Assignment 3, due: Apr. 8, 6pm in CBY A608 (hard copy, + scanned pdf via email)

Note 1: Late submissions will not be accepted. Please also keep an extra copy of your submission for your own record.

Note 2: Submission format: can be hand-written or printed (handwriting must be readable), letter-sized paper, stapled, cover page must include course code/title, assignment number, your name and student number, date of submission, all pages numbered. Scanned pdf must be 100% identical to the hard copy. Please include all details/steps in your answers (e.g. derivations, analytical steps, etc.) not just final results. All symbols used must be clearly defined.

Note 3: Before doing the assignment, read relevant chapters/sections of the textbook (or other recommended books), study carefully all examples, attempt some end of chapter problems. Extra references are given below. No submission is needed for this part (this is very helpful for your studies though).

1. (a) Find out optimal Rx for the BPSK signal operating over AWGN channel using a single baseband pulse $p(t)$. Sketch its block diagram and clearly explain the functions of all its elements. Include a detailed proof of optimality this Rx and its block diagram (using the tools of signal space analysis we studied before for baseband systems).

   (b) How does your answer in the previous Part change if the system uses 4-PAM baseband modulation, combined with the carrier modulation ? Explain carefully the difference in details.

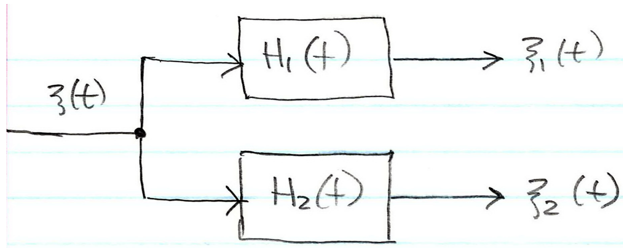
   (c) For Parts (a) and (b), find the PSD of modulated signal assuming that a pulse train is transmitted with iid bit stream. What is its bandwidth if $p(t) = \Pi(t/T_s)$ and $T_s = 0.1\mu s$?

   (d) Explain how the answer of Part (a) changes if the system is transmitting a pulse train of 1000 pulses, instead of a single pulse. Clearly explain the difference in the optimal Rx in this case.

2. Prove eq. (42)-(45) in Lecture 7. Include all the details, explain key steps.

3. Prove 2nd equality in eq. (46) of Lecture 7. Pay attention to the fact that its left-hand side includes $\boldsymbol{r}_1$ but its right-hand side – $r(t)$. Provide a detailed analysis, not just verbal statements.

4. In Lecture 7, we discarded $\boldsymbol{r}_2$ claiming it is irrelevant. In this question, we elaborate on this issue.
   (a) Consider the following setup consisting of 2 filters whose frequency responses do not overlap, i.e. $H_1(f)H_2(f) = 0$ for any f. Its input is AWGN noise $\xi(t)$. Prove that the outputs $\xi_1(t_1)$ and $\xi_2(t_2)$ are independent of each other, for any t_1 and t_2.
   (b) Assuming that the input PSD is N_0, find the output powers P_1 and P_2 and the input power P_0.



5. Consider a 4-PAM signal with $p(t) = \Pi(t/T_s)$ and the allowed amplitude levels are $[-3A, -A, A, 3A]$.
   (a) Explain how to change the block diagrams of optimal Rx on slide 33 of Lecture 7 to accommodate such modulation.
   (b) What is the symbol error rate for this modulation format? Find it and plot it in a figure similar to that in slide 35.

6. Bonus (optional) question. Consider Question 4 but assume that the input noise is not white anymore but instead has PSD $P_\xi(f)$. Prove that the outputs are still independent of each other if $P_\xi(f) = const$ for any f such that $|H_1(f)| + |H_2(f)| \neq 0$ and may not be constant otherwise.

For all relevant questions, please include Matlab codes in your submission.

References

[1] J.M. Wozencraft, I.M. Jacobs, Principles of Communication Engineering, Wiley: New York, 1965.

[2] S. Haykin, Digital Communication Systems, Wiley, 2014.

[3] R.E. Ziemer, W.H. Tranter, Principles of Communications, Wiley, 2009.

[4] A. Lapidoth, A Foundation in Digital Communication, Cambridge University Press, 2017.

Note on plagiarism

Plagiarism (presenting somebody's else solution/report as your own, where "somebody" means any source, including Internet (Google, ChatGPT, etc.), "cut-and-paste" from a student to a student, other forms of "borrowing" the material) is absolutely unacceptable and will be penalized. Each student must submit his own solutions. If two (or more) identical or almost identical sets of solutions are found (including those from Internet), each student involved receives 0 (zero) for that particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be send to the Dean's office for further investigation and further penalties.