On the Use of Cann's Model for Nonlinear Behavioral-Level Simulation

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***Abstract*—The use of Cann's model for modeling nonlinear circuits and systems is discussed in this paper. It is shown that this model has a nonphysical behavior for the small-signal regime in most cases (it has no some derivatives and, correspondingly, cannot be expanded in a Taylor series) and gives incorrect predictions for nonlinear products. When nonlinearities are to be taken into account in the simulation (intermodulation, adjacent channel power, etc.), the use of Cann's model should be avoided or at least it should be used with extreme care.**

***Index Terms*—Behavioral-level simulation, Cann's model, nonlinear modeling.**

I. Introduction

BEHAVIORAL-LEVEL techniques are presently very popular tools for the nonlinear simulation of mobile communication circuits and systems. Simulation accuracy and, consequently, the efficiency of a simulation technique as a design tool depends substantially on accurate modeling of nonlinear elements. Cann's model for a nonlinear input–output transfer characteristic [1] was used for the nonlinear modeling of amplifiers [intermodulation products (IMPs), spectral regrowth, etc.] in several recent publications [2]–[4]. However, as a detail consideration shows, this model gives inaccurate results in many cases and should consequently be used with extreme care when nonlinear effects (IMPs, harmonics, etc.) are analyzed. In this paper, we consider the use of Cann's model for the nonlinear product prediction and show the reason why this model fails to predict correctly the levels of these products in the small-signal regime (for weakly nonlinear circuits).

II. Cann's Model of Input–Output Transfer Characteritic

In this model, the input–output transfer characteristic is described by the following [1], [3], [4]:

$$y(x) = \frac{L \cdot \mathrm{sgn}(x)}{\left[1 + \left(\dfrac{l}{|x|}\right)^s\right]^{1/s}} = \frac{\dfrac{L}{l} \cdot x}{\left[1 + \left(\dfrac{|x|}{l}\right)^s\right]^{l/s}} \qquad (1)$$

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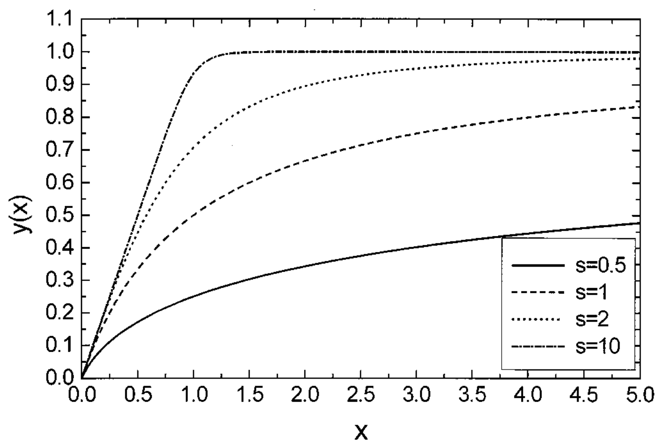

Fig. 1. Cann's model for various values of knee sharpness parameter $s(s = 0.5,\ 1,\ 2$, and $10)$; $L = l = 1$.

where

- x input voltage;
- y output voltage;
- L output limit (asymptote as $|x| \to \infty$) level;
- l input limit level;
- s knee sharpness parameter.

Fig. 1 shows various transfer characteristics described by (1). The modulus function in the denominator of (1) and the sign function $\mathrm{sgn}(x)$ in the numerator are required if various values of s are allowed, not only even integer ones, since x can be negative as well as positive (bipolar signals), and the transfer characteristic is an odd one $y(-x) = -y(x)$ (we consider now first-zone products only). A slightly different form of Cann's model was adopted in [2], but if we accept the considerations given above, then it must be transformed to (1). Changing parameters L, l, and s, we can approximate by (1) a large number of real-world amplifier input–output transfer characteristics [1], [3]. Note that s need not be an integer.

III. Some Strange Properties of Cann's Model in the Small-Signal Regime

As it seems from Fig. 1, Cann's model can approximate many real-world voltage transfer characteristics quite satisfactory and it is indeed used for such a purpose. The input–output curves seem to be quite smooth (see Fig. 1) so one might expect that it predicts nonlinear product levels quite accurately if its parameters are accurately determined (using a curve-fitting technique). However, as mentioned for the first time in [3] (to the best of the author's knowledge),

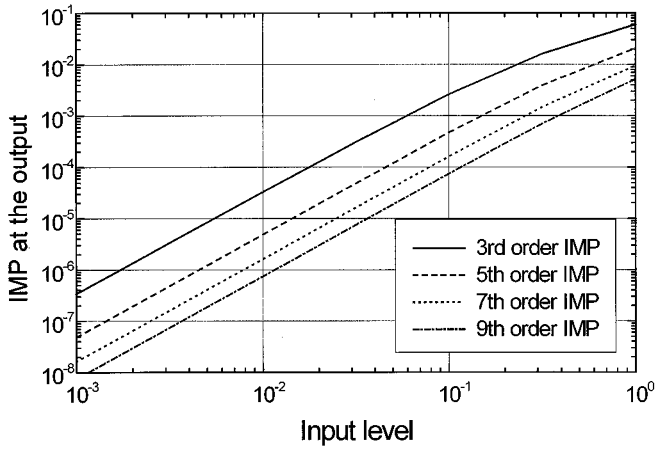

Fig. 2. Third-, fifth-, seventh-, and ninth-order IMPs at the output of Cann's model $(L = l = 1,\ s = 1)$.

Cann's model predicts the slope of third-order intermodulation products equal to $s+1$ (for equal input tone amplitudes. Henceforth, we shall consider only this case) on the decibel scale for the small-signal regime (see Fig. 2. Cann's model was used for the envelope input–output transfer characteristic $s = 1$). However, as is well known from theory and extensive practical design experience (e.g., [5]–[7]), the third-order IMP slope must be equal to three (60 dB per decade in decibel units) in the small-signal regime. Therefore, Cann's model gives correct predictions only if $s = 2$. It is most probably that $s \neq 2$ if this parameter is determined using measured data and a curve fitting technique. Thus, Cann's model will give incorrect results for the third-order IMP (as well as for higher order IMPs and harmonics), at least for the small-signal regime. As we also see from Fig. 2, the fifth-, seventh-, and ninth-order IMPs have the same slope as the third-order IMP, what is also incorrect for the small-signal regime. Thus, the next question arises: What is the reason for Cann's model incorrectness in this case? The reason proposed in [3] is, "However, since these models are only characterized to yield correct single-tone measurements it is not expected that the magnitude of these intermodulation products are correct." However, as was shown in many publications (e.g., [8]–[10]), the quadrature modeling technique that is characterized from single-tone measurements can nevertheless predict multiple-tone nonlinear products (IMPs, adjacent channel power, etc.) quite accurately. Thus, the above statement does not provide the true reason.

The real reason does not lie in the simulation or the characterization approach, but in the model itself (in its particular mathematical form). Examining carefully (1), one may expect some problems with the derivatives of $y(x)$ at $x = 0$ due to $|x|$ because $|x|$, as is well known [11], [12], has no derivatives at $x = 0$. Detail analysis (using left-side and right-side derivatives) shows that the first derivative exists and the problems appear for higher-order derivatives. Table I shows the detail results for the first-order to the seventh-order derivatives. From this table, as well as from (1), we may conclude that there exist no problems with derivatives when $s =$ *even integer*. However, even in this case the behavior of the Cann's model in the small-signal regime is very strange. For instance, when $s = 4$ (or, in a more general case, when $s > 2$), there is no third-order nonlinearity in this characteristic and, consequently, the slope of IMP_3 will not be equal to three in the small-signal regime. When $s =$ *odd integer*, this model also experiences the problems with derivatives. In particular, if $s = 2k+1$ (where k is an integer), there does not exist the $(2k+2)$th-order derivative and, consequently, we cannot use the Taylor series expansion of the Cann's model. This is also a very undesirable feature because the Taylor series expansion is a very useful and widely accepted tool for the analysis of weakly nonlinear circuits (without memory effects; when memory effects are to be taken into account, Volterra series expansion is used) [7], [13], [14], and the transfer characteristic of any weakly nonlinear circuit (like amplifier in the small-signal regime) can be expanded in the Taylor series (when memory effects are not taken into account). When $s \neq integer$, some

TABLE I

DERIVATIVES OF $y(x)$ AT $x = 0$ (ODD-ORDER DERIVATIVES ARE EQUAL TO $+\infty$ OR $-\infty$ DEPENDING ON A SPECIFIC VALUE OF s)

Order	Derivative of $y(x)$ at $x=0$
1	L/l
2	$\begin{cases} not\ \ exist, & if\ \ 0 < s \le 1 \\ 0, & if\ \ s > 1 \end{cases}$
3	$\begin{cases} \pm\infty, & if\ \ 0 < s < 2\ \ (s \neq 1) \\ 6L/l^3, & s = 1 \\ -3L/l^3, & s = 2 \\ 0, & s > 2 \end{cases}$
4	$\begin{cases} not\ \ exist, & if\ \ 0 < s \le 3\ \ (s \neq 2) \\ 0, & if\ \ s > 3\ \ or\ \ s = 2 \end{cases}$
5	$\begin{cases} \pm\infty, & if\ \ 0 < s < 4\ \ (s \neq integer) \\ 120L/l^5, & s = 1 \\ 45L/l^5, & s = 2 \\ -30L/l^5, & s = 4 \\ 0, & s > 4\ \ or\ \ s = 3 \end{cases}$
6	$\begin{cases} not\ \ exist, & if\ \ 0 < s \le 5\ \ (s \neq 2,3,4) \\ 0, & if\ \ s > 5\ \ or\ \ s = 2,3,4 \end{cases}$
7	$\begin{cases} \pm\infty, & if\ \ 0 < s < 6\ \ (s \neq integer) \\ 5040L/l^7, & s = 1 \\ -1575L/l^7, & s = 2 \\ 1120L/l^7, & s = 3 \\ -840L/l^7, & s = 6 \\ 0, & s > 6\ \ or\ \ s = 4,5 \end{cases}$

odd-order derivatives are infinite and some even-order derivatives do not exist. Hence, in this case, one also cannot expand the Cann's model in the Taylor series. Thus, the only case when all derivatives exist and are finite is $s =$ *even integer* (however, as it was mentioned above, IMP levels in the small-signal regime are also incorrectly predicted in this case). If one uses a curve-fitting technique to compute s from measured data, s is most probably will not be equal to an even integer and, consequently, it will not be possible to expand (1) in a Taylor series. Note that in the limiting case of $s \to \infty$ Cann's model reduces to the ideal limiter [1] that can be expanded in the Taylor series (which consists of only the first-order term) for $x < l$. In this case, as Table I shows, second- and higher order derivative are equal to zero and the Taylor expansion exists.

Let us now consider the binomial series expansion of (1) [8]

$$y(x) = \frac{L}{l} x \left(1 + a_1 \left(\frac{|x|}{l} \right)^s + a_2 \left(\frac{|x|}{l} \right)^{2s} + a_3 \left(\frac{|x|}{l} \right)^{3s} + \cdots \right) \tag{2}$$

where a_i are the expansion coefficients. We see that this characteristic is an odd one $(y(-x) = -y(x))$, as one would expect, but if $s = 1$, for example, it also contains "even-power nonlinearities" [more precisely, products of even powers of x and $\operatorname{sgn}(x)$, one obtains these terms when the brackets in (2) are removed. They are nonlinear functions of strictly even power in x for $x \geq 0$ or for $x \leq 0$, but not for both]. This is very strange because usually even-order powers are associated with even characteristics $(y(-x) = y(x))$, not with an odd one. Hence, the properties of (2) are also very different from those of a Taylor series expansion.

Thus, in the general case (when $s \neq$ *even integer*) we cannot expand (1) in a Taylor series and, consequently, there is no such thing as a small-signal regime (weak nonlinearity) for (1)—any signal, no matter how small it can be, remains in the large-signal area when using Cann's model. As mentioned in [1] and [3], one attribute for a transfer characteristic model is that "it has to be linear for small signals." But (1) is not linear for small signals, it only looks like it is linear when we plot it and it has a very strange behavior in the expected "small-signal" region. The necessary and sufficient condition for a function $f(x)$ to be linear is that its second- and higher order derivatives equal to zero

$$\frac{d^k}{dx^k} f(x) = 0, \quad k \geq 2. \tag{3}$$

But how can we decide whether the function is linear (or close to linear) if its higher order derivatives do not exist? The real criterion for small-signal linearity is not the function's plot, but its derivatives and the possibility to expand it in a converging Taylor series that has a dominant first-order term. Thus, we obtain the nonexpected slope for the third-order IMP because the device we analyze operates in the large-signal regime where the slope is not necessarily equal to three (the same is true for higher order IMPs and other nonlinear products). As is also well known [16]–[18], nonlinear product levels are determined by higher order derivatives of the transfer function and since we have troubles with these derivatives, we also have troubles with nonlinear products.

Let us now consider the case of $|x| \ll l$. This is an "expected" small-signal region, but, as considerations given above show, there is no small-signal region (in the sense that the model is weakly nonlinear and, consequently, there exists the Taylor series expansion and IMPs have correct slopes) for Cann's model in most cases. Detailed analysis shows that IMP levels of all orders (not only the third-order one) are determined by the second term in brackets in (2) in this case, if s is not an even integer. When $s = 2$, this term determines only third-order IMP, higher order IMPs are determined by higher order terms and have correct slope. One may say that this case is the only one when Cann's model predicts correctly the nonlinear products (IMPs and harmonics) of a weakly nonlinear circuit. If $s = 4$, e.g., there is no third-order IMP (more precisely, there is a spectral component at $2f_1 - f_2$, but its slope is five, thus, this is the fifth-order nonlinearity). The same is true for $s = 6, 8, 10 \ldots$. Furthermore, when we compare the two cases of $s = 2$ and, say, $s = 1.9$, we find that in the first case IMPs have correct slope and in the second one all the IMPs have the same slope, i.e., small variation in s changes significantly the qualitative properties of the model. Thus, Cann's model has unstable physical behavior with respect to changes in s.

It should be noted that Cann's model can be used for both the instantaneous transfer characteristic (broad-band or instantaneous nonlinearity) and the envelope transfer characteristic (bandpass or envelope nonlinearity). However, it has the undesirable properties discussed above in both cases and also in the case of $s \neq \text{integer}$.

IV. Conclusion

We can conclude that the main reason of incorrect predictions of IMP levels (and also other nonlinear products) by Cann's model is its nonphysical behavior at $x = 0$, namely, the nonexistence of many derivatives at this point in most cases. Thus, constructing a model for an input–output transfer characteristic, it is not enough to consider only the characteristic values themselves (its plot), but its derivatives are also of great importance when a nonlinear behavior is involved. Special care must also be paid to stable physical behavior of the model with respect to parameter changes over reasonable ranges. The use of Cann's model in the simulation of communication systems should be avoided (or, at least it should be used with extreme care) when nonlinear products are to be taken into account. The main advantage of Cann's model is that it is described by a closed-form expression that is very convenient for analytical calculations. Presently, numerical methods are extensively used, thus, this advantage is not so important any longer. A better choice in this case is to use some series expansions or splines to approximate the measured transfer characteristic [8], [10], [19], [20].

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