CORRELATION AND MIMO COMMUNICATION ARCHITECTURE (INVITED)

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Abstract: In this paper, we give an overview of MIMO architecture and the impact of correlation on its operation using the correlation matrix approach. First, we derive a universal upper bound on the MIMO channel capacity, which is not limited to a particular scenario, using the Jensen's inequality. This bound accounts for both transmit and receive branch correlation in such a way that the impact of these branches can be estimated separately, which simplifies the procedure substantially. Some simple analytical results, which quantify the impact of correlation on the MIMO capacity in an explicit way, are given. We show that correlation increase is equivalent to SNR decrease in some cases. The concept of MIMO effective dimensionality is further introduced. Using a block correlation matrix model, we show that the effect of correlation is to decrease the effective dimensionality. We also discuss the paradox of zero correlation and provide a statistical explanation for it. We demonstrate why zero mean correlation is not a guarantee of high capacity. Finally, we introduce the concept of adaptive MIMO architecture and discuss the fading reduction provided by it.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) communication architecture has recently emerged as a new paradigm for very efficient wireless communications in rich multipath environments [1-5]. Using multi-element antenna arrays (MEA) at both transmitter and receiver, which effectively exploits the third (spatial) dimension in addition to the time and frequency dimensions, this architecture achieves channel capacities far beyond those of traditional techniques. Fig. 1 shows the MIMO architecture. Incoming bit stream $b_1b_2b_3$ is splitted into three substreams and transmitted by corresponding antennas. At the receiver, a vector signal processor is employed to extract the bits from the received signal, which is, in fact, the mixture of all the transmitted substreams. The function of the vector signal processor is to diagonalize the channel matrix. After it, the channel looks like n parallel independent subchannels, where n is the number of transmit/receive antennas (provided that the complete diagonalization is possible – see discussion below). Detail technical description of the MIMO architecture is available in [3].

In uncorrelated Rayleigh channels the MIMO capacity scales linearly as the number of antennas [1,2,5]in contrast to conventional systems where it scales logarithmically. Thus, substantial increase (order of magnitude) in capacity

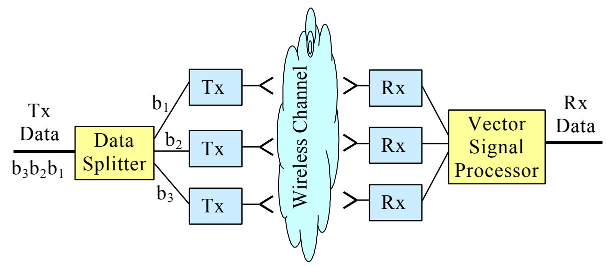

Figure 1. Multiple-input multiple-output communication architecture.

is possible. Using a prototype system, as high capacities as 30-40 bit/s/Hz has been demonstrated in the laboratory environment [4], which is simply unattainable using traditional techniques.

However, there are several limitations to the performance of this architecture in real-world conditions [1,2,6,7]. One of the major limitations is the correlation of individual sub-channels, i.e. links between one transmitter and one receiver antennas, of the matrix channel, which may result in severe degradation of MIMO performance [8-11]. We analyze the effect of correlation using the correlation matrix approach [9-12, 14, 15]. In Section II, we derive the universal upper bound on the MIMO capacity of a stochastic channel using Jensen's inequality. In Section III, we give some approximate analytical results, which quantify the effect of correlation in an explicit way and provide useful insight. In Section IV, we discuss the concept of effective dimensionality, which is introduced through a comparison of the MIMO capacity for correlated and uncorrelated channels. In Section V, we discuss the paradox of zero correlation and give a statistical explanation for it. In Section VI, we introduce the concept of adaptive MIMO architecture.

II. UNIVERSAL UPPER BOUND ON MIMO CHANNEL CAPACITY: CORRELATION MATRIX APPROACH

For a fixed linear $n \times n$ matrix channel with additive white gaussian noise and when the transmitted signal vector is composed of statistically independent equal power components each with a gaussian distribution, the channel capacity is [1]:

$$C = \log_2 \det\left(\mathbf{I} + \frac{\rho}{n} \mathbf{H} \cdot \mathbf{H}^{+} \right) \text{ bits/s/Hz} , \quad (1)$$

where n is the number of transmit/receive antennas (we consider here the case when the number of transmit and receive antennas are equal), ρ is the signal-to-noise ratio (SNR), **I** is n×n identity matrix, **H** is the normalized channel matrix, which is considered to be frequency independent over the signal bandwidth, and "+" means transpose conjugate. We adopt here the following normalization condition:

$$\sum_{i,j=1}^{n} \left| h_{ij} \right|^2 = n \ , \quad (2)$$

where h_{ik} denotes the components of **H** (h_{ij} is the transfer factor between j^th transmit antenna and i^th receive antenna**).** Hence, ρ/n is the average per-branch SNR, i.e. ρ is the ratio of total received power (in all branches) to the per-branch noise level. Some other kinds of the normalization can also be used, but in this case ρ/n will have a slightly different meaning.

When the channel is random (stochastic), then the capacity is random, too. The mean (ergodic) capacity can be defined in this case as [5]:

$$\left\langle C \right\rangle = \left\langle \log_2 \det\left[\delta_{ij} + \frac{\rho}{n} \cdot r_{ij} \right] \right\rangle \ , \quad (3)$$

where r_{ij} is "instantaneous" correlation matrix,

$$r_{ij} = \sum_k h_{ik} h_{jk}^* \ , \quad (4)$$

δ_{ij} is Kroneker's delta, < > is the expectation over the channel matrix. Note that Eq. (3) does take into account correlation occurring at both the transmit and receive ends. This equation can be used for statistical (Monte-Carlo) simulations to evaluate $\left\langle C \right\rangle$ for some specific models of the channel matrix. However, these matrix numerical computations can be very lengthy, especially when the number of antennas is very large. Here we propose to use Jensen's inequality to obtain an upper bound on $\left\langle C \right\rangle$. According to this inequality and concavity of *log det* function [17], one obtains:

$$\left\langle C \right\rangle \le \overline{C_R} = \log_2 \det\left[\delta_{ij} + \frac{\rho}{n} \cdot r_{ij}^R \right] \quad (5)$$

where r_{ij}^R is the correlation matrix of receive branches,

$$r_{ij}^R = \sum_k \left\langle h_{ik} h_{jk}^* \right\rangle \ , \quad (6)$$

Note that this correlation matrix does not capture the correlation of transmit branches (since k in (6) represents the transmit antenna index and it is the same for both factors). Thus, the upper limit in (5) can be close to the mean capacity when the correlation of receive branches is much higher than the correlation of transmit branches and, consequently, the effect of transmit branch correlation can be ignored. However, if the transmit correlation is higher than the receive one, then the upper bound in (5) is not an accurate approximation of the mean capacity. Therefore, in order to have an upper bound that is as close as possible to the mean capacity, one must also account for transmit correlation. To this end, the reciprocity of (1) can be used in the following way. First, we note that the MIMO capacity given by (1) is invariant under the transformation $\mathbf{H} \to \mathbf{H}^{\mathrm{T}}$ ("T" means transpose). This in effect is equivalent to reversing the direction of information transmission by interchanging transmit and receive ends. Thus, (3) still holds true if we define r_{ij} as:

$$r_{ij} = \sum_k h_{ki} h_{kj}^* \ , \quad (7)$$

Hence, one obtains the second upper bound (the transmit bound),

$$\left\langle C \right\rangle \le \overline{C_T} = \log_2 \det\left[\delta_{ij} + \frac{\rho}{n} \cdot r_{ij}^T \right] \quad (8)$$

where r_{ij}^T is the correlation matrix of transmit branches,

$$r_{ij}^T = \sum_k \left\langle h_{ki} h_{kj}^* \right\rangle \ , \quad (9)$$

Note that the upper bound in (8) does not capture the receive correlation. Therefore, this upper bound will be close to the mean capacity when the transmit correlation is higher than the receive one. However, if the opposite is true, then this upper bound is not an accurate approximation of the mean capacity.

From inequalities (5) and (8) it is clear that a tighter upper bound of the mean channel capacity can be obtained by combining them. Thus, we form the compound upper bound by taking minimum of the two bounds defined above,

$$\overline{C_{cmp}} = \min\left[\overline{C_R}, \overline{C_T} \right] \quad (10)$$

This upper bound is much tighter than the receive or transmit bound considered separately when the transmit and receive branch correlations are significantly different.

Let us now consider an illustrative example of correlated Rayleigh channel. The components of **H** are taken to be identically distributed complex gaussian variables (real and imaginary parts are identically distributed and independent, i.e. the phase is uniformly

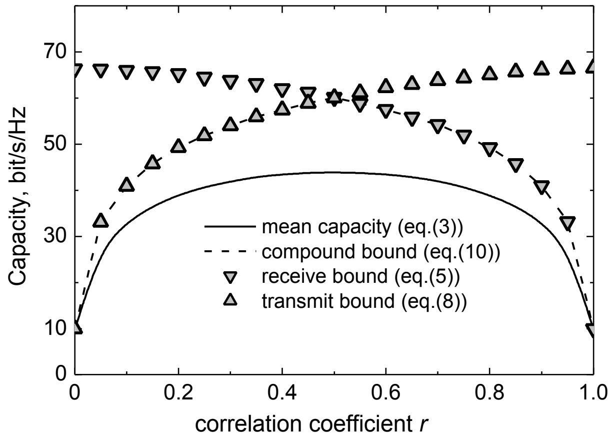

Figure 2. MIMO channel capacity and its upper bounds versus correlation coefficient

distributed over $[0,2\pi]$) with zero mean and unit variance. The correlation matrix of **H** is assumed to be of the following form:

$$R_{ij,km} = \langle h_{ik} h_{jm}{}^* \rangle = R_{ij}^R \cdot R_{km}^T, \tag{11}$$

where R_{ij}^R and R_{ij}^T are uniform correlation matrixes of the receive and transmit branches correspondingly,

$$R_{ij}^R = \begin{cases} r, & i \neq j \\ 1, & i = j \end{cases}, \quad R_{km}^T = \begin{cases} 1-r, & i \neq j \\ 1, & i = j \end{cases}, \tag{12}$$

where $0 \le r \le 1$. In fact, (11) assumes that the receive and transmit branches are correlated independently on each other (which may be justified by the presence of local scatterers near both ends). Fig. 2 shows the mean capacity of this channel, obtained by extensive numerical simulations (Eq. 3), and the receive (Eq. 5), transmit (Eq. 8) and compound (Eq. 10) bounds. In this example, $r = 0$ corresponds to uncorrelated receive branches and full correlation of the transmit ones; $r = 1$ corresponds to full correlation of receive branches and uncorrelated transmit ones. The compound bound provides a good approximation to the mean capacity while the receive or transmit bounds alone are not accurate for the whole range of *r*. It is also interesting to note that the maximum capacity is achieved for $r = 0.5$. This indicates that decrease in capacity is usually due to that side (transmit or receive) which has higher correlation. Thus, a rough estimation of the capacity may be obtained by considering only the higher correlated side.

Note that the compound upper bound accounts for the Tx and Rx correlations in such a way that their impact can be estimated separately. Thus, a conclusion can be made as to which site contributes more to capacity reduction, which is not easy to do using the mean capacity or the capacity-versus–outage distribution.

III. Some Analytical Results

Using the upper bound on MIMO channel capacity derived above, one may apply the analytical results on the MIMO capacity of a deterministic channel [9-12] to the case of random channel, i.e. to obtain the upper bound. For simplicity, we assume here that the transmit branches are not correlated and the correlation impact is due to the receive branch correlation. Obviously, the impact of transmit branch correlation can be estimated in a similar way and the combination of the results is trivial. It is also assumed that the receive power is identical for all the receive branches. In this case,

$$\sum_j \left|h_{ij}\right|^2 = 1 \tag{13}$$

and r_{ij} in (4) is the normalized correlation matrix, $\left|r_{ij}\right| \le 1$. The effect of unequal received powers can be considered in a straightforward way [9].

We start with the uniform correlation matrix model, when all the correlation coefficients are equal and real [10],

$$r_{ij} = \begin{cases} r, & i \neq j \\ 1, & i = j \end{cases} \qquad \mathrm{Im}[r] = 0 \tag{14}$$

This case is somewhat artificial because one expects that the correlation of neighbouring branches is larger than that of distant branches. However, the case of equal correlation coefficients provides a worst-case estimation and some insight into MIMO operation in correlated channels, so it deserves to be considered (besides, one may interpret *r* as an "average" correlation coefficient). After some mathematical development for a practically-important case of $0 \le r < 1$ and $\rho / n >> 1$, we present the upper bound (5) in the following form:

$$\overline{C} \approx n \cdot \log_2 \left(1 + \frac{\rho}{n} (1-r) \right) \tag{15}$$

As a detail analysis shows, the channel capacity decreases substantially only for $|r| \ge 0.5 - 0.8$, what agrees well with the recent measurements of the MIMO channel [16]. In the limiting case of $n \to \infty$, one obtains from (15):

$$\overline{C_\infty} \approx \frac{\rho(1-r)}{\ln 2} \tag{16}$$

When *r=0*, the last two equations reduce to the well-known formulas (in this case, **H**=**I**) [1]:

$$\overline{C} = n \cdot \log_2 \left(1 + \frac{\rho}{n} \right) \text{ and } \overline{C_\infty} = \frac{\rho}{\ln 2} \tag{17}$$

Comparison of (15) and (16) with (17) clearly indicates that the effect of the channel correlation is equivalent to the decrease in the SNR. Hence, for example, *r=0.5* is equivalent to 3 dB reduction in SNR. Another interpretation of (15) and (16) is that the correlation of individual sub-channels gives an increase in the noise level because for each particular sub-channel all the other sub-channels are just the sources of interference.

Let us now consider the case of exponential correlation matrix [12],

$$r_{ij} = \begin{cases} r^{j-i}, & i \le j \\ r_{ji}^*, & i > j \end{cases}, \quad |r| \le 1 \tag{18}$$

Obviously, (18) may be not an accurate model for some real-world scenarios but this is a simple single-parameter model which allows one to study the effect of correlation on the MIMO capacity in an explicit way and to get some insight. It remains to be investigated whether this model is applicable or not to some specific scenarios. Note, however, that this model is physically reasonable in the sense that the correlation decreases with increasing distance between receive antennas. Thus, it should be more accurate than the uniform model above. In a practically-important case of high SNR ($\rho/n >> 1$), *n>>1* and *r<1*, (5) can be reduced to:

$$\overline{C} \approx n \cdot \log_2\left(1 + \frac{\rho}{n}\left(1 - |r|^2\right)\right) \tag{19}$$

In the limiting case of $n \longrightarrow \infty$, one obtains from (19):

$$\overline{C_\infty} \approx \frac{\rho}{\ln 2}\left(1 - |r|^2\right) \tag{20}$$

Comparison of (19) and (20) with (17) clearly indicates that the effect of the channel correlation is equivalent to the SNR loss, the same as for the uniform model above. Hence, for example, *r=0.7* is equivalent to 3 dB reduction in the signal-to-noise ratio. Note also that the channel capacity does not depend on the correlation coefficient phase.

Fig. 3 shows the upper bound of MIMO channel capacity versus the correlation coefficient evaluated by the full matrix computation (Eqs. (5) and (18)) and by (19) for *n*=10 and 50, and ρ=30 dB. The MIMO channel capacity evaluated using the uniform correlation matrix model (see (14)) is also shown for comparison. As one may see from this figure, the accuracy of approximate formulas is quite good. It should be noted that the accuracy decreases as *n* and ρ/*n* decreases. The uniform model predicts lower capacity, as it should be (because it is the worst case model – the correlation between distant receive branches is the same as between neighbouring ones). We also see that the MIMO capacity decreases significantly for *r>0.5-0.8*, that agrees well with the know results on the spatial diversity techniques [18] and with recent measurements of the MIMO

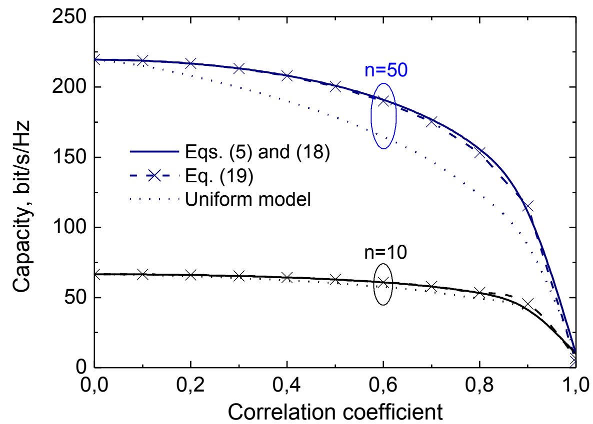

Figure 3. MIMO capacity upper bound versus correlation coefficient for exponential and uniform models.

channel [16]. Detailed analysis using Monte-Carlo simulations shows that the mean capacity is approximately 20 to 40% smaller then the upper bound above.

IV. Effective dimensionality of MIMO system

For simplicity, we further consider the case of equal received powers in every receive branch (see (13)). In this case, (1) simplifies to [10, 11]:

$$C(\mathbf{R},n) = \log_2 \det\left(\mathbf{I} + \frac{\rho}{n}\mathbf{R}\right), \tag{21}$$

where **R** is the normalized channel correlation matrix $\left(|r_{ij}| \le 1\right)$ whose components are given by (4). Eq. (21) emphasizes that the MIMO channel capacity *C*(**R**,*n*) is a function of the correlation matrix **R** and of the number *n* of antennas. We define the MIMO effective dimensionality n_e from the following equation:

$$C(\mathbf{R},n) = C(\mathbf{I},n_e), \tag{22}$$

where $C(\mathbf{I},n_e)$ is given by (17) for $n = n_e$. Thus, the effective dimensionality is the number of dimensions of a MIMO system operating over an uncorrelated channel, which has the same channel capacity as the actual system operating over the actual correlated channel (the signal-to-noise ratio ρ being the same for both cases). The ED shows how efficiently we use the actual receive and transmit branches and is, hence, a system performance parameter. In general, (22) is a transcendental equation and cannot be solved analytically for n_e. Numerical methods can be applied to solve it. No convergence problems are anticipated since both sides of (22) are monotonous functions of *n*. For some specific cases, analytical techniques can be used which allow us to gain some insight and to obtain simple analytical solutions for practically-important cases. A similar concept to the ED has been introduced in [8] as the number of effective degrees of freedom of a MIMO system:

$$EDOF = \frac{d}{d\delta} C(2^{\delta}\rho)\Big|_{\delta=0} \tag{23}$$

In some cases, the ED and the EDOF give very close values while in some other cases their values are very different. We give below a comparative analysis of the ED and the EDOF for some important cases. Similar parameters have also been considered in [2].

Let us now consider the case where the correlation matrix $\mathbf{R}$ has a block diagonal structure:

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n-k} \end{bmatrix} \tag{24}$$

where $\mathbf{I}_{n-k}$ is the $(n-k)\times(n-k)$ identity matrix, $\mathbf{0}$ is the zero matrix, and $\mathbf{R}_k$ is a $k \times k$ correlation sub-matrix with non-zero components. In this model, only k receive branches are correlated; the rest (n-k) branches are independent. In order to demonstrate how the ED concept works, we adopt here the exponential model for $\mathbf{R}_k$ given by (18). Other models of $\mathbf{R}$ (using, for example, electromagnetic simulation) can also be implemented. Using (21) and (24), $C(\mathbf{R}, n)$ can be presented in the following form:

$$C(\mathbf{R}, n) = \log_2\left(\left(1+\frac{\rho}{n}\right)^n \det \overline{\mathbf{R}}_k\right), \tag{25}$$

where $\overline{\mathbf{R}}_k$ is the following $k \times k$ matrix:

$$\left[\overline{\mathbf{R}}_k\right]_{ij} = \begin{cases} \beta r^{j-i}, & i \le j \\ 1, & i = j \\ \beta\left(r^{i-j}\right)^*, & i > j \end{cases} \tag{26}$$

and

$$\beta = \frac{\rho}{n}\left(1+\frac{\rho}{n}\right)^{-1} \tag{27}$$

Unfortunately, it is impossible to obtain a closed-form expression for the $\det \overline{\mathbf{R}}_k$ in general cases. Still, for some practically-important cases, a simplified expression may be obtained. For a system having a large signal-to-noise ratio ($r/n >> 1$) and a large number of antennas (n>>1), after some transformations which do not change the determinant, we obtain [12]:

$$\det\left[\overline{\mathbf{R}}_k\right] \approx \left(1-\beta|r|^2\right)^{k-1} \tag{28}$$

Approximate solution of (22) then takes the simple form:

$$n_e \approx n - \gamma(k-1) \tag{29}$$

where

$$\gamma = 1 - \frac{\log_2\left(1+\frac{\rho}{n}\left(1-|r|^2\right)\right)}{\log_2\left(1+\frac{\rho}{n}\right)} \tag{30}$$

For $|r| = 1$, one obtains $\gamma = 1$ and, consequently, $n_e \approx n - k + 1$. Thus, the reduction in system effective dimensionality Δn due to the strong correlation of k receive branches is $\Delta n = n - n_e \approx k - 1$. This is a physically reasonable conclusion because we cannot transmit information independently over these k branches but have to use them as only one branch. For r=0, $\gamma = 0$ and $\Delta n = 0$, and the system has full dimensionality as it should be. Fig. 4 shows Δn as a function of $|r|$ for different values of k computed using (29) and (30) and by numerical solution of (22) using (24). As one may see, eqs. (29) and (30) provide quite a good approximation. An interesting question however is how strong should the correlation be for $\Delta n \approx k - 1$. A detailed analysis (as well as an examination of Fig. 4) gives the following rough estimation: $|r| \ge 1 - n/(2\rho)$. For the scenario of Fig. 4, one obtains: $|r| \ge 0.995$. Thus, for almost all practical cases (when $\rho/n >> 1$) Δn will be smaller than $k-1$.

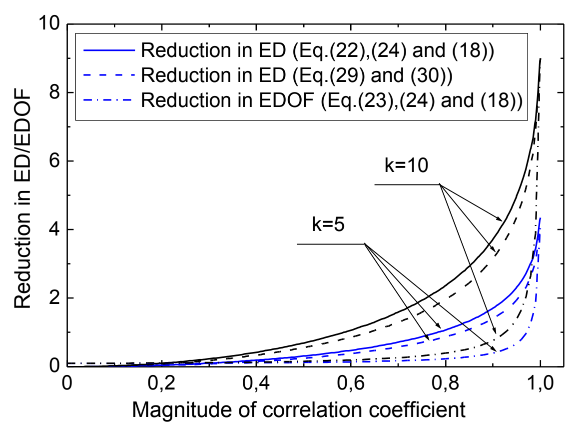

Figure 4. Reduction in ED/EDOF versus the magnitude of correlation coefficient, *n*=10, r=30 dB.

Let us now compare the ED and the EDOF concepts for a practically-important case of $\rho/n >> 1$ and n>>1. A detail analysis (as well as Fig. 4) shows that for $|r| = 1$ and $|r| = 0$ the ED and the EDOF give approximately the same prediction. But in between these two extreme cases their values are different, as Fig. 4 illustrates. One may wonder: what is the reason for this difference? Both parameters have very similar physical meaning and both characterize the system performance from the same viewpoint. However, in the ED concept the real system performance (channel capacity) is compared to the ideal system performance, which operates over uncorrelated parallel subchannels, for the same total transmitted power. The ideal system is taken as a reference. Thus, one may conclude that in the ED concept the total transmitted power of the ideal system is distributed between the effective dimensions only, not between the actual number of transmitters. On the other hand, EDOF is determined through variation in SNR, i.e. in the transmitted power, for fixed channel correlation. There exists no explicit reference (ideal) system in this case and the total transmitted power is always distributed between the actual number of transmitters. Both concepts can be used for estimating MIMO system performance but from different viewpoints. In the EDOF concept, the effect of SNR is emphasized and, hence, it is more relevant when one wants to know how the actual system performance varies with transmitted power for fixed correlation. On the contrary, the effect of channel correlation is emphasized in the ED concept and, consequently, it is more relevant when one wants to know how the system performance varies with channel correlation for fixed power.

It should be noted that we assumed in this Section that the channel is deterministic. If it is random (i.e., Rayleigh fading) then the capacity is random as well. In this case, one needs to use the upper bound (10). Thus, the results above hold for the capacity upper bound of a random ergodic channel. However, as detailed analysis shows, the upper about above is usually only 20-40% larger than the mean capacity (this is roughly the same as for conventional SISO systems). The type of channel coefficient distribution has no major impact on capacity, the main impact is due to channel correlation.

V. Paradox of Zero Correlation

MIMO channel capacity is usually thought of as limited by correlation: it is low for highly correlated channel, and it is high when the correlation between individual sub-channels (i.e. links between one transmit and one receive antenna) of the matrix (MIMO) channel is zero. However, an elegant example has been presented in [19], which demonstrates that zero correlation is not a guarantee of high capacity, i.e. the channel may have zero correlation and still only a single degree of freedom (these are so called degenerate channels or keyholes). However, no explanation has been provided to this phenomenon. In this section, we provide a statistical explanation of this phenomenon and, in particular, we emphasize that one should distinguish between "instantaneous" and "mean" (or conventional) correlation. We also present a general statistical criterion for the channel to be degenerate and propose a method to estimate the capacity of those channels.

Let us now consider 2x2 deterministic MIMO channel. (1) takes the following form in this case:

$$C = \log_2\left(\left(1+\frac{\rho}{2}\right)^2 - \left(\frac{\rho}{2}\right)^2 |R_{12}|^2\right) + \log_2(r_{11}r_{22}) \quad (31)$$

where R_{12} is the normalized correlation coefficient ($|R_{12}| \le 1$),

$$R_{12} = \frac{r_{12}}{\sqrt{r_{11}r_{22}}} \quad (32)$$

In fact, r_{11} and r_{22} represent the normalized received power in 1st and 2nd branch correspondingly. Hence, the last term in (31) describes the effect of SNR. The eigenvalues λ of r_{ij} can be obtained from the following equation:

$$\lambda^2 - (r_{11}+r_{22})\lambda + r_{11}r_{22}\left(1-|R_{12}|^2\right) = 0 \quad (33)$$

The singular values of h_{ij} are square roots of λ [5]. Thus, correlation has the major impact on the number of degrees of freedom (i.e., non-zero singular or eigenvalues): there are two degrees of freedom when $|R_{12}| < 1$ and only one when $|R_{12}| = 1$, as long as the received powers are not zero.

When the channel is random, the mean (ergodic) capacity may be defined using the expectation over the channel matrix in (31) (see eq. (3)) [5]. Detailed analysis using Monte-Carlo simulations of the correlated Rayleigh channel and of the channel in [19] shows that the impact of the second term in (31) on the mean capacity is much smaller than that of the first term for $\rho >> 1$. The same conclusion may be obtained using Jensen's inequality. The second term mainly accounts for varied received powers. On the contrary, correlation has the major impact on the mean capacity. Hence, to isolate and study the effect of correlation, we neglect the second term. In this case, the mean capacity depends on R_{12} only, which is "instantaneous" correlation coefficient, and can be presented as follows:

$$\langle C \rangle = \int_{DR_{12}} C(R_{12}) \cdot f(R_{12}) dR_{12} \quad (34)$$

where $f(R_{12})$ is the probability density function (PDF) of R_{12}, and DR_{12} is the range of R_{12}. Thus, the mean capacity depends on the PDF of R_{12}, not only on its mean value. In general, the mean correlation is not a reliable tool in estimating the MIMO capacity of a random channel.

Let us now consider an illustrative example, when $R_{12} = \pm 1$ with equal probability. Obviously, the mean correlation is zero but the mean capacity is low and there is only one degree of freedom just because $|R_{12}| = 1$. The next example is provided in [19]. In that case,

$$R_{12} = e^{j(\varphi_1 - \varphi_2)} \tag{35}$$

where $\varphi_1 = \arg(b_1)$, $\varphi_2 = \arg(b_2)$, and b_1 and b_2 are scattering coefficients (see [19] for detail discussion). Again, $\langle R_{12} \rangle = 0$ because φ_1 and φ_2 are independent and uniformly distributed, but the mean capacity is low and there is only one degree of freedom because $|R_{12}| = 1$. From the considerations above, we may conclude the following:

- The capacity of deterministic channel is maximum when $R_{12} = 0$ (see (31)). However, as the examples above show, it is wrong to state the same about random channel using the mean correlation, i.e. zero mean correlation of a random channel is not a sufficient condition of maximum mean capacity. Using inequality (5), we conclude that it is the necessary condition (i.e., if the mean correlation is high, than the mean capacity is necessarily low).
- Referring to eq. (31), we conclude that the sufficient condition of high capacity is low mean magnitude correlation. For example, if $\langle |R_{12}| \rangle = 0$, then there are two degrees of freedom and the mean capacity is maximum simply because $R_{12} = 0$ in this case.

The general conditions for a channel to be degenerate are,

$$\langle R_{12} \rangle = 0 \quad \text{and} \quad |R_{12}| = 1 \tag{36}$$

From practical viewpoint, $|R_{12}|$ may not be equal to 1 but be close to it. The capacity will be low in this case as well. In particular, according to the results of Section III, it will be low when $|R_{12}| \geq 0.5 - 0.8$. In the case of degenerate channels, the mean correlation does not provide an accurate estimation of the capacity. Figure 5 illustrates the channel capacity for PDF of R_{12} of the following form:

$$f(R_{12}) = c \cdot \exp\left(\frac{|R_{12}|}{\alpha}\right), \quad |R_{12}| \leq 1 \tag{37}$$

where c is normalizing constant, and α determines the root-mean-square (RMS) value of R_{12} (note that α may be negative as well as positive). Obviously, the mean correlation is zero and its use for estimating the capacity will give an incorrect result. As figure 5 indicates, a more accurate estimation of the capacity of degenerate channels can be obtained using RMS or mean magnitude correlation.

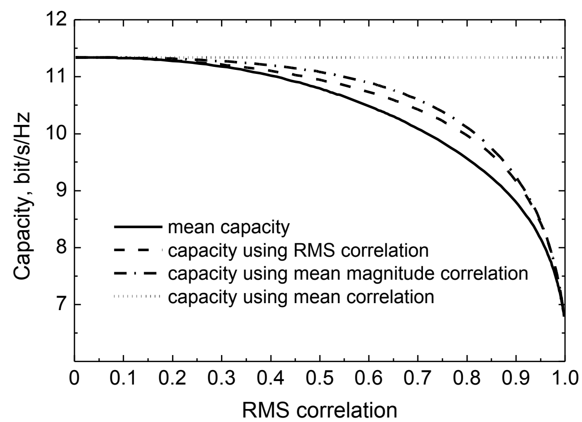

Figure 5. MIMO capacity using RMS, mean magnitude and mean correlation versus RMS correlation for ρ=20 dB.

It is interesting to note that the eigenvalue approach, which is widely used for the MIMO system analysis, is more formal mathematically and does not provide this insight.

VI. Fading and Adaptive MIMO Architecture

In general, MIMO architecture can provide four advantages: (1) high channel capacity, (2) low fade depth, (3) low cochannel interference, (4) highly secure communication. However, all these advantages cannot be achieved at the same time. Thus, an adaptive MIMO architecture can be built, which operates in one of the four modes: (1) high capacity mode (few tens or even hundreds bit/s/Hz), (2) low fading mode (10-30 dB reduction in fading), (3) low interference mode (5-15 dB reduction in intereference), (4) high security mode.

In the low fading mode, the fade depth level for MIMO architecture is substantially smaller than for SISO or SIMO systems because the diversity order of MIMO system is n^2, and SISO and SIMO systems – 1 and n correspondingly (however, some space-time coding is required for MIMO system to achieve this diversity order). Besides, the MIMO system efficiently exploits diversity at both Tx and Rx sites. Thus, for example, no any advantage is provided by SISO or SIMO if fading is correlated at Rx site, but MIMO provides fading improvement even in this case (if Tx site fading is not correlated). Hence, the MIMO system availability is twice that of SIMO, if Tx and Rx fadings are not correlated. The outage probability versus fade depth curve slope is $10n^2$ dB/decade for MIMO, compared to $10n$ and 10 dB/decade for SIMO and SISO correspondingly. Hence, the advantage of using MIMO is high even for moderate n.

VII. CONCLUSION

In this paper, we have given a review of MIMO architecture and the impact of wireless channel correlation on its operation. The use of Jensen inequality allows one to estimate the MIMO channel capacity through the upper bound on it. Using the original and transposed channel matrix, the compound upper bound is formed, which accounts for both transmit and receive branch correlation in such a way that the impact of these branches can be estimated separately, which simplifies the procedure substantially. Extensive numerical simulations confirm that this bound is a quite accurate estimation of the mean MIMO capacity. The statistics of amplitude distribution of the matrix channel coefficients has no significant impact on the capacity – the main impact is due to correlation.

Using the bound above, we applied the results on MIMO capacity of deterministic channels to a random channel, i.e. estimated the capacity upper bound using the uniform and exponential correlation matrix models. These estimations agree well with the recent measurements of the MIMO radio channel. We have also shown that the increase in correlation is equivalent to the decrease in SNR.

The concept of MIMO effective dimensionality has been introduced and studied. Its purpose is to compare the actual system performance to that of the ideal system, whose channel capacity is maximum, and to characterize in this way the effect of correlation. Roughly speaking, ED is a factor in front of *log*(1+SNR) after correlation has been taken into account.

We have discussed the paradox of zero correlation and provided a statistical explanation for it. In particular, we have shown that one should distinguish between “average” (conventional) and “instantaneous” correlation. High magnitude correlation is the solution to this paradox. Zero average correlation is not a guarantee of high capacity. On the contrary, zero or low mean magnitude correlation is indeed a guarantee of high capacity. Mean magnitude or RMS correlation should be used for the capacity estimation of degenerate channels.

Finally, we have introduced the concept of adaptive MIMO architecture, which can achieve one or combination of some of the advantages provided by the MIMO architecture, and discussed fading in MIMO systems.

References

[1] Foschini, G.J., Gans M.J.: ‘On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas’, Wireless Personal Communications, vol. 6, No. 3, pp. 311-335, March 1998. (available for download in [3])

[2] Rayleigh, G.G., Cioffi, J.M.: "Spatio-Temporal Coding for Wireless Communications," IEEE Trans. Commun., v.46, N.3, pp. 357-366, 1998.

[3] www.bell-labs.com/project/blast

[4] G.D. Golden, G.J. Foschini, R.A. Valenzuela, P.W. Wolniansky, ‘Detection Algorithm and Initial Laboratory Results Using V-BLAST Space-Time Communication Architecture’, Electronics Letters, vol. 35, No. 1, pp.14-16, 7th January 1999.

[5] I.E. Telatar, "Capacity of Multi-Antenna Gaussian Channels," AT&T Bell Lab. Internal Tech. Memo., June 1995 (European Trans. Telecom., v.10, N.6, Dec.1999)

[6] Hochwald, B.M., Marzetta, T.L.: "Unitary Space-Time Modulation for Multiple-Antenna Communication Systems," IEEE Trans. Information Theory, v.46, N. 2, Mar. 2000, pp. 543-564.

[7] Driessen, P.F., Foschini, G.J.: "On the Capacity Formula for Multiple Input-Multiple Output Wireless Channel: A Geometric Interpretation", IEEE Trans. Communications, v. 47, N. 2, Feb. 1999, pp. 173-176.

[8] Shiu, D.S., Foschini, G.J., Gans, M.J., Kahn, J.M., 'Fading Correlation and Its Effect on the Capacity of Multielement Antenna Systems,' IEEE Trans. on Communications, v. 48, N. 3, Mar. 2000, pp. 502-513.

[9] Loyka, S.L., 'Channel Capacity of Two-Antenna BLAST Architecture,' Electronics Letters, vol. 35, No. 17, pp. 1421-1422, 19th Aug. 1999.

[10] Loyka, S.L., Mosig, J.R.: 'Channel Capacity of N-Antenna BLAST Architecture,' Electronics Letters, vol. 36, No.7, pp. 660-661, Mar. 2000.

[11] Loyka, S.L., Mosig, J.R.: 'Spatial Channel Properties and Spectral Efficiency of BLAST Architecture,' AP2000 Millennium Conference on Antennas & Propagation, Davos, Switzerland, 9-14 April, 2000.

[12] Loyka, S.L.: ' Channel Capacity of MIMO Architecture Using the Exponential Correlation Matrix', IEEE Communicatons Letters, 2000, submitted for publication

[13] V.A. Aalo, "Performance of Maximal-Ratio Diversity Systems in a Correlated Nakagami-Fading Environment," IEEE Trans. Commun., vol. 43, N. 8, Aug. 1995, pp.2360-2369.

[14] S. Loyka, A. Kouki, New Compound Upper Bound on MIMO Channel Capacity, IEEE Communications Letters, 2001, submitted

[15] S. Loyka, A. Kouki, On the Use of Jensen’s Inequality for MIMO Channel Capacity Estimation, 2001 Canadian Conference on Electrical and Computer Engineering (CCECE 2001), 13-16 May, Toronto.

[16] C.C. Martin, J.H. Winters, N.S. Sollenberger, Multiple-Input Multiple-Output (MIMO) Radio Channel Measurements, IEEE VTC’2000 Fall Conference, Sept. 24-28 2000, Boston, USA.

[17] T.M. Cover, J.A. Thomas, Elements of Information Theory, John Wiley & Sons, New York, 1991

[18] Jakes, W.C. Jr.: ‘Microwave Mobile Communications’, John Wiley and Sons, New York, 1974.

[19] D. Chizhik, G.J. Foschini, R.A. Valenzuela, ‘Capacities of multi-element transmit and receive antennas: Correlations and keyholes’, Electronics Letters, vol. 36, No. 13, pp.1099-1100, 22nd June 2000