SPATIAL CHANNEL PROPERTIES AND SPECTRAL EFFICIENCY OF BLAST ARCHITECTURE

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INTRODUCTION

The Bell Labs Layered Space-Time (BLAST) architecture has been recently proposed as an extremely efficient tool for wireless communications in rich multipath environment [1-4]. Using multi-element antenna arrays and signal processing in the spatial dimension, this architecture achieves channel capacity as high as 10-100 times that of traditional architectures in certain applications (indoor communication, local-area networks, cellular systems in large cities etc.). Such a tremendous increase in the channel capacity will stimulate in future, without any doubts, widespread use of such communication systems. The main idea of the BLAST architecture is to exploit rather than suppress multipath propagation. For a classical communication architecture with multi-element antenna arrays, channel capacity (Shannon's channel capacity limit) depends linearly on the logarithm of n (the number of array elements) and for the BLAST architecture – linearly on n (for uncorrelated spatial paths). Thus, it grows much faster in the latter case as n increases. An experimental setup was built to validate the new architecture and bit rates as high as 40 bits/s/Hz has been achieved for indoor communications [4].

The BLAST channel capacity in the case of independent (uncorrelated) Rayleigh faded paths between antennas has been extensively investigated [1, 2]. In the general case of $n \times n$ matrix channel and when the transmitted signal vector is composed of statistically independent equal power components each with a gaussian distribution, the channel capacity is given by the following [2]:

$$C = log_2\, det\left(\mathbf{I} + \frac{\rho}{n}\mathbf{H} \cdot \mathbf{H}^+\right) \text{ bits/s/Hz}, \tag{1}$$

where n – is the number of transmit/receive antennas, ρ - is the signal-to-noise ratio, $\mathbf{I}$ – is n×n identity matrix, $\mathbf{H}$ – is the normalized channel matrix (for our consideration, we don't need a non-normalized channel matrix so we shall not consider it), and "$^+$" means transpose conjugate. In the case of n independent identical channels $\mathbf{H}=\mathbf{I}$ and

$$C = n \cdot log_2\left(1 + \frac{\rho}{n}\right). \tag{2}$$

For large n, this channel capacity is substantially higher than that of a traditional 1×1 channel (with the same total radiated power). However, the practical value of the BLAST channel capacity depends substantially on the correlation between individual channels (spatial paths) and may be much smaller in the case of highly correlated channels. Much work must be done on the spatial path correlation and antenna coupling in order to get realistic values of the channel capacity under real-world conditions.

In this paper, we first investigate the channel capacity of n-antenna (both at the transmitter and at the receiver) BLAST architecture for the case of arbitrary (however linear and time-independent or slowly-varying) correlated channels. An explicit dependence of the channel capacity on the channel correlation coefficients is given. We consider next the case of n-antenna architecture having real and equal correlation coefficients and give a simple formula for its channel capacity. A correlation coefficient as high as 0.5-0.8 is admissible in this case (note that this result compares well with known results on the spatial diversity techniques [5]). When the correlation coefficient is an increasing function of n

(that is the case when one locates more and more antennas into a given space), there exists the maximum (optimum number of antennas) in the channel capacity as a function of n. It allows one to define the notion of channel capacity of a given space (by analogy with the classic Shannon formula which defines the channel capacity of a given bandwidth). This results in a very symmetric picture (in the spirit of spatial relativity): the channel capacity depends not only on a given bandwidth (time or frequency domain) but also on a given space (space domain). Thus, time (or frequency) and space are included in this notion of channel capacity on the equal basis.

CHANNEL CAPACITY OF BLAST ARCHITECTURE IN ARBITRARY CORRELATED CHANNELS

Let us now consider the case of *n*-antenna architecture in correlated channels:

$$\mathbf{Y} = \mathbf{H} \cdot \mathbf{X} \ , \qquad (3)$$

where $\mathbf{X} = (x_1, x_2 \ldots \ \ x_n)^{\mathrm{T}}$ and $\mathbf{Y} = (y_1, y_2 \ldots \ \ y_n)^{\mathrm{T}}$ are transmitted and received signal vectors (complex envelopes) respectively, and "T" means transpose. Since $\mathbf{H}$ is normalized,

$$\sum_{i,j=1}^{n} \left| h_{ij} \right|^2 = n \ , \qquad (4)$$

$\mathbf{Y}$ and $\mathbf{X}$ are also normalized (h_{ij} denotes the components of $\mathbf{H}$). Following [2], we further assume that the transmitted signal vector is composed of statistically independent equal power components each with a gaussian distribution (thus, (1) is true). Since the transmitted signals are non-correlated, the correlation coefficient of the received signals is

$$r_{ij} = \frac{\langle y_i \cdot y_j^* \rangle}{\sqrt{\langle y_i \cdot y_i^* \rangle \langle y_j \cdot y_j^* \rangle}} = \frac{\sum_k h_{ik} h_{jk}^*}{\sqrt{\sum_k \left| h_{ik} \right|^2 \sum_k \left| h_{jk} \right|^2}} \ , \qquad (5)$$

where "< >" denotes time average, "*" denotes complex conjugate. Note that $\mathbf{H}$ is assumed to be time-independent or varying slowly with time (for more detail consideration, see [2]). The cause of correlation of received signals is the correlation of individual channels. Thus, r_{ij} is also the correlation coefficient of individual channels (note that it depends on the channel matrix only). Denoting the matrix under determinant sign in (1) as $\mathbf{Z}$ and using (5), we obtain the following equation for the components of $\mathbf{Z}$:

$$z_{ij} = \delta_{ij} + \rho \sqrt{\beta_i \beta_j} \cdot r_{ij} \ , \text{ where } \beta_i = \frac{\langle y_i \cdot y_i^* \rangle}{\sum_k \langle y_k \cdot y_k^* \rangle} = \frac{1}{n} \sum_k \left| h_{ik} \right|^2 \ , \qquad (6)$$

β_i is the relative received power in i-th channel (without the noise contribution), and δ_{ij} is Kroneker's delta. Using (6), we obtain the following expression for the determinant:

$$det(\mathbf{Z}) = \sum_{i_1, i_2 \ldots i_n} \varepsilon_{i_1 i_2 .. i_n} \prod_{k=1}^{n} \left(\delta_{k i_k} + \rho \sqrt{\beta_{i_k} \beta_k} \cdot r_{k i_k} \right) , \qquad (7)$$

where $\varepsilon_{i_1 i_2 .. i_n} = 1$ if $[i_1, i_2, .. i_n]$ is an even permutation of $[1,2,..n]$, $\varepsilon_{i_1 i_2 .. i_n} = -1$ if $[i_1, i_2, .. i_n]$ is an odd permutation of $[1,2,..n]$, and $\varepsilon_{i_1 i_2 .. i_n} = 0$ otherwise. Eqs. (1) and (7) give an explicit dependence of the BLAST channel capacity on the individual channel correlation. In the case of equal received powers $\beta_i = 1/n$. Hence, Eq. (7) can be presented in the following form:

$$det(\mathbf{Z}) = \sum_{k=0}^{n} a_k \left(\frac{\rho}{n} \right)^k \left(1 + \frac{\rho}{n} \right)^{n-k} \ , \qquad (8)$$

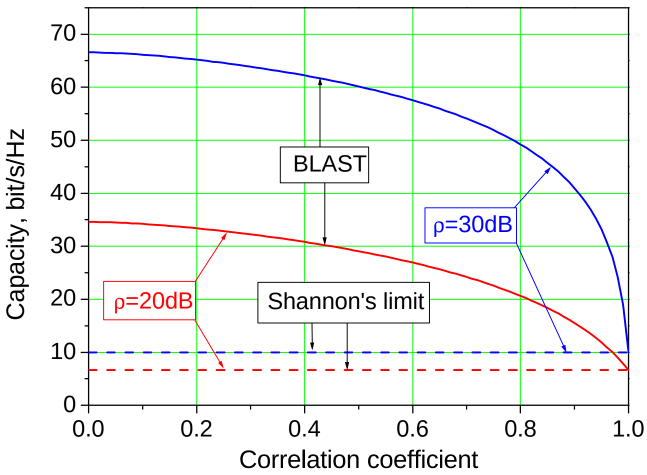

Fig. 1. BLAST channel capacity versus correlation coefficient, $n=10$.

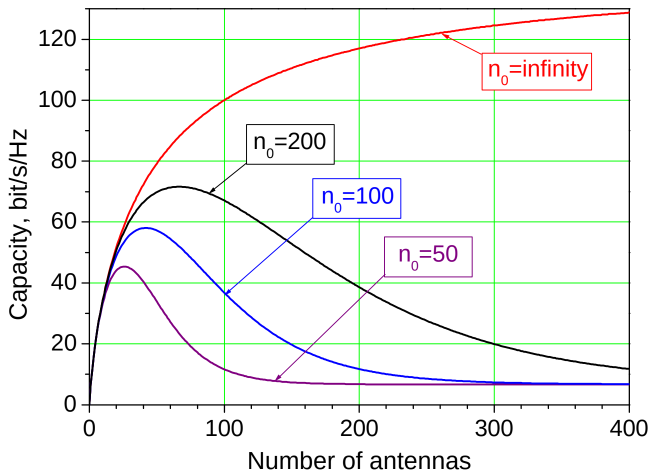

Fig. 2. BLAST channel capacity versus number of antennas, $\rho=20$ dB.

where a_k depends on the correlation coefficients only. This proves the first hypothesis in [6]. Few first a_k can be expressed as:

$$a_0 = 1, \quad a_1 = 0, \quad a_2 = -\sum_{i_1<i_2} \left| r_{i_1 i_2} \right|^2, \quad a_3 = 2 \cdot \sum_{i_1<i_2<i_3} Re\left(r_{i_1 i_2} r_{i_2 i_3} r_{i_3 i_1} \right). \tag{9}$$

In general, the following estimation holds: $a_k \sim r^k$ $(k \geq 2)$. However, it is not true to conclude that in the case of $r << 1$ the channel capacity depends only on the second-order term in (8) – higher-order terms also give a substantial contribution. As a detailed analysis shows, higher-order terms may be neglected only when $r << 1/\rho$.

Let us now consider the case when all the correlation coefficients are equal and real: $r_{ij} = r$, $Im(r) = 0$. This case is somewhat artificial because one expects that the correlation of neighboring antennas is larger than that of distant antennas. However, the case of equal correlation coefficients provides a worst-case estimation and some insight into BLAST operation in correlated channels, so it deserves to be considered. In this case, (8) reduces to

$$C = n \cdot log_2\left(1 + \frac{\rho}{n}(1-r) \right) + log_2\left(1 + \rho \cdot r \cdot \left(1 + \frac{\rho}{n}(1-r) \right)^{-1} \right). \tag{10}$$

For n=2, (10) reduces to (8) in [6]. Fig. 1 shows the channel capacity as a function of the correlation coefficients for various signal-to-noise ratios. As it can be seen from this figure, the channel capacity decreases substantially for $|r| \geq 0.5 - 0.8$ what agrees well with known results on spatial diversity techniques [5]. It should be noted that the second term in (10) is essential only when the correlation coefficient is close to 1. However, in this case the advantage of the BLAST architecture over traditional techniques is very small (the channel capacity is close to Shannon's limit) and it is not reasonable to use it. Thus, when the BLAST architecture provides a substantial advantage, its channel capacity can be estimated as

$$C \approx n \cdot log_2\left(1 + \frac{\rho}{n}(1-r) \right) \tag{11}$$

Comparing (11) with (2), we find that the effect of the channel correlation is equivalent to the decrease in the signal-to-noise ratio.

When one locates more and more antennas in a given space, the correlation between individual channels will increase (due to decrease in antennas' separation). In general, the channel correlation will be a non-monotonic function (with a lot of oscillations) of the number of antennas. For a large space and a small number of antennas, the correlation will be close to zero, and for a large number of antennas and a small space, the correlation will be close to unity. For the sake of simplicity, we approximate it by the following:

$$r(n) = tanh(n/n_0), \tag{12}$$

where n_0 depends on the space size occupied by antennas. Using (10) and (12) together, we find that $C(n)$ has the maximum (see Fig. 2), i.e. it increases with the increase in n up to some specific value (n_{max}). Further increase in n (over n_{max}) results in a decrease rather than an increase of $C(n)$. This fact has an important consequence. The existence of the maximum value of $C(n)$ means that the information transmission rate (per Hz) is limited for the given space by exactly $C(n_{max})$, which is "spatial channel capacity". Thus, one may control the channel capacity by not only the channel bandwidth (and, of course, by signal-to-noise ratio), but also by a spatial arrangement of the system. In fact, $C(n_{max})$ is the maximum information transmission rate that can be realized for the given space. This gives a spatial analog of the Shannon's limit.

It should be noted that the BLAST architecture relies substantially on active array technology (it is impossible in principle to realize such a system using passive arrays) [4], i.e. it employs active circuitry in every transmit and receive paths, and operates in rich multi-signal and multipath environment. In this case, nonlinear behavior of the active circuitry has profound impact on the system operation and should be carefully taken into account [7]. In fact, this is one more limitation on the channel capacity of BLAST architecture.

CONCLUSION

The channel capacity of the BLAST architecture depends substantially on not only the channel bandwidth and signal-to-noise ratio (as it is the case for the classic Shannon's limit), but also on the channel spatial properties. The fundamental limitation on channel capacity increase by employing more and more antennas is the correlation of individual spatial paths (channels). The more antennas one employs in a given space, the larger is the channel correlation that, in turn, decreases the channel capacity. Thus, the optimum number of antennas exists, for which the channel capacity achieves its maximum value. This gives the spatial analog of the Shannon's limit and the notion of "spatial channel capacity".

In this paper, we developed a simple mathematical model of the spatial channel in order to investigate the communication theory aspect of the problem. This model provides the worst-case estimation and allows us to apply the known results on spatial diversity techniques to the present problem. More accurate analysis of the BLAST architecture requires more precise electromagnetic models of the spatial channel and, in particular, of the electromagnetic environment, the antennas and their coupling.

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