Statistical Approach to MIMO Capacity Analysis in a Fading Channel

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Abstract—**Theoretical analysis of the MIMO outage capacity distribution for an uncorrelated Rayleigh fading channel reveals that the Gaussian distribution is a good approximation, especially for large a number of antennas. While the capacity analysis itself was done in a mathematically rigorous way, the validation of this approximation was done by visually comparing the capacity graphs only. The mean and outage capacity of many measured MIMO channels have been also reported without accompanying statistically-rigorous analysis. Hence, the question as to whether the measured channel capacity distribution is close to the theoretical one has not been answered yet in a satisfactory way. We address this problem by developing a statistically-rigorous procedure (in terms of hypothesis testing) for the analysis of the mean and outage capacities of MIMO channels (both, theoretical models and measured) with the main goal being to compare the theoretical and measured capacity distribution, especially the validity of Gaussian approximation for the measured outage capacity. As we demonstrate, there is a tight lower bound on the amount of measured data necessary to provide a unique answer with high confidence probability. Additionally, based on the procedure above, we develop guidelines for future measurements and Monte-Carlo simulations in terms of accuracy.**

Keywords- wireless communications, MIMO channel capacity, statistical analysis.

I. INTRODUCTION

Multiple-antenna systems (MIMO) promise significant improvement in capacity over multipath channels. The capacity of an uncorrelated Rayleigh MIMO channel grows proportionally to the channel rank and linearly depends on a number of receive antennas when the number of transmit antennas is asymptotically large [1].

The distribution of the outage capacity of a Rayleigh MIMO channel is studied extensively in recent years. The explicit expressions for its characteristic function and moments in uncorrelated and semicorrelated (i.e. with spatial correlation either at a transmitter or a receiver but not both) Rayleigh channels are derived by Smith *et al* [2] and Chiani *et al* [3]. However the expressions obtained are complicated and as the result the impact of various factors, such as correlation, *SNR* etc', on the outage capacity is difficult to see.

Simplier and closed-form expressions are derived by Hochwald *et al* [4], where the authors give the asymptotic distribution of the outage capacity of an uncorrelated MIMO Rayleigh channel using the properties of Wishart matrices and the Central Limit Theorem. In particular, it is shown that the distribution is asymptotically Gaussian when the number of either transmit or receive or both antennas approach infinity. Using Monte-Carlo simulations, Hochwald *et al* [4] and Smith *et al* [5] show that the outage capacity distribution of an uncorrelated Rayleigh MIMO channel converges very fast to Gaussian and becomes “virtually indistinguishable” from the Gaussian distribution when the rank of the channel matrix greater than five [5]. Smith *et al* [5] also notice that the convergence is faster as signal-to-noise-ratio (*SNR*) becomes smaller. Finally, Moustakas *et al* in [6] derive the distribution moments for a correlated Rayleigh MIMO channel and demonstrate that the Gaussian distribution is a good approximation for the correlated channel as well even when the number of transmit and receive elements is not that large e.g. at least three antennas at each side.

While the capacity distribution analysis was a mathematically rigorous one, the validation of the approximations involved was done visually. Most of the measured results on MIMO capacity and other channel parameters were not a subject to a rigorous statistical analysis, and the comparison of the measured distributions to the theoretical models was done with no strictly defined criteria. As a result, different conclusions were reported by different authors. Validity of the Gaussian approximation for the outage capacity has also not been a subject to the rigorous statistical analysis. Rather visual comparison of the capacity plots was done, which can neither account for the statistical error due to the limited amount of data nor for the confidence probability of the conclusions. These limitations are especially pronounced for measured channels, where the number of data points available is typically limited to 100 – 200 at most [7]. Hence we pose the following question: Is the discrepancy between theoretical and measured capacities due to their inherent difference or it just a statistical error due to the limited amount of data available? Clearly, only the rigorous statistical analysis is able to provide a credible answer.

II. METHODS OF STASTISTICAL ANALYSIS

In general, there are two hypotheses considered against each other in any statistical test: an assumption on some property of the measured data (the null hypothesis H_0) against the possibility that this assumption is not true (the alternative hypothesis H_1). For this purpose, a test statistics T_n (a function applied on the measured data) is calculated using n observations and compared to some critical value ε. The meaning of ε depends on the meaning of T_n in each particular test. If $|T_n| \le \varepsilon$, H_0 is accepted; otherwise, it is rejected. We should stress, that if H_0 is accepted it does not mean that the measured data possesses the assumed property; it simply means that the test performed does not find any statistically significant difference between the observed and assumed properties. Apparently, there are two probabilities associated with T_n and ε:

$$\alpha = P\{|T_n| > \varepsilon | H_0\} \quad (1)$$

where α is the miss probability or significance level; i.e. the probability to reject H_0 given it is true. $\beta = P\{|T_n| \le \varepsilon | H_1\}$ is the false alarm probability; i.e. the probability to accept H_0 given it is not true.

Unlike the computer-based Monte-Carlo simulations, the common problem of any measurement is a limited number of observations available. Therefore, it is important to choose α and β (test parameters) properly with accordance to the data size, especially when the size is small.

To show how α and β should be chosen, let us consider T_n of a *monotonically consistent* statistical test. Then the following is true: i) for any ε $\lim_{n\to\infty} P\{|T_n| > \varepsilon | H_0\} = 0$ and ii) for any given α and ε there is only one n, which satisfies (1). Apparently, as follows from i) and ii), for any $m > n$, $\alpha > P\{|T_m| > \varepsilon | H_0\}$. Moreover, due to the additive property of the probability measure, for any n $P\{|T_n| > \varepsilon | H_0\}$ is a non-increasing function of ε. Therefore, if ε is small, either α or n should be large. This is a general conclusion that is true regardless of any specifics.

On the other hand, let us consider β. Its exact value depends on the actual distribution of the measured data, which is unknown in most practical cases. However, in general, due to the additive property of the probability measure, β is a non-decreasing function of ε. Thus, if β is low, the corresponding α would be high for given n. Therefore, the only way to keep the equality in (1) with decreasing α for fixed β would be to increase the size of the acquired data. While the relationship given in (1) between n, ε and α is general for any *monotonically consistent* statistical test, specific values of n, ε and α depend on a particular test to be used. Further, we use three statistical tests to analyze the measured MIMO channel: 1) Pearson χ^2 test, for statistical hypothesis on distributions; 2) generalization of the T-test of correlation coefficients, to check whether the measured channel correlation is statistically different from zero; and 3) generalization of the F-test (variance ratio test), to check whether the variances of two sample sets are statistically identical. It can be shown that all three tests are *monotonically consistent*. Moreover, since the test statistics distributions of these tests are known given H_0 is true, (1) for the χ^2 test can be written as:

$$\alpha = 1 - \gamma(0.5(K-m-1), 0.5n \cdot \varepsilon) \cdot \Gamma^{-1}(0.5(K-m-1)) \quad (2)$$

where $\gamma(a,x)$ is the incomplete Gamma function, $\Gamma(a) = \gamma(a,\infty)$ is the Gamma function, K is the number of intervals of the observed data, and m is the number of moments to be estimated. The meaning of ε in (2) is a critical mean relative deviation of the observed histogram from the expected one. For the generalized t-test, (1) is given by:

$$\alpha = \exp\left\{-0.5 \cdot \varepsilon^2 \cdot (2n-2) / (1-\varepsilon^2)\right\} \quad (3)$$

where ε is a critical sample correlation. For the generalized F-test, (1) is:

$$\alpha = 1 - \frac{\Gamma(2n-1)}{\Gamma^2(n-0.5)} \cdot \int_{1-\varepsilon}^{1+\varepsilon} \frac{w^{(n-1.5)}}{(1+w)^{(2n-1)}} dw \quad (4)$$

where ε is a critical deviation of a ratio of two sample variances from one. To demonstrate the general relationship between n, ε and α, α vs. n in (2) for different ε is plotted in Figure 1. Clearly, decreasing α for given n results in increasing ε, which, in turn, increases β. The only way to decrease α while keeping β low is to increase n. This is a statistically-rigorous representation of error due to the limited amount of the data available.

III. STATISTICAL ANALYSIS OF THE RAYLEIGH CHANNEL

The subject of this section is to describe the rigorous statistical analysis of the outage capacity distribution of a correlated flat-fading Rayleigh MIMO channel. To account for the correlation, the channel matrix $\mathbf{H}$ is given by the Kronecker model [6]:

$$\mathbf{H} = \mathbf{R}^{1/2}\mathbf{G}\mathbf{T}^{1/2} \quad (5)$$

where $\mathbf{T}$ and $\mathbf{R}$ are the $[t \times t]$ transmit and $[r \times r]$ receive correlation matrices respectively; $\mathbf{G}$ is an $[r \times t]$ matrix whose entries are i.i.d. complex Gaussian, $CN\{\mathbf{0}, \mathbf{I}\}$. We also use the exponential correlation matrix model to represent both $\mathbf{T}$ and $\mathbf{R}$ through complex correlation coefficients r_T and r_R respectively [8], i.e.:

$$\mathbf{T}_{i,j} = \begin{cases} r_T^{j-i}; i \le j \\ \mathbf{T}_{ji}^*; i > j \end{cases}; |r_T| \le 1 \quad (6)$$

The elements of $\mathbf{R}$ are given by similar expression via r_R. From [1] the outage capacity C of the channel is determined from the following for a given outage probability P_{out}:

$$P_{out} = \Pr\{C \geq \log_2 \det(\mathbf{I} + \rho/t \cdot \mathbf{H} \cdot \mathbf{H}^{+})\} \qquad (7)$$

where $\mathbf{I}$ is the $[r \times r]$ identity matrix; ρ is an *SNR* defined as the total average power at one receive antenna over the noise power at that antenna, and $\mathbf{H}^{+}$ is a transpose conjugate of $\mathbf{H}$.

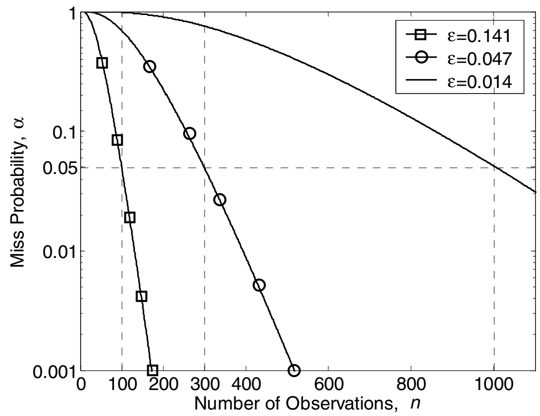

Figure 1. α vs. n in the χ^2 test (K=10, m=2)

We generated the sets of 100, 300 and 1000 matrices $\mathbf{H}$ with orders up to 8x8 for different r_T and r_R. The χ^2 test with $K = 10$ intervals was then applied on the computed standardized outage capacity (i.e. capacity shifted by its mean and normalized by its standard deviation) for different ρ. We chose $\alpha = 0.05$ regardless of the number of the channel realizations to allow fair comparison. As follows from (2), this α corresponds to the critical values of ε equal to 0.141, 0.047 and 0.014 for n equal to 100, 300 and 1000 respectively (see also Figure 1). Following the Gaussian approximation [4], we assume as H_0 that the standardized outage capacity distribution is Gaussian distributed with zero mean and unit variance. For the sets of 100 realizations, the χ^2 test did not provide any credible conclusions. For the wide range of tested ρ and regardless of r_T and r_R, H_0 was accepted already for a 1x1 channel which contradicts the most of the published results [1, 4, 5]. In fact, the critical value $\varepsilon = 0.141$, which corresponds to $n = 100$ is too large and the corresponding false alarm probability is too high. On the contrary, for $n = 300$ the test results are in good agreement with the existing theory. Some of these results are given in Table I. Each cell in Table I contains the value of the χ^2 test statistics $|T_n|$. If $|T_n| \leq \varepsilon$, the H_0 is accepted and the corresponding cell is shadowed. The table rows (r) and columns (t) represent the number of receive and transmit antennas of the tested channel, i.e. the MIMO order. The χ^2 test results for $n = 1000$ were found quite similar to those of $n = 300$ and therefore are not shown.

As can be seen in Table I, the Gaussian distribution is a good approximation for the correlated Rayleigh channel starting from orders 2x2 and 3x3 where the capacity is already *statistically* Gaussian. Moreover, as follows from Figure 2, where the χ^2 statistics of the 2x2 MIMO channel vs. correlation coefficient is shown, the convergence rate to the Gaussian distribution with respect to the MIMO order is faster for low ρ, i.e. for low correlations, $|T_n|$ is smaller for lower ρ. This fact coincides well with the conclusion made in [5]. Similarly to the observation reported in [3], the results presented in Figure 2 do not reveal any significant change in the outage capacity statistics for correlation coefficients < 0.5. However the increase in correlation has a similar effect on the capacity distribution with respect to the Gaussian distribution as decrease in ρ. When the correlation coefficient increases (0.5 and up) the corresponding $|T_n|$ for $\rho = 20dB$ and $30dB$ decrease and approach that of $\rho = 5dB$. Moreover from Table I, the 3x3 MIMO channel outage capacity distribution is statistically Gaussian in the two tables where we deliberately increase ρ along with increasing correlation coefficients r_T and r_R. The above might be an analogy to the results reported in [8], where it is analytically shown that the effect of increase in correlation on the mean capacity is equivalent to decrease in SNR. Another observation which follows from Figure 2 is that when the correlation coefficient goes close to the unity, $|T_n|$ sharply increases regardless of ρ. Indeed, when the spatial correlation is close to the unity, the MIMO channel degenerates, i.e. it's order reduces to 1x1, and as a results, the outage capacity distribution of that channel is far from Gaussian.

TABLE I. RESULTS OF THE χ^2 TEST FOR DIFFERENT MIMO ORDERS (r×t) PERFORMED ON 300 SPATIAL REALIZATIONS (ε=0.047)

r \ t	1.	2.	3.	4.	5.	6.	7.	8.
1.	0.11	0.11	0.04	0.06	0.05	0.05	0.07	0.11
2.	0.11	0.07	0.05	0.06	0.03	0.02	0.01	0.02
3.	0.05	0.04	0.01	0.01	0.01	0.01	0.05	0.06
4.	0.06	0.12	0.03	0.02	0.03	0.01	0.06	0.04
5.	0.05	0.04	0.02	0.03	0.02	0.04	0.01	0.05
6.	0.05	0.02	0.03	0.05	0.01	0.03	0.01	0.01
7.	0.07	0.04	0.02	0.03	0.06	0.03	0.02	0.04
8.	0.12	0.04	0.07	0.02	0.04	0.02	0.01	0.01

a) ρ=5dB, r_T=r_R=0

r \ t	1.	2.	3.	4.	5.	6.	7.	8.
1.	0.07	0.05	0.04	0.01	0.04	0.08	0.01	0.04
2.	0.05	0.06	0.02	0.06	0.01	0.04	0.01	0.01
3.	0.07	0.03	0.02	0.03	0.01	0.02	0.02	0.04
4.	0.02	0.04	0.01	0.03	0.03	0.01	0.04	0.02
5.	0.02	0.03	0.03	0.02	0.01	0.02	0.02	0.03
6.	0.05	0.03	0.02	0.02	0.01	0.02	0.05	0.02
7.	0.01	0.01	0.03	0.03	0.02	0.03	0.02	0.02
8.	0.03	0.03	0.02	0.02	0.03	0.01	0.05	0.01

a) ρ=10dB, r_T=r_R=0.8

The behavior of the real physical channel is different in many cases from the theoretical models. Hence we want to apply the same rigorous statistical analysis as above on a measured channel and assess the validity of the Gaussian approximation on it. This issue is addressed in the next section.

IV. STATISTICAL ANALYSIS OF THE MEASURED CHANNEL

In this section we analyze the experimental data based on the measurements of the 8x8 5.2 GHz indoor MIMO channel reported in [7]. However, the procedure is general enough to be applied to any channel. The MIMO channel was measured at $F=193$ frequency bins equally spread over 120MHz frequency band at the central frequency of 5.2GHz. At each frequency bin, $n=130$ spatial realizations of the 8x8 MIMO complex channel matrix were taken at 8 different locations (Rx1, Rx2,…, Rx7, and Rx9) and 3 different directions (D1, D2, and D3) in each location. As a result we have (3x8x130x193x8x8) 6-dimensional complex channel transfer matrix (for details see [7]).

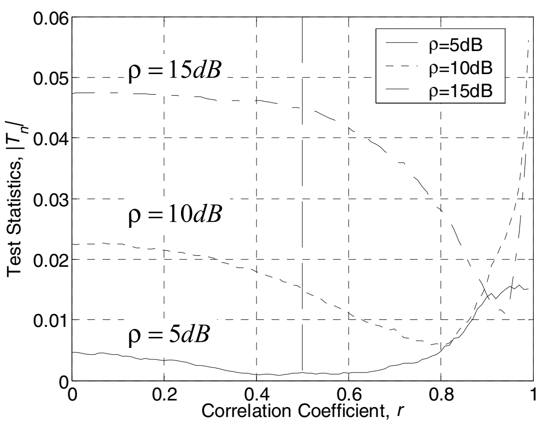

Figure 2. 2x2 MIMO channel χ^2 statistics vs. correlation coefficient: $r=r_T=r_R$

Below we compare the outage capacity distribution of the measured channel to the Gaussian model in [4] in a statistically-rigorous way. For this purpose, we initially test whether the channel is Rayleigh distributed, uncorrelated, frequency flat and non-degenerated. At the end, we study how fast the outage capacity distribution of the measured channel converges to the Gaussian model with respect to different MIMO system orders $(r \times t)$ and *SNR*.

TABLE II. RESULTS OF χ^2 TEST IN RX6D2 FOR DIFFERENT MIMO ORDERS ($r \times t$), ρ=5dB AND ε=0.092.

r \ t	1.	2.	3.	4.	5.	6.	7.	8.
1.	0.14	0.06	0.10	0.12	0.13	0.03	0.06	0.13
2.	0.10	0.02	0.07	0.07	0.06	0.05	0.06	0.10
3.	0.11	0.05	0.03	0.11	0.17	0.08	0.06	0.07
4.	0.04	0.05	0.06	0.14	0.08	0.07	0.05	0.11
5.	0.04	0.02	0.06	0.11	0.16	0.18	0.13	0.11
6.	0.05	0.14	0.14	0.08	0.08	0.13	0.09	0.05
7.	0.04	0.03	0.10	0.03	0.01	0.03	0.05	0.04
8.	0.06	0.03	0.05	0.06	0.07	0.03	0.06	0.06

A. Channel Gain Distribution: The χ^2 test was applied on the sets of the measured complex channel gains. As H_0, it was assumed that the gains are Rayleigh distributed. As a compromise between low α and not very big ε we chose $\alpha = 0.05$, which corresponds to $\varepsilon = 0.119$ for $n=130$ and $K=10$ intervals we used in the test.

For every considered configuration, H_0 passed the test. We also noticed that the measured channel does not have line-of-sight (LOS) component, since the estimated LOS factors are very low (around -30dB) in each considered configuration.

B. Tx and Rx correlation: In order to test Tx and Rx correlations, we estimated sample correlations between different Tx and Rx antennas at different frequencies, locations and directions. Then, the generalized T-test was applied, where H_0 means no correlation in the measured channel. The miss probability was $\alpha = 0.05$, which corresponded to $\varepsilon = 0.054$.

In most cases the test showed that there is a statistically significant correlation (in some cases >0.75). We also observed much more severe correlation at the Rx than at the Tx. We explain this by the fact that the angular spread is smaller in the transmitter rather than in the receiver.

C. Channel frequency response: To test the channel frequency response, we considered the ratio of the channel power gains measured at different frequencies in each location and direction. H_0 meant a frequency-flat channel. The generalized F-test of the variance ratio was applied with $\alpha = 0.1$ ($\varepsilon = 0.205$ for $n=130$). For all considered configurations, H_0 was rejected, i.e. the channel has different power gains at different frequencies. Therefore, the channel is statistically *frequency selective* within the considered frequency band. Hence, measurements at different frequencies have to be analyzed *separately*.

D. Outage Capacity Distribution: First, the measured channel rank was determined to be eight. That means that the measured channel is *non-degenerated* or it has no "keyholes". Further, the χ^2 test with $K=10$ intervals was applied to the standardized outage capacity distribution computed in different locations, directions, ρ, different channel orders and at all frequency bins. To test different orders ($r \times t$) the right-upper corners with appropriate size were picked up from the 8x8 measured channel matrices.

Following [4], H_0 is that the standardized outage capacity distribution is Gaussian distributed with zero mean and unit variance. Since the number of spatial observations in each tested location and direction is $n=130$, we chose $\alpha = 0.1$, which corresponds to $\varepsilon = 0.092$ in (2). Some of the results are presented in Table II, where the χ^2 test statistics $|T_n|$ in Rx6D2 computed at the central frequency bin for different orders and $\rho = 5dB$ are given. As above, if $|T_n| \le \varepsilon$, H_0 was accepted and the corresponding cell is shadowed.

Following the Gaussian approximation [4], H_0 is expected to be more frequently accepted for higher orders as well as for lower ρ, as suggested in [5]. However, as Table II demonstrates, as the order of the MIMO channel increases the χ^2 test does not follow systematically this expectation. The same was observed when ρ decreases. Since $\alpha = 0.1$, on average in 6.4 tests out of 64 H_0 will be rejected given it is true. However, we were encountering significantly more test fails in the high order regions, i.e. after the orders for which

H_0 was first accepted. It does not mean that H_0 is not true, since the probability to get more than 6.4 failures in the test is significant. To increase the credibility of the test, α needs to be reduced. Another reason for the observed inconsistency could be a result of the large ε, as in our case for $n = 130$ As indicated above, the only way to reduce α and ε simultaneously is to increase n. Thus, to avoid insufficient statistics we split each 8x8 spatial realization of the channel into four spatially independent 2x2 realizations. This allowed us to get in total $n = 520$ realizations, which following (2) corresponds to $\varepsilon = 0.012$ for $\alpha = 0.05$ against $\varepsilon = 0.092$ for $\alpha = 0.01$ in the preceding test. Then we applied the χ^2 test with Gaussian H_0 on the computed standardized outage capacity given in (7) for the obtained 2x2 MIMO channel measured in different locations, directions, frequencies and ρ. We found that as expected the 2x2 outage capacity distribution strongly depends on ρ and basically it is *statistically* Gaussian for $\rho < 10dB$. We also noticed that the average 2x2 outage capacity measured over all 120MHz frequency band is *statistically* Gaussian as well for $\rho \le 30dB$.

However, the χ^2 test gives an integral evaluation of the discrepancy between measured and expected distributions (i.e. for the entire range). This has some disadvantages: contribution for high outage probability region is included, which is not of interest from practical viewpoint (since this is low quality of service region). From the practical perspective, it is important to know the outage capacity distribution on the distribution tales, where the outage probability P_{out} is low, i.e. in the region of high quality of service, and the capacity distribution for high P_{out} is not that important. In order to compare the measured capacity to its Gaussian approximation in low P_{out} region, we plot the 2x2 MIMO standardized outage capacities for $n = 520$ and $\rho = 5dB$ computed at the central frequency bin in Figure 3 together with $\pm\sigma$ confidence intervals (evaluated based on the measured data). Despite the fact that the measured capacity distribution is *statistically* Gaussian, as affirmed by the χ^2 test, in some locations the difference between the measured capacity distribution and the Gaussian approximation exceeds the $\pm\sigma$ error range for low P_{out}. This deviation is especially large on the tales for $P_{out} < 0.02$.

In conclusion we can state that following the rigorous statistical analysis done, the outage capacity distribution of the measured indoor MIMO channel is described well enough by the Gaussian distribution starting already from 2x2 order for $\rho < 10dB$. The difference between the measured capacity and the Gaussian model on the distribution tales may be still significant and should be taken in account when low P_{out} is important. In addition, the presented study shows that in order to avoid insufficient statistics, future experiments should make at least $n = 300$ measurements at the same frequency and environment. This will grant $\alpha = 0.05$ and $\varepsilon = 0.047$, which are shown to be small enough to provide credible conclusions.

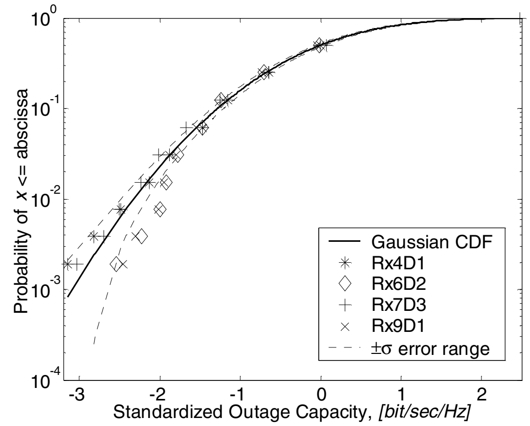

Figure 3. 2x2 MIMO channel standardized outage capacity sample CDF in different locations (ρ=5dB)

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