ELG5375: Digital Communications

Lecture 10

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Digital Communications: fundamental limits

- from information theory[1]
- single user: channel capacity: [bit/s] or [bit/ch. use]
- benchmark for actual system performance
- optimal system design (Tx, Rx)
- much less is known about networks

[1] T.M. Cover, J.A. Thomas, Elements of Information Theory, Wiley, 2006.

Fundamental Limit to Reliable Communication

- from information theory[2]
- **objectives**:
  - maximize data rate
  - keep low (minimize) error rate
  - under constrained Tx power & bandwidth
- Shannon'48: **channel capacity** = the largest possible rate of reliable communication over that channel

[2] T.M. Cover, J.A. Thomas, Elements of Information Theory, 2006

Fundamental Limit: Channel Capacity

- largest data rate s.t. power & reliability constraints
- AWGN channel:

$$R < C = \Delta f \log(1+\gamma) \text{ [bit/s]} \tag{1}$$

  - C = channel capacity [bit/s]
  - Δf = channel bandwidth [Hz]
  - $\gamma = P_s/P_n$ = analog SNR
  - power constraint: Tx power $\leq P_s$
  - reliability constraint: arbitrary-low error probability
  - no limits on Tx/Rx/complexity
- C: fundamental limit due to nature, not technology

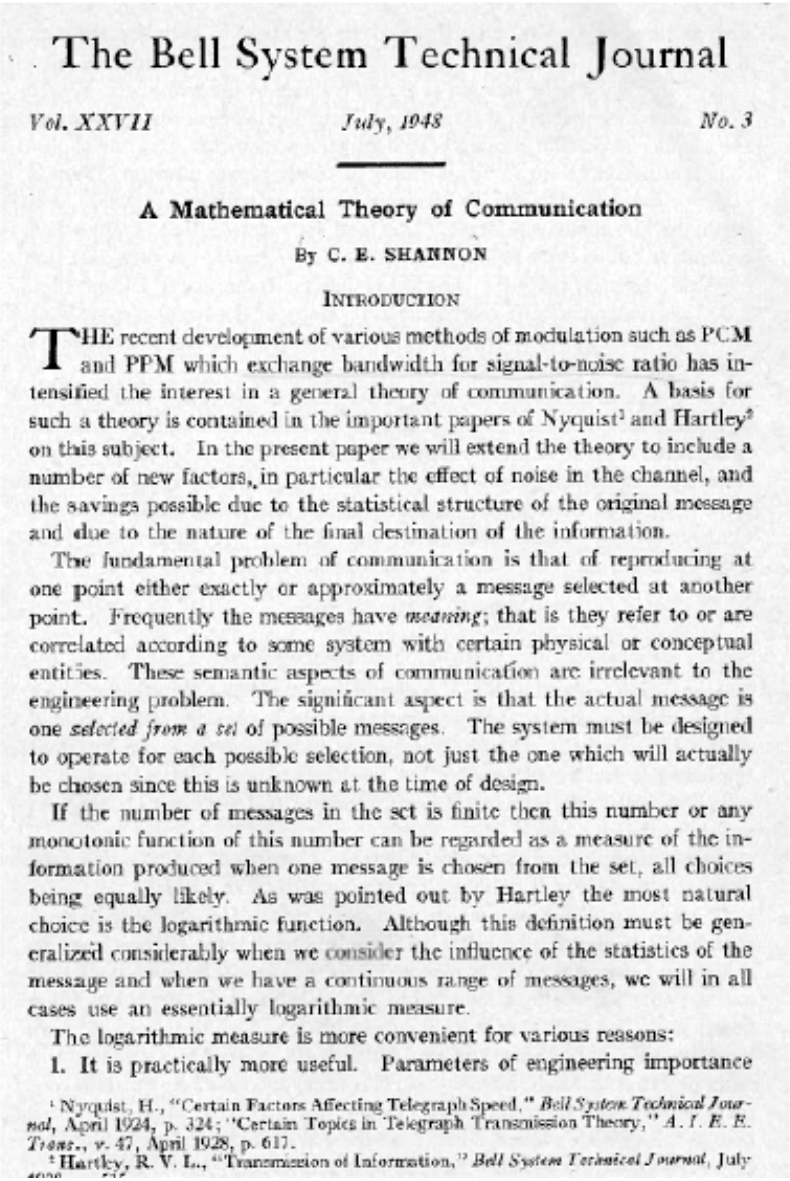
Fundamental Limit: Channel Capacity

- largest data rate s.t. power & reliability constraints

$$R < C = \Delta f \log(1+\gamma) \text{ [bit/s]} \tag{2}$$

- **the most fundamental result**
- best Tx/Rx strategies
- (almost) error-free transmission:
  - possible if $R < C$
  - not possible otherwise

Claude Shannon, Farther of Information Theory [1948]



The Bell System Technical Journal

Vol. XXVII July, 1948 No. 3

A Mathematical Theory of Communication

By C. E. SHANNON

INTRODUCTION

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist[1] and Hartley[2] on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.

The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one *selected from a set* of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.

If the number of messages in the set is finite then this number or any monotonic function of this number can be regarded as a measure of the information produced when one message is chosen from the set, all choices being equally likely. As was pointed out by Hartley the most natural choice is the logarithmic function. Although this definition must be generalized considerably when we consider the influence of the statistics of the message and when we have a continuous range of messages, we will in all cases use an essentially logarithmic measure.

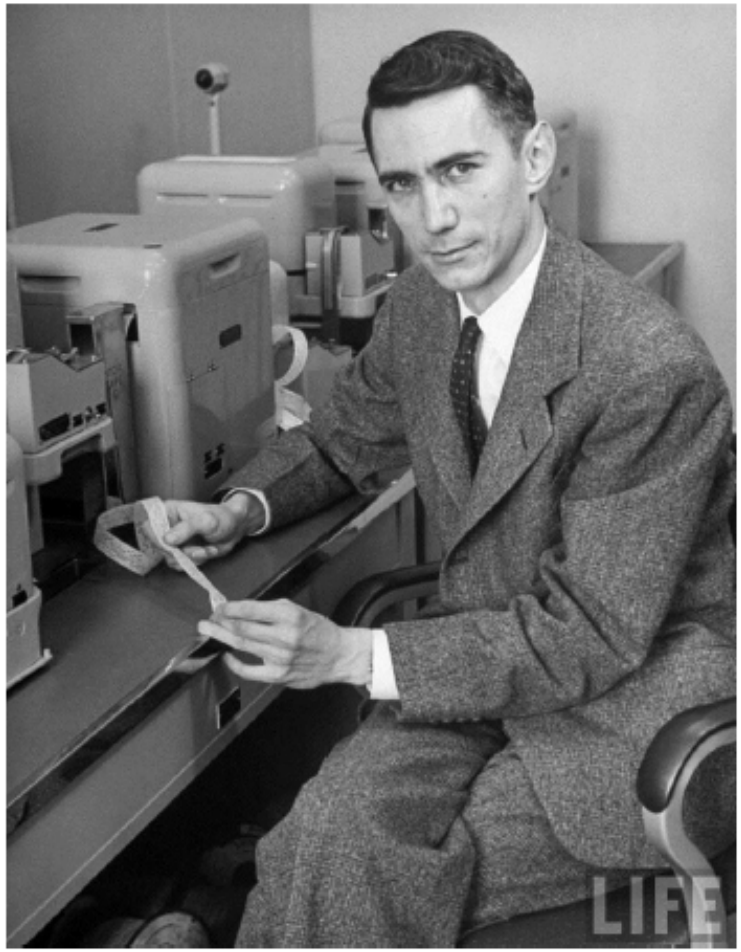
The logarithmic measure is more convenient for various reasons:

1. It is practically more useful. Parameters of engineering importance

[1] Nyquist, H., "Certain Factors Affecting Telegraph Speed," *Bell System Technical Journal*, April 1924, p. 324; "Certain Topics in Telegraph Transmission Theory," *A. I. E. E. Trans.*, v. 47, April 1928, p. 617.

[2] Hartley, R. V. L., "Transmission of Information," *Bell System Technical Journal*, July

- Farther of Information Theory
- **Born:** 30. Apr. 1916, Michigan, USA
- **Died:** 24 Feb. 2001, Massachusetts, USA



Channel Capacity: two fundamental resources

From the capacity expression,

$$C = \Delta f \log(1 + \gamma) \text{ [bit/s]} \tag{3}$$

C can be increased by increasing

1. bandwidth Δf (expensive in wireless)
2. power P_s, via the SNR $\gamma = P_s/P_n$
3. anything else?

Fundamental Limit: Channel Capacity

- largest data rate s.t. power & reliability constraints

$$R < C = \Delta f \log(1+\gamma) \text{ [bit/s]} \tag{4}$$

- Q1: **why** C (or R) is limited?
- Q2: **why** by Δf and γ?

Channel Capacity & M-ary Modulation

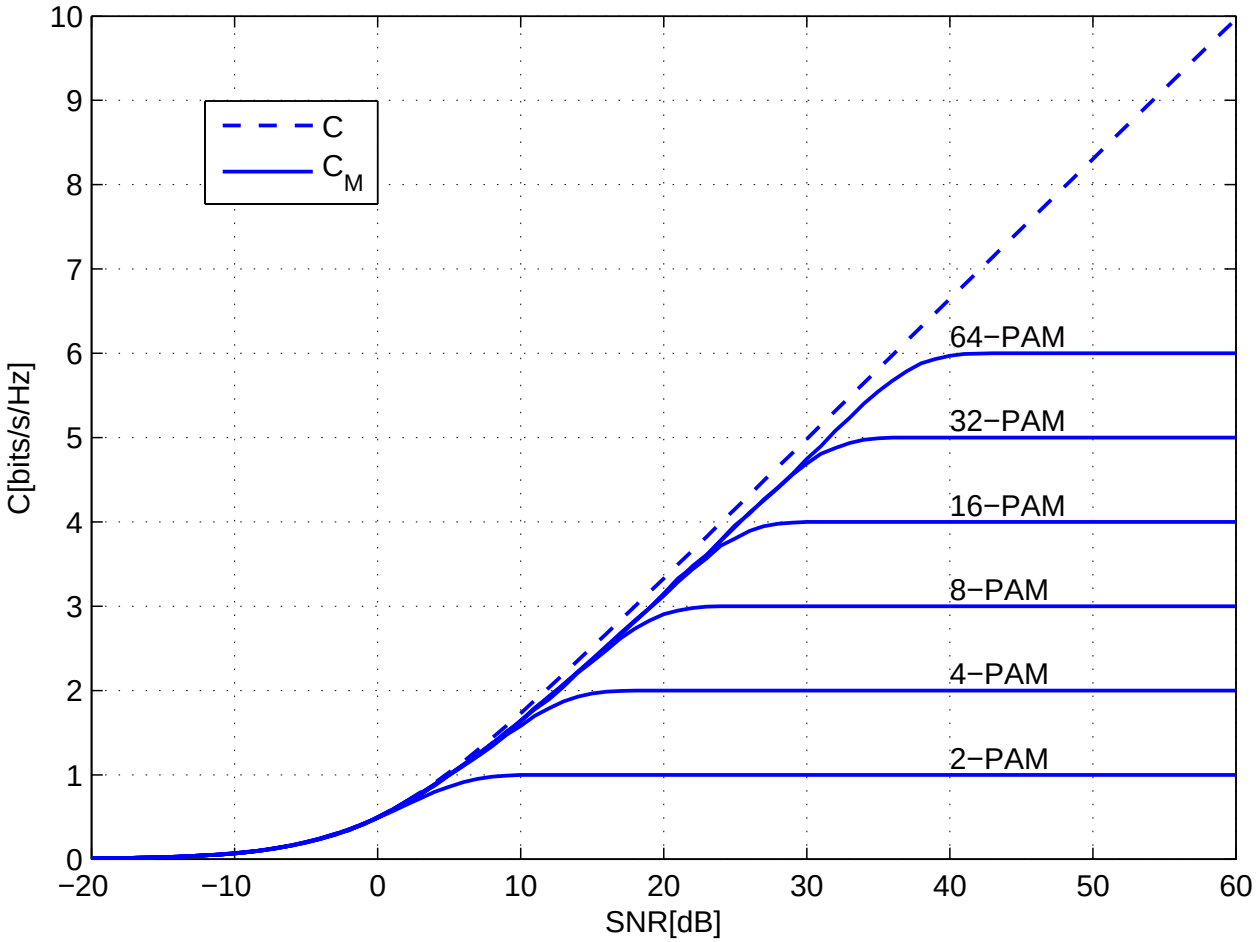


Figure: The constellation capacity of M-PAM[3]

[3] Y. Han, Optimization of Modulation Constrained Digital Transmission, MS Thesis, University of Ottawa, 2017.

Fundamental Limit: Channel Capacity

- how to approach C? $\rightarrow$ best modulation/coding[4]
- $R = C$ is not possible, but R can be close to C
- in practice,

$$R = \log(1 + \gamma/\Gamma) \text{ [bit/s/Hz]} \tag{5}$$

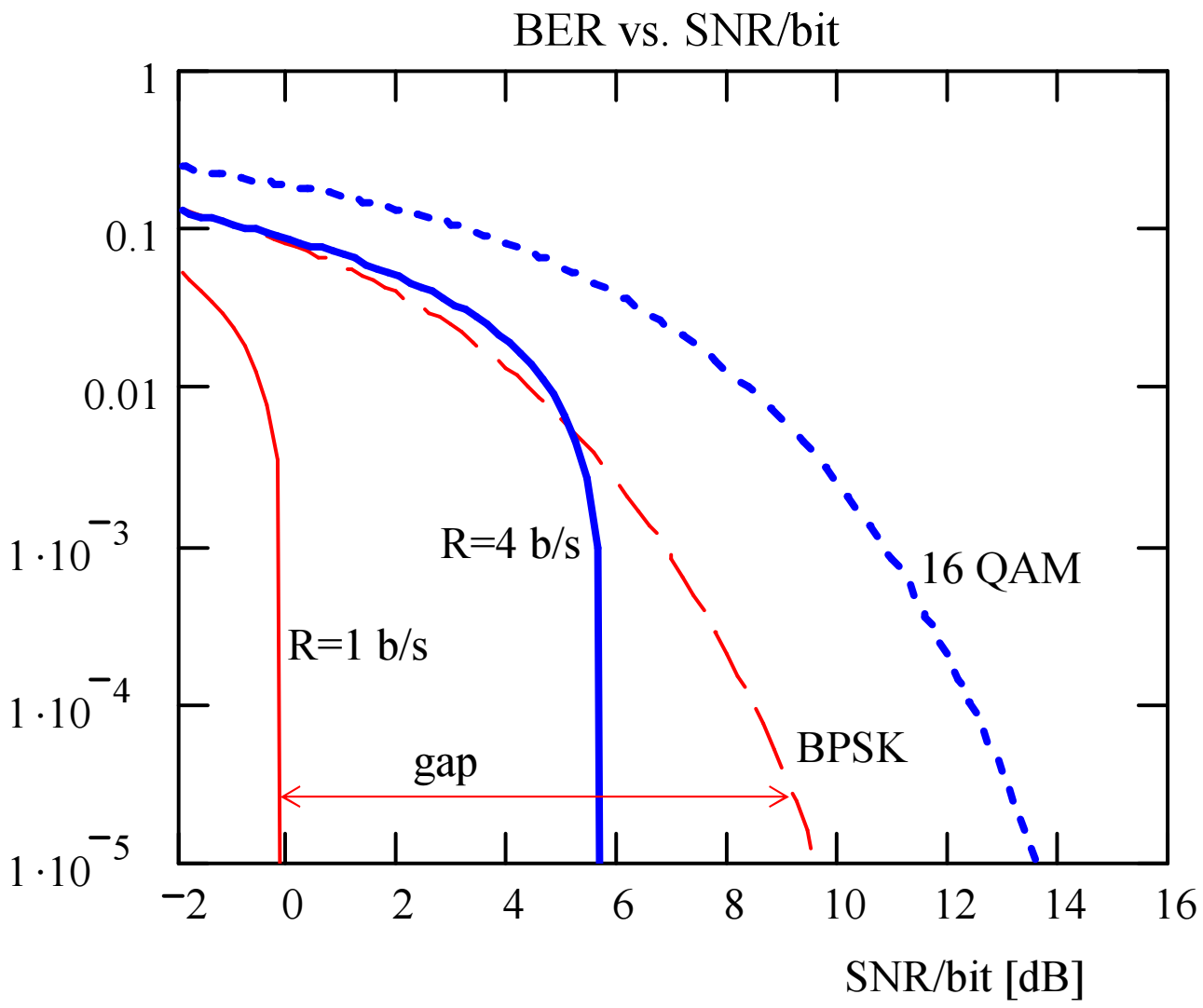
  $\Gamma > 1$ is the SNR gap to capacity
- $\Gamma \rightarrow 1$ for good (capacity-approaching) system
- depends on modulation and coding[5][6]

[4] D.J. Costello and G.D. Forney, Channel coding: The road to channel capacity, *Proc. IEEE*, vol. 95, no. 6, Jun. 2007.

[5] G.D. Forney, G. Ungerboeck, Modulation and Coding for Linear Gaussian Channels, IEEE Trans. Info. Theory, v. 44, no. 6, pp. 2384–2415, Oct. 1998.

[6] J. Cioffi, EE 379A - Digital Communication: Signal Processing, Stanford University.

SNR-Gap-to-Capacity in Practice[7]



[7] D.J.C. MacKay, Information Theory, Inference and Learning Algorithms, Cambridge University Press, 2003, p.15, 162.

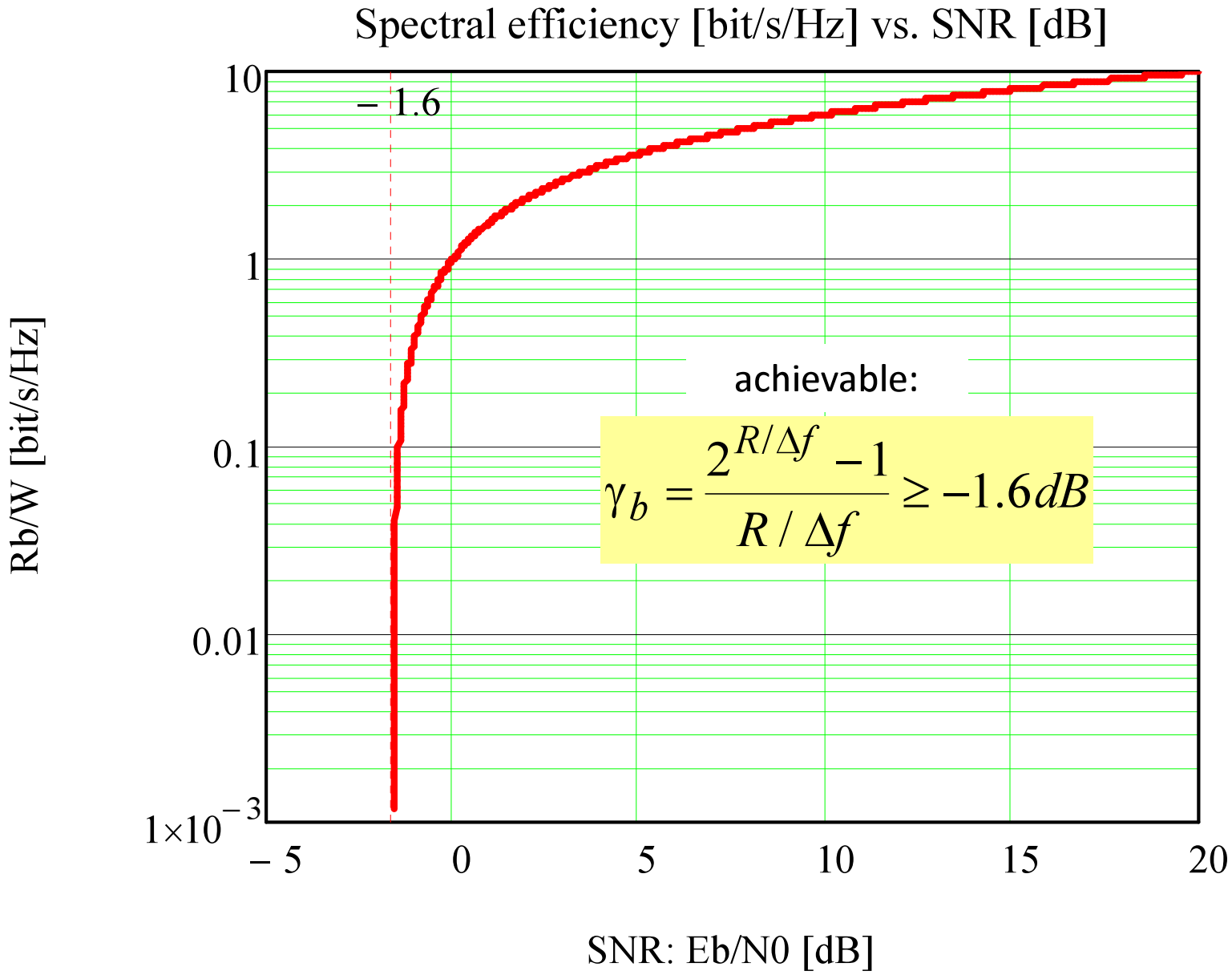
Spectral/Power Efficiency: Fundamental Tradeoff

- power efficiency: $\gamma_b = \text{SNR(energy)/bit}$
- spectral efficiency: $R/\Delta f$ [bit/s/Hz]
- the tradeoff:

$$\gamma_b \geq \frac{2^{R/\Delta f} - 1}{R/\Delta f} \geq \ln 2 = -1.6 \text{ dB} \tag{6}$$

- follows from $R < C = \Delta f \log(1 + \gamma)$ with $E_b = P_s T_b$, $\gamma_b = \gamma \Delta f / R$
- LB: $\uparrow$ $R/\Delta f$ (SE)
- min LB $= -1.6$ dB/bit
- **Q:** prove (6) and properties

Spectral/Power Efficiency: Fundamental Tradeoff



achievable:

$$\gamma_b = \frac{2^{R/\Delta f} - 1}{R/\Delta f} \geq -1.6dB$$

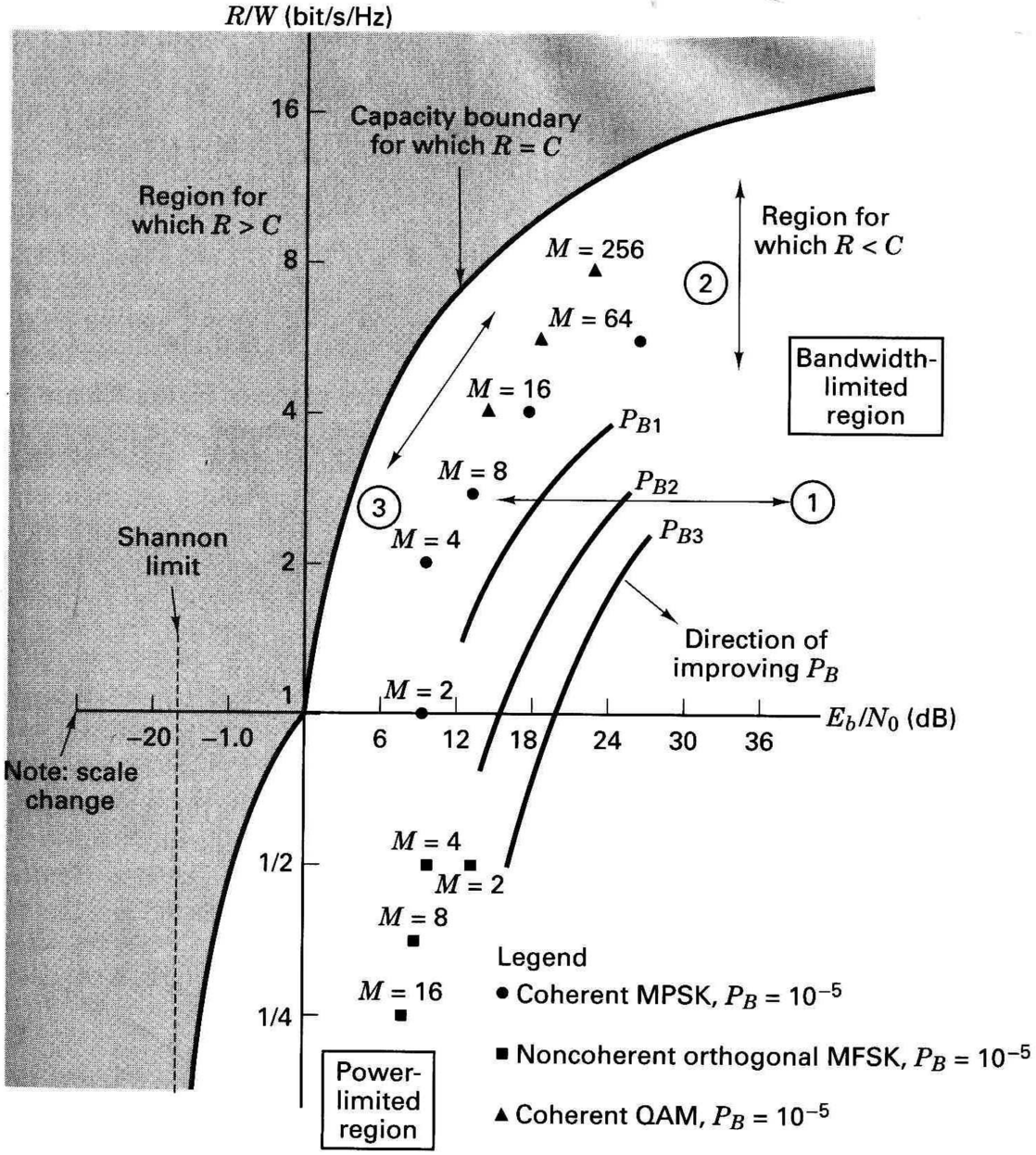


Figure 9.6 Bandwidth-efficiency plane.

B. Sklar, Digital Communications, Prentice Hall, 2001

From Shannon to Practice

- Shannon (1948): capacity is achievable
- But his proof was **non-constructive** (existence only)
- Question: can we build practical codes that approach capacity?

Answer (after 5 decades): **yes**

Progress Towards the Capacity[8]

Progress toward the Shannon limit

The original turbo codes: about **0.7 dB** from capacity

C. Berrou, A. Glavieux, and P. Thitimajshima, Near Shannon limit error-correcting coding and decoding: Turbo codes, *IEEE Int. Communications Conference*, 1993.

Irregular LDPC codes: about **0.1 dB** from capacity

T.J. Richardson and R. Urbanke, The capacity of low-density parity-check codes, *IEEE Transactions on Information Theory*, February 2001.

How about 0.01 dB from capacity? And 0.001 dB?

J. Boutros, G. Caire, E. Viterbo, H. Sawaya, and S. Vialle, Turbo code at 0.03 dB from capacity limit, *IEEE Symp. Inform. Theory*, July 2002.

S-Y. Chung, G.D. Forney, Jr., T.J. Richardson, and R. Urbanke, On the design of low-density parity-check codes within 0.0045 dB of the Shannon limit, *IEEE Communications Letters*, February 2001.

Conclusion: ***For all practical purposes, Shannon's puzzle has been now solved and Shannon's promise has been achieved!***

[8] A.Vardy, What's New and Exciting in Algebraic and Combinatorial Coding Theory? Plenary Talk at ISIT-06.

Timeline of Modern Codes

- 1950s–1990s: algebraic codes (Reed–Solomon, BCH)
- 1993: **Turbo codes**
- 1990s–2000s: **LDPC codes** (rediscovered)
- 2009: **Polar codes** (Arikan)

All of these can operate very close to capacity.

Key Idea: Iterative Decoding

- Modern codes use **probabilistic decoding**
- Messages are refined iteratively
- Often implemented via message passing

Instead of hard decisions, decoders work with probabilities.

Modern Coding: Landmark Papers

- C. Berrou, A. Glavieux, and P. Thitimajshima, Near Shannon Limit Error-Correcting Coding and Decoding: Turbo Codes, *ICC*, 1993
- R. G. Gallager, *Low-Density Parity-Check Codes*, 1963 (PhD thesis)
- E. Arikan, Channel Polarization: A Method for Constructing Capacity-Achieving Codes, *IEEE Trans. Info. Theory*, 2009

References on coding/modulation

- E. Biglieri, *Coding for Wireless Channels*, Springer, 2005.
- W.E. Ryan and S. Lin, *Channel Codes: Classical and Modern*, Cambridge, U.K.: Cambridge Univ. Press, 2009
- T.K. Moon, *Error Correction Coding–Mathematical Methods and Algorithms*, New York, NY, USA: Wiley, 2005.
- I.B. Djordjevic, W. Ryan, and B. Vasic, *Coding for Optical Channels*, New York, NY, USA: Springer, 2010.
- I.B. Djordjevic, *Advanced Optical and Wireless Communications Systems*, Springer, 2018.

References on coding/modulation

- D.J. Costello and G.D. Forney, Channel coding: The road to channel capacity, *Proc. IEEE*, vol. 95, no. 6, pp. 1150–1177, Jun. 2007.
- D.J. Costello *et al.*, Applications of error-control coding, *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2531–2560, Oct. 1998.
- G.D. Forney and G. Ungerboeck, Modulation and coding for linear Gaussian channels, *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2384–2415, Oct. 1998.
- A. Leven and L. Schmalen, Status and recent advances on forward error correction technologies for lightwave systems, *IEEE J. Lightwave Technol.*, vol. 32, no. 16, pp. 2735–2750, Aug. 2014.
- M. Nakazawa et al., Extremely higher-order modulation formats, *Optical Fiber Telecommunications: Systems and Networks*, 2013, vol. B, pp. 297–336.

References on modern coding

- IEEE ComSoc: Best Readings in Polar Coding, available online.
- Recent Advances in Capacity Approaching Codes, IEEE JSAC, Feb. 2016.
- V. Bioglio et all, Design of Polar Codes in 5G New Radio, IEEE Comm. Surveys & Tutorials, vol. 23, no. 1, 2021.
- M. Sy, Demystifying 5G Polar and LDPC Codes: A Comprehensive Review and Foundations, Feb. 2025.
- `https://en.wikipedia.org/wiki/Low-density_parity-check_code`
- `https://en.wikipedia.org/wiki/Polar_code_(coding_theory)`
- `https://en.wikipedia.org/wiki/Turbo_code`

State-of-the-art (in fiber optical comm.)

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Record-High 17.3-bit/s/Hz Spectral Efficiency Transmission over 50 km Using Probabilistically Shaped PDM 4096-QAM

Samuel L.I. Olsson[1,*], Junho Cho[1], Sethumadhavan Chandrasekhar[1], Xi Chen[1], Ellsworth C. Burrows[1], and Peter J. Winzer[1]

[1]*Nokia Bell Labs, Holmdel, New Jersey, 07733, United States*

16384-QAM TRANSMISSION AT 10 GBD OVER 25-KM SSMF USING POLARIZATION-MULTIPLEXED PROBABILISTIC CONSTELLATION SHAPING

Xi Chen, Junho Cho, Andrew Adamiecki, and Peter Winzer

Nokia Bell Labs, Holmdel, New Jersey, United States

State-of-the-art (in fiber optical comm.)

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10.66 Peta-Bit/s Transmission over a 38-Core-Three-Mode Fiber

Georg Rademacher[(1)], Benjamin J. Puttnam[(1)], Ruben S. Luís[(1)], Jun Sakaguchi[(1)], Werner Klaus[(1)], Tobias A. Eriksson[(1,2)], Yoshinari Awaji[(1)], Tetsuya Hayashi[(3)], Takuji Nagashima[(3)], Tetsuya Nakanishi[(3)], Toshiki Taru[(3)], Taketoshi Takahata[(4)], Tetsuya Kobayashi[(4)], Hideaki Furukawa[(1)], and Naoya Wada[(1)]

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[(2)]Royal Institute of Technology (KTH), AlbaNova University Center, 106 91 Stockholm, Sweden
[(3)]Sumitomo Electric Industries, Ltd., 1 Taya-cho, Sakae-ku, Yokohama 244-8588, Japan
[(4)]Optoquest Co. Ltd., 1335 Haraichi, Ageo, Saitama 362-0021, Japan

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State-of-the-art (in fiber optical comm.)

"World Record Optical Fiber Transmission Capacity Doubles to 22.9 Petabits per Second", NICT, 2023.

49th European Conference on Optical Communications (ECOC 2023) Glasgow, UK 1–5 October 2023

22.9 Pb/s Data-Rate by Extreme Space-Wavelength Multiplexing

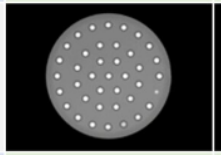
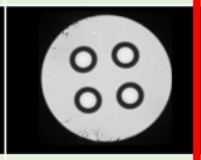
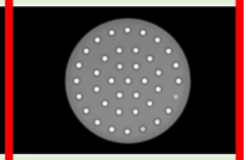
B. J. Puttnam(1), M. van den Hout(1,2), G. Di Sciullo(1,3), R. S. Luis(1), G. Rademacher(1)*, J. Sakaguchi(1), C. Antonelli(3), C. Okonkwo(2), and H. Furukawa(1)

(1) NICT, 4-2-1, Nukui Kitamachi, Koganei, 184-8795 Tokyo, Japan, E-mail:ben@nict.go.jp
(2) Eindhoven University of Technology, The Netherlands (3) University of L'Aquila and CNIT, 67100, L'Aquila, Italy. * now with INT, University of Stuttgart, Pfaffenwaldring 47, 70569 Stuttgart, Germany.

Abstract *We transmit 750 wavelength channels covering 19 THz bandwidth over a few-mode multi-core fiber with 114 spatial channels, recording a total GMI-estimated data-rate of 24.7 petabit/second and 22.9 Pb/s after LDPC decoding, both exceeding 200 Tb/s per spatial channel.*

State-of-the-art (in fiber optical comm.)

"World Record Optical Fiber Transmission Capacity Doubles to 22.9 Petabits per Second", NICT, 2023.

			This study
Fiber type	38 core, 3 mode	4 core	38 core, 3 mode
	Mar. 2020	May 2022	Oct. 2023
Fiber cross section			
Spatial channels	114	4	114
Data rate (Pb/s)	10.66	1.02	22.9
Transmission distance (km)	13	51.7	13
Bandwidth (THz), Wavelength band	9.2 C, L	20 S, C, L	18.8 S, C, L
QAM order	64 /256	256	256

Summary

- Fundamental limits in digital communications
- Channel capacity
- Spectral/power efficiency tradeoff
- How to approach the capacity ?
- SNR gap to capacity

Reading

- T.M. Cover, J.A. Thomas, Elements of Information Theory, Wiley, 2006.
- S. Haykin, Digital Communication Systems, Wiley, 2014.
- J.M. Wozencraft, I.M. Jacobs, Principles of Communication Engineering, Wiley, 1965.
- Other books/papers (see the reference list).

Note: Do <u>not</u> forget to do end-of-chapter problems. Remember the learning efficiency pyramid!