rate allocation), the weakest stream dominates the performance: $C_u = m \max_R \{R \leq C_i, i = 1 \ldots m\} = m \min_i C_i$, which is equivalent to (15). ■

Proposition 2. *The outage probabilities of the FWF, the WF, the IRA and the uniform power/rate allocation are ordered in an arbitrary-fading channel as follows:*

$$P_{out}^{FWF} \leq P_{out}^{WF} \leq P_{out}^{IRA} \leq P_{out}^{u}$$

with the equality if $m = 1$. In the i.i.d. Rayleigh-fading channel, the equalities are achieved only if $m = 1$.

Proof: Each inequality follows from the fact that its left-hand side corresponds to optimization over a feasible set that is larger compared to that of its right-hand side. ■

Based on Proposition 1, the outage probabilities can now be characterized in a more specific way.

Corollary 1. *For any R and any γ_0, the outage probabilities of the coded V-BLAST with the uniform power/rate allocation, the IRA, the WF and the FWF in the i.i.d. Rayleigh fading channel are bounded as follows:*

$$F_n^m(z) \leq P_{out}^{FWF} \leq F_n^m(z_m/m) \tag{16}$$

$$\prod_{i=1}^{m} F_{n-m+i}(z) \leq P_{out}^{WF} \leq \prod_{i=1}^{m} F_{n-m+i}(z_m/m) \tag{17}$$

$$\prod_{i=1}^{m} F_{n-m+i}(z) \leq P_{out}^{IRA} \leq \prod_{i=1}^{m} F_{n-m+i}(z_m) \tag{18}$$

$$P_{out}^{u} = 1 - \prod_{i=1}^{m} (1 - F_{n-m+i}(z)) \tag{19}$$

where $z = (e^R - 1)/\gamma_0$, $z_m = (e^{mR} - 1)/\gamma_0$, $F_k(x) = 1 - e^{-x} \sum_{l=0}^{k-1} x^l/l!$ is the outage probability of k-th order MRC.

Proof: Observe that $|\mathbf{h}_i|^2 \sim \chi^2_{2n}$, all independent of each other, and $|\mathbf{h}_{i\perp}|^2 \sim \chi^2_{2(n-m+i)}$, and also independent of each other [10]. For $X \sim \chi^2_{2k}$, we have $\Pr\{X < x\} = F_k(x)$. Using these facts, the bounds of Corollary 1 follow from the bounds of Proposition 1. ■

B. Low outage probability regime

The diversity gains can now be characterized in the low-outage regime based on Corollary 1. The diversity gain can be found from [6]

$$d = -\lim_{\gamma_0 \to \infty} \ln P_{out} / \ln \gamma_0. \tag{20}$$

or by inspection when a closed-form low-outage approximation of P_{out} is available.

Corollary 2. *For fixed R, the diversity gains of the unoptimized V-BLAST, the IRA, the WF and the FWF are related as follows:*

$$d_u = n - m + 1 \leq d_{WF} = d_{IRA} = \sum_{i=1}^{m} (n - m + i) \leq d_{FWF} = nm$$

The equality is achieved for $m = 1$ only, i.e. only the FWF achieves the full MIMO channel diversity nm for $m > 1$[3].

Proof: Using the well-known approximation $F_k(x) = x^k/k! + o(x^k)$, $x \to 0$, in the upper and lower bounds to P_{out} in each equation of Corollary 1 and substituting it into (20), one observes that the lower and upper bounds give the same diversity gain, which is therefore the diversity gain. ■

The instantaneous rate allocation is the most efficient of all the techniques in terms of incremental improvement as it brings significant diversity gain and keeps the rate close to the capacity. When $m > 1$, the full MIMO channel diversity is achieved by the FWF only.

C. Any SNR, low rate (wideband) regime

The $R \ll 1$ regime here is also known as the wideband regime [11] (since R is in [nat/s/Hz], i.e. the rate per unit bandwidth), which is a popular solution for many systems (e.g. CDMA).

Theorem 3. *For any γ_0 and low rate $R \ll 1$, the outage probabilities of the coded V-BLAST with the uniform power/rate allocation, the IRA, the WF and the FWF in the i.i.d. Rayleigh fading channel are given by*[4]

$$P_{out}^{u} = 1 - \prod_{i=1}^{m} (1 - F_{n-m+i}(x)) \approx \frac{x^{n-m+1}}{(n-m+1)!}, \tag{21}$$

$$P_{out}^{IRA} \approx F_{d_{IRA}}(mx) \approx \frac{1}{d_{IRA}!} (mx)^{d_{IRA}}, \tag{22}$$

$$P_{out}^{WF} \approx \prod_{i=1}^{m} F_{n-m+i}(x) \approx \frac{x^{d_{IRA}}}{\prod_{i=1}^{m} (n-m+i)!} \tag{23}$$

$$P_{out}^{FWF} \approx F_n^m(x) \approx \frac{x^{nm}}{(n!)^m} \tag{24}$$

where the second approximation in each case holds at the low outage regime, $x = R/\gamma_0 \ll 1$.

Proof: Follows from Corollary 1 by applying the low rate approximation $e^{mR} - 1 \approx mR$ and observing that the lower and upper bounds coincide. ■

Note that the FWF not only has a higher diversity gain, but also an SNR gain of $\prod_{i=1}^{m} n!/(n-m+i)!$ over the WF.

D. Low SNR regime

The next result characterizes the optimization strategies at the low SNR regime. We emphasize that low SNR does not imply high error rate in coded systems, unlike uncoded ones. For example, the outage probability of the coded system is small at low SNR as long as $R/\gamma_0 \ll 1$. Many practical systems (e.g. CDMA) operate in this regime [6][11].

Theorem 4. *In the low SNR regime, $m\gamma_0 \max_i |\mathbf{h}_i|^2 \ll 1$, the instantaneous capacities of the regular (unoptimized) V-*

[3] This conclusion is not in contradiction to Corollary 9 of [8] since the latter (as well as Theorem 7 in [8]) requires all streams to be active, which is not the case for fixed R and $\gamma_0 \to \infty$.

[4] to the best of our knowledge, it is the first time when the outage probability of the waterfilling algorithms is found in an explicit, closed form.

BLAST and of the IRA, the WF, and the FWF are given by

$$C_u \approx m\gamma_0 \min_i |\mathbf{h}_{i\perp}|^2 \tag{25}$$

$$C_{IRA} \approx \gamma_0 \sum_{i=1}^{m} |\mathbf{h}_{i\perp}|^2 \tag{26}$$

$$C_{WF} \approx m\gamma_0 \max_i |\mathbf{h}_{i\perp}|^2 \tag{27}$$

$$C_{FWF} \approx m\gamma_0 \max_i |\mathbf{h}_i|^2 \tag{28}$$

and these capacities are attained by the following power/rate allocations:

• (26) *is attained by* $R_i^{IRA} = \gamma_0 |\mathbf{h}_{i\perp}|^2$,

• (27) *is attained by* $\alpha_{i_{max}}^{WF} = m$, $R_{i_{max}}^{WF} = m\gamma_0 |\mathbf{h}_{i_{max}\perp}|^2$, *and 0 otherwise, where* $i_{max} = \arg\max_i |\mathbf{h}_{i\perp}|^2$ *is the strongest projected channel,*

• (28) *is attained by* $\alpha_{i_{max}}^{FWF} = m$, $R_{i_{max}}^{FWF} = m\gamma_0 |\mathbf{h}_{i_{max}}|^2$ *and 0 otherwise, where* $i_{max} = \arg\max_i |\mathbf{h}_i|^2$ *is the strongest unprojected channel.*

Proof: In the capacity expressions of Proposition 1, apply the approximation $\ln(1+x) \approx x$, which is valid for $x \ll 1$, and observe that the lower and upper bounds coincide. ■

It is clear from (27), (28) that the unordered FWF performs as well as the WF combined with the optimal ordering.

Corollary 3. *In the low SNR regime, the following holds:*

1) $C_u = C_{IRA} = C_{WF} = m\gamma_0 |\mathbf{h}_{1\perp}|^2$ *if and only if* $|\mathbf{h}_{1\perp}|^2 = \ldots = |\mathbf{h}_{m\perp}|^2$.
2) $C_u = C_{WF}$ *if* $C_u = C_{IRA}$*, i.e. if there is no advantage in the IRA (compared to the unoptimized system), there is no advantage in the WF either, only the FWF may bring an improvement.*
3) $C_{WF} = C_{FWF}$ *if and only if* $i_{max} = \arg\max_i |\mathbf{h}_{i\perp}|^2 = m$ *or* $\mathbf{h}_{i_{max}} \perp \{\mathbf{h}_{i_{max}+1}, \ldots, \mathbf{h}_m\}$.

Additionally, it follows from Theorem 4 that the FWF significantly outperforms the WF, $C_{WF} \ll C_{FWF}$, when $\max_i |\mathbf{h}_{i\perp}| \ll \max_i |\mathbf{h}_i|$ and their performance is close otherwise.

V. Example

Let us consider 2x2 V-BLAST in the i.i.d. Rayleigh fading channel under the optimization strategies discussed above. Outage probabilities of the V-BLAST with these strategies obtained by Monte-Carlo simulations and the approximations above are shown in Fig. 1. As follows from the analysis, both the IRA and the FWF provide a significant improvement over the unoptimized system. As per Corollary 2, the conventional WF fails to achieve the minimum outage probability for a given total rate and to provide any diversity gain over the IRA (both have the diversity gain of 3), while the FWF achieves the full MIMO diversity of 4, outperforming both the IRA and the WF by a wide margin. Note also a significant advantage of the instantaneous optimizations over the average ones. The low-rate approximations in Theorem 3 are remarkably accurate.

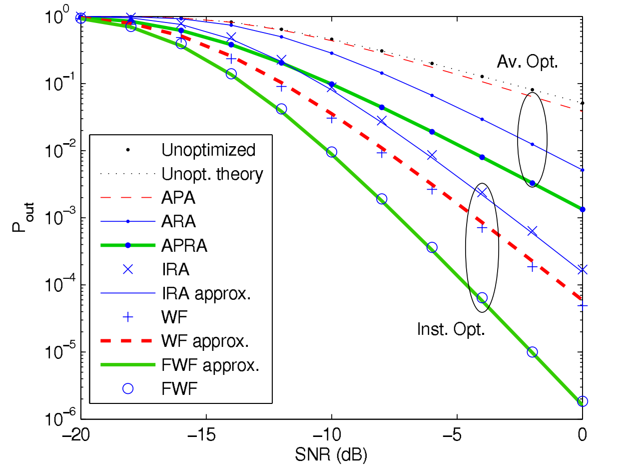

Fig. 1. Outage probability vs. average SNR of the instantaneous optimization strategies for 2×2 V-BLAST in i.i.d. Rayleigh fading channel, $R = 0.1$ [nat/sec/Hz], the approximations (lines) from Theorem 3 and Monte-Carlo simulations (symbols); average power, rate and power/rate (APA, ARA and APRA) allocations are also shown for comparison.

VI. Conclusion

Optimum instantaneous rate and joint power-rate allocations for the coded V-BLAST have been studied. A number of closed-form bounds and approximations have been given to study the performance of the IRA, the FWF and the WF. While the conventional WF algorithm fails to maximize the capacity of the coded V-BLAST, the recently proposed FWF maximizes its capacity and outperforms the other strategies by a wide margin, achieving the full MIMO channel diversity.

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