27th URSI General Assembly, Maastricht, The Netherlands, Aug. 17-24, 2002.

MIMO Channel Capacity: Electromagnetic Wave Perspective

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Abstract

- MIMO capacity is crucially affected by radio propagation channel. Uncorrelated high-rank channel -> maximum capacity (roughly linear in the number of Tx/Rx antennas)
- Channel correlation decreases the capacity and, at some point, it is the dominant effect. Analysis methods: the eigenvalue (or SVD) decomposition approach and the correlation matrix approach.
- What are *the constrains imposed by the laws of electromagnetism on achievable MIMO capacity in its most general form*?
- *Spatial capacity* is defined as the maximum of the conventional MIMO capacity over possible propagation channels, subject to some constrains.
- This maximum is shown to exist. It provides new insight into the notion of channel capacity and the link between information theory and the laws of electromagnetism.

Introduction

- Propagation channel affects crucially MIMO system performance.
- Ideal case: the channel is uncorrelated and of high rank -> MIMO capacity is maximum and scales roughly linearly as the number of Tx/Rx antennas.
- Bad (realistic) case: channel is correlated -> capacity is low.
- Maxwell equations control EM field (waves).
- *What is, if any, the impact of Maxwell equations on the notion of information in general and on channel capacity in particular?*

MIMO Channel Capacity

- AWGN fixed channel, Rx knows the channel -> celebrated Foschini-Telatar formula:

$$C = \log_2 \det\left(\mathbf{I} + \frac{\rho}{n} \mathbf{G} \cdot \mathbf{G}^{+} \right)$$

- *n* is the numbers of Tx/Rx antennas, ρ is the average SNR, **I** is n×n identity matrix, **G** is the normalized channel matrix
- MIMO channel capacity crucially depends the propagation channel **G** !
- The impact of Maxwell equations comes through **G** .

The Laws Of Electromagnetism

- Maxwell equations:

$$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

- Fields in source-free region -> wave equation:

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0, \quad \nabla^2 \mathbf{H} - \frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2} = 0$$

- There are 6 field components (“polarization degrees of freedom”). Anyone can be used for communication.
- Only two of them “survive” in free space (“poor” scattering) .

The Laws of Electromagnetism

- Frequency-domain representation:

$$\phi(\mathbf{r},\omega) = \int \phi(\mathbf{r},t)e^{-j\omega t}dt \Rightarrow \nabla^2\phi(\mathbf{r},\omega) + \left(\omega / c\right)^2 \phi(\mathbf{r},\omega) = 0$$

- where ϕ is any of the components of **E** or **H** .
- Plane-wave spectrum expansion:

$$\phi(\mathbf{k},\omega) = \int \phi(\mathbf{r},\omega)e^{j\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}$$

$$\phi(\mathbf{r},t) = \frac{1}{(2\pi)^4}\iint \phi(\mathbf{k},\omega)e^{j(\omega t-\mathbf{k}\cdot\mathbf{r})}d\mathbf{k}d\omega \Rightarrow \left(|\mathbf{k}|^2 - \left(\frac{\omega}{c}\right)^2\right)\phi(\mathbf{k},\omega) = 0$$

- Since EM waves are used to carry information, the channel matrix entries must satisfy the same wave equation!

Spatial Capacity

- MIMO channel capacity C -> maximum (over Tx vector) mutual information. Depends on **G**.
- Next step – maximize C over **G** -> spatial capacity!
- Maximization over **G** is subject to some constrains.
- Maxwell equations is one of them !(i.e., **G** must satisfy the wave equation)
- Does this maximum exists? If so, what is it ? What are the main factors that have an impact on it?
- To answer these questions, one has to unite information theory and electromagnetic wave theory.
- Reduced ("practical") version of the problem: Tx & Rx antennas are limited to given apertures.

Spatial Capacity: Fundamental Limit

- How to find the fundamental limit, which is due to the laws of electromagnetism?
- Get rid of all design-specific details!
- The following assumptions are adopted:
  - *limited region of space* is considered (similar to limited power!)
  - the *richest scattering*: infinite number of ideal scatterers, uniformly distributed, which do not absorb the EM waves
  - Tx & Rx antenna elements are *ideal field sensors*, with no size and no mutual coupling
- Capacity is linear in the number of antennas -> use as many antennas as possible!
- Is there any limit to this?

Spatial Capacity: Fundamental Limit

- Increasing the number of antennas increases capacity at first.
- Later, one has to reduce antenna spacing to accommodate more antennas within limited space.
- This increases correlation and decreases capacity!
- Some minimum antenna spacing must be respected in order to avoid loss in capacity.
- 2-D analysis shows that this limit is about half a wavelength:

$$d_{\min} \approx \lambda / 2$$

- 3-D case – no rigorous results, but intuitively (extrapolate 2-D) it should be around half a wavelength as well.

Spatial Capacity: Fundamental Limit

- Limited region of space -> limited number of antennas (due to the minimum spacing!)
- Use "sphere packing" argument to estimate it:

$$n_{opt} \approx \frac{6V}{V_S} = \frac{288V}{\pi\lambda^3} \quad \Rightarrow \quad C_{\max} \approx n_{opt} \log_2\left(1 + \rho / n_{opt}\right)$$

- where *V* is the volume of the space region, ? is SNR, and factor 6 is due to 6 "polarizational" degrees of freedom.
- C_{max} is the maximum capacity the region of space of volume *V* is able to provide.

Conclusions

- MIMO capacity depends on propagation channel.
- Is there any fundamental limit on it?
- Spatial capacity is defined as the maximum MIMO capacity over channel matrix **G**.
- Fundamental limit is imposed on the capacity by the laws of electromagtnetism.
- This limit comes in a form of minimum antenna spacing.
- This limits spatial capacity.

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