MIMO CHANNEL CAPACITY: ELECTROMAGNETIC WAVE PERSPECTIVE

Sergey Loyka

School of Information Technology and Engineering (SITE), University of Ottawa
161 Louis Pasteur, Ottawa, Ontario, Canada, K1N 6N5, Email: sergey.loyka@ieee.org

ABSTRACT

Radio propagation channel has a profound impact on the MIMO channel capacity. Under favourable conditions (i.e., uncorrelated high-rank channel or rich scattering), the MIMO capacity scales roughly linearly as the number of Tx/Rx antennas. However, the channel correlation decreases the capacity and, at some point, it is the dominant effect. This effect received considerable attention in recent years. Two main approaches have been adopted to study it and to predict the system performance in realistic scenarios: the eigenvalue (or SVD) decomposition approach [1, 2] and the correlation matrix approach [3-8]. Some practical design guidelines have been developed as well. In this paper, we consider the effect of correlation from a completely different perspective. Specifically, we consider the constrains imposed by the laws of electromagnetism on achievable MIMO capacity in its most general form. To accomplish this, we define the spatial capacity as the maximum of the conventional MIMO capacity (per unit bandwidth, i.e. bits/s/Hz) over possible propagation channels, subject to some constrains. We show that, at least in some scenarios, this maximum exists and it provides new insight into the notion of channel capacity when electromagnetic waves are used as the carrier of information.

INTRODUCTION

The wireless propagation channel has a profound impact on the performance of multiple-input multiple-output (MIMO) communication architecture. In ideal conditions, i.e. when the channel is uncorrelated (independent fading at both Tx and Rx ends, i.e. rich scattering) and of high rank (no keyholes), the MIMO capacity is maximum and it scales roughly linearly as the number of Tx/Rx antennas (strictly speaking, the minimum of Tx and Rx antenna numbers, but, for the sake of simplicity, we consider the case when these numbers are equal). The effect of channel correlation is to decrease the capacity and, at some point, this is the dominant effect. It should be pointed out that this effect depends on the propagation channel as well as on the antennas themselves (which are, in a sense, a part of the propagation channel). Two main approaches have been adopted in recent years to study this effect: the eigenvalue decomposition (or singular value decomposition) approach [1,2] and the correlation matrix approach [3-8]. While the former is a powerful mathematical technique, the later provides more physical insight and is more close to engineering practice. Many practically-important scenarios have been studied and some design guidelines have been proposed as well.

Our viewpoint in this paper is completely different. Electromagnetic waves (either guided (wired) or not (wireless)) are used as the primary carrier of information. The basic electromagnetism laws, which control the electromagnetic field behaviour, are expressed as Maxwell equations. Hence, we ask a question: What is, if any, the impact of Maxwell equations on the notion of information in general and on channel capacity in particular? As to the best of our knowledge, only two attempts have been made so far to answer questions similar to the stated above [9,10]. In this paper, we try to answer the second question. In other words, do the laws of electromagnetism impose any limitations on the achievable channel capacity? We are not targeting in particular scenarios, rather, we are going to look at fundamental limits that hold in any scenario. Analyzing MIMO channel capacity allows one, in our opinion, to come very close to answering this question.

MIMO CHANNEL CAPACITY

There are several definitions of the MIMO channel capacity, depending on the scenario considered. The main differences between these definitions are due to the following. Channel state information (CSI): may be available at the receiver (Rx), transmitter (Tx), both or not at all (if CSI is available at Tx, water filling is possible). Ergodicity assumption: when channel is random, its capacity is random too; mean ergodic capacity may be defined if the ergodicity assumption is employed. Another possibility is to consider outage capacity. MIMO network capacity may also be defined when there are several users which interfere with each other. Since the arguments presented in this paper hold true for most definitions, we do not discuss in detail these differences. To be specific, we employ the following expression for the MIMO channel capacity, which is valid for a fixed linear $n \times n$ matrix channel with additive white gaussian noise and when the transmitted signal vector is composed of statistically independent equal power components each with a gaussian distribution and the receiver knows the channel [11],

$$C = \log_2 \det\left(\mathbf{I} + \frac{\rho}{n}\mathbf{H}\cdot\mathbf{H}^{+}\right) \text{ bits/s/Hz}, \tag{1}$$

where n is the numbers of transmit/receive antennas, ρ is the average signal-to-noise ratio, **I** is n×n identity matrix, **H** is the normalized channel matrix, which is considered to be frequency independent over the signal bandwidth, and "+" denotes transpose conjugate. When the channel is random (stochastic), then the capacity is random, too. The expectation over the channel matrix can be employed in this case in order to define the mean (ergodic) capacity as follows [7]:

$$\langle C\rangle = \left\langle \log_2 \det\left(\mathbf{I} + \frac{\rho}{n}\mathbf{H}\cdot\mathbf{H}^{+}\right)\right\rangle \le \overline{C} = \log_2 \det\left[\mathbf{I} + \frac{\rho}{n}\mathbf{r}\right] \tag{2}$$

where $\langle C\rangle$ is the mean capacity and $\langle\ \rangle$ is expectation over the channel matrix. The inequality in (2) follows from the Jensen's inequality and concavity of *log det*, where **r** is the correlation matrix with components

$$r_{ij} = \sum_k \langle h_{ik} h_{jk}^{*}\rangle, \quad i, j, k = 1,\ldots,n \tag{3}$$

and h_{ij} denotes the components of **H**. It should be noted that the upper bound in (2) accounts for the Rx end correlation only. However, the Tx end correlation can be accounted for in a similar way [8]. Thus, without loss of generality, we use (2) for further discussion. It should be noted that the upper bound in (2) is quite close to the mean capacity and, hence, we can use it as an estimation of the mean capacity (we use this approach in (8)).

It follows from (1)-(3), that the MIMO channel capacity crucially depends the propagation channel **H** . Since electromagnetic waves are used as the carrier of information, the laws of electromagnetism must have an impact on the MIMO capacity. They ultimately determine behaviour of **H** in different scenarios. Since we are interested in the fundamental limitations imposed by these laws on the MIMO capacity, we outline in the next section the laws of electromagnetism in their general form.

THE LAWS OF ELECTROMAGNETISM

In they most general form, the laws of electromagnetism are expressed as Maxwell equations (differential form):

$$\nabla\cdot\mathbf{D} = \rho \quad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \quad \nabla\cdot\mathbf{B} = 0 \quad \nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t} \tag{4}$$

where ρ and **J** are charge and current sources correspondingly, **E** and **H** are electric and magnetic field vectors, and **D** and **B** are electric and magnetic flux densities ($\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$, ε and μ are permittivity and permeability of the media correspondingly). **H** denotes the magnetic field vector in this section only and should not be confused with the channel matrix **H** in (1). (4) is a system of second-order partial differential equations. Appropriate boundary conditions must be applied in order to solve it. We are interested in application of (4) to find the channel matrix (i.e., **H** in (1)). Since Tx and Rx antennas are located in different points in space, we need to solve (4) in the area outside of the sources only, where $\rho = 0$ and $\mathbf{J} = 0$ (i.e., electromagnetic waves). In this case, (4) simplifies to the system of two decoupled wave equations:

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0 \quad \nabla^2\mathbf{H} - \frac{1}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} = 0 \tag{5}$$

where c is the speed of light. Since the carrier frequency is usually much larger the signal bandwidth, one may further employ the harmonic wave assumption. In this case, (5) simplifies to

$$\nabla^2\tilde{\mathbf{E}} - \gamma^2\tilde{\mathbf{E}} = 0 \quad \nabla^2\tilde{\mathbf{H}} - \gamma^2\tilde{\mathbf{H}} = 0 \tag{6}$$

where $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ are complex magnitudes of time-varying electric and magnetic field vectors, and γ is the propagation constant. In the next section, we discuss how to apply (5)-(6) to get the fundamental limit imposed by Maxwell equations on the MIMO channel capacity. It should also be noted that there are 6 field components (or "polarizational degrees of freedom") associated with (5) and/or (6) (three for electric and three for magnetic fields), which can be used for communication in rich-scattering environment. Of course, only two of them survive in free space ("poor scattering”). Hence, in a generic scattering case the number of polarizational degrees of freedom varies between 2 and 6.

MIMO CAPACITY OF A GIVEN SPACE (SPATIAL CAPACITY)

The MIMO channel capacity is defined as the maximum mutual information, the maximum being taken over all possible transmitted vectors. Under some conditions, this results in (1). The next step would be to try to maximize the capacity by changing **H** (within some limits), for example, by appropriate positioning of antennas. Using the analogy with the channel capacity definition, one can call this maximum (if it exists) “capacity of a given space” (since we have to vary channel during this maximization the name “channel capacity” seems to be inappropriate simply because channel is not fixed. On the other hand, we vary channel within some limits, i.e. within given space. Thus, the term “capacity of a given space” seems to be appropriate). The question arises: what is this maximum and what are the main factors which have an impact on it?

To answer this question, we define a spatial MIMO capacity as a maximum of the conventional MIMO channel capacity (per unit bandwidth, i.e. in bits/s/Hz) over possible propagation channels (including Tx & Rx antenna locations and scatterers’ distribution), subject to some possible constraints. Using the ray optics arguments and the recent result on the MIMO capacity, we further demonstrate that there exists an optimal distribution of scatterers and Tx/Rx antennas that provides the maximum possible capacity in a given region of space. Hence, the MIMO capacity per unit space volume can be defined in a fashion similar to the traditional definition of the channel capacity per unit bandwidth.

Considering a specific scenario would not allow us to find a fundamental limit simply because the channel capacity would depend on too many specific parameters. For example, in outdoor environments the Tx and Rx ends of the system are usually located far away from each other. Hence, any MIMO capacity analysis (and optimization) must be carried out under the constrain that the Tx and Rx antennas cannot be located close to each other. However, there exists no fundamental limitation on the minimum distance between the Tx and Rx ends. Thus, this maximum capacity would not be a fundamental limit. In a similar way, a particular antenna design may limit the minimum distance between the antenna elements but it is just a design constrain rather than a fundamental limit. Similarly, the antenna design has an effect on the signal correlation (due to the coupling effect), but this effect is very design-specific and, hence, is not of fundamental nature. In other words, the link between the wave equations (5) or (6) and the channel matrix **H** is very implicit since a lot depends on Tx and Rx antenna designs and many other details.

In order to avoid the effect of design-specific details, we adopt the following assumptions. Firstly, we consider a limited region of space. All the system components are located inside of this region. As it is well-known, a rich scattering environment is required to order to achieve high MIMO capacity. Thus, secondly, the rich scattering assumption is adopted in its most abstract form. Specifically, it is assumed that there is infinite number of randomly and uniformly-located ideal scatterers (the scattering coefficient equals to unity), which form a uniform scattering medium in the entire space (including the space region considered) and which do not absorb EM field. Thirdly, antenna array elements are considered to be ideal field sensors with no size and no coupling between the elements in the Rx (Tx) antenna array. Our goal is to find the maximum MIMO channel capacity in such a scenario (which posses no design-specific details) and the limits imposed by the electromagnetism laws. It should be emphasized that the effect of electromagnetism laws is already implicitly included in some of the assumptions above. In general, Maxwell equations describe the EM field inside and outside of the filed sources (i.e., (4)). Since the Tx and Rx antennas are located in different positions, we need the field only outside of the sources. In this case, Maxwell equations simplify to two vector wave equations (5) or (6). In general, we should use these wave equations (together with appropriate boundary conditions) for the analysis. The channel matrix **H** must also satisfy (5) or (6),

$$\nabla^2\mathbf{H} - \frac{1}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} = 0 \quad \text{or} \quad \nabla^2\tilde{\mathbf{H}} - \gamma^2\tilde{\mathbf{H}} = 0 \tag{7}$$

(we remind that **H** means here the channel matrix). Hence, the formal definition of the spatial capacity is the maximum of C (given by (1)) over **H** subject to the constraint (7) and the total power constraint. In order to simplify analysis further, we use the ray (geometrical) optics approximation (this justifies the ideal scattering assumption above).

Knowing that the capacity increases with the number of antennas, we try to use as many antennas as possible. It there any limit to it? Since antennas have no size (by the assumption above), the given region of space can accommodate the infinite number of antennas. However, if antennas are located close to each other the channel correlation increases and, consequently, the capacity decreases. A certain minimum distance between antennas must be respected in order to avoid capacity decrease, even in ideal rich scattering [7]. Hence, when we increase the number of antennas the capacity at first increases. But at some point, due to the space limitation, we have to decrease the distance between adjacent antennas to accommodate new antennas. When the adjacent antenna spacing decreases, the capacity increase slows down and finally, when the antenna spacing is less than the minimum distance, the capacity begins to decrease. Hence, there is an optimal number of antennas, for which the capacity is maximum. An argument like this one has already been presented earlier [5]. However, the optimal number of antennas has not been evaluated. To accomplish this, we use the MIMO channel model presented in [7]. Strictly speaking, the model in [7] is a two-dimensional one. However, it can be applied to both orthogonal planes and, due to the symmetry of the problem (no preferred direction), the same result should hold in 3D as well. We note that, under the assumptions above, the angle-of-arrival (AOA) of multipath components is uniformly distributed over $\left[0,2\pi\right]$ in both planes. Thus, the model above can be applied and the minimum distance is half a wavelength. Due to the assumption of uniform scattering media, all the antennas experience the same multipath environment. Using the argument and the minimum distance above, we easily estimate the maximum number of antennas ("sphere packing") and, hence, the maximum MIMO capacity of a given region of space:

$$n_{opt} \approx \frac{6V}{V_S} = \frac{288V}{\pi\lambda^3} \text{ and } C_{\max} \approx n_{opt} \log_2\left(1+\frac{\rho}{n_{opt}}\right) \tag{8}$$

where V is the volume of the space region considered, and λ is the wavelength, and factor 6 is due to 6 polarizational degrees of freedom. The last equality holds due to the rich scattering assumption (i.e., the channel is uncorrelated).

Some conclusions from this analysis are as follows. The maximum capacity depends on the size of the space region considered as well as on the wavelength. Thus, a spatial capacity can be defined as the maximum capacity of a unit space region (bits/s/Hz/m^3), similar to the conventional capacity of a unit bandwidth (bits/s/Hz). Hence, space can be considered as a capacity-bearing object, similar to the bandwidth. The time (or frequency) domain and the space domain are included in this notion of capacity on an equal basis, which is in good agreement with the basic physical concepts (i.e., the special relativity). While the traditional approach allows us to define the capacity of a unit bandwidth (i.e., 1 Hz), the new limit defines the capacity of a unit space region (i.e., 1 m^3). We believe this partially answers the second question stated at the beginning. Our final remark is that while the limit above may be far away from any practically-achievable bit rate in a foreseeable future, it does provide some insight in the relation between the laws of electromagnetism and the laws of information theory.

ACKNOWLEDGEMENTS

The author wishes to thank Dr. C. Charalambous for many fruitful discussions.

References

[1] D.S. Shiu, G.J. Foschini, M.J. Gans, J.M. Kahn, Fading Correlation and Its Effect on the Capacity of Multielement Antenna Systems, IEEE Trans. on Communications, v. 48, N. 3, Mar. 2000, pp. 502-513.
[2] A. M. Sengupta, P. P. Mitra, "Capacity of multivariate channels with multiplicative noise: I. Random matrix techniques and large-N expansions for full transfer matrices," Bell Labs., Tech. Rep., 2001.
[3] S.L. Loyka, Channel Capacity of Two-Antenna BLAST Architecture, Electronics Letters, vol. 35, No. 17, 19th Aug. 1999, pp. 1421-1422.
[4] S.L. Loyka, J.R. Mosig, Channel Capacity of N-Antenna BLAST Architecture, Electronics Letters, vol. 36, No.7, pp. 660-661, Mar. 2000.
[5] S.L. Loyka, J.R. Mosig, Spatial Channel Properties and Spectral Efficiency of BLAST Architecture, AP2000 Millennium Conference on Antennas & Propagation, Davos, Switzerland, 9-14 April, 2000.
[6] S.L. Loyka, Channel Capacity of MIMO Architecture Using the Exponential Correlation Matrix, IEEE Communication Letters, v.5, N. 9, pp. 369 –371, Sep 2001.
[7] S. Loyka, G. Tsoulos, Estimating MIMO System Performance Using the Correlation Matrix Approach, IEEE Communication Letters, v. 6, N. 1, pp. 19-21, Jan. 2002.
[8] S. Loyka, A. Kouki, New Compound Upper Bound on MIMO Channel Capacity, IEEE Communications Letters, v. 6, N. 3, pp. 96-98, Mar. 2002.
[9] A. L. Moustakas et al, Communication Through a Diffusive Medium: Coherence and Capacity, Science, v. 287, pp. 287-290, Jan. 2000.
[10] M.R. Andrews, P.P. Mitra, R. deCarvalho, Tripling the Capacity of Wireless Communications Using Electromagnetic Polarization, Nature, v.409, pp. 316-318, 18 Jan. 2001.
[11] G.J. Foschini, M.J Gans: 'On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas', Wireless Personal Communications, vol. 6, No. 3, pp. 311-335, March 1998.