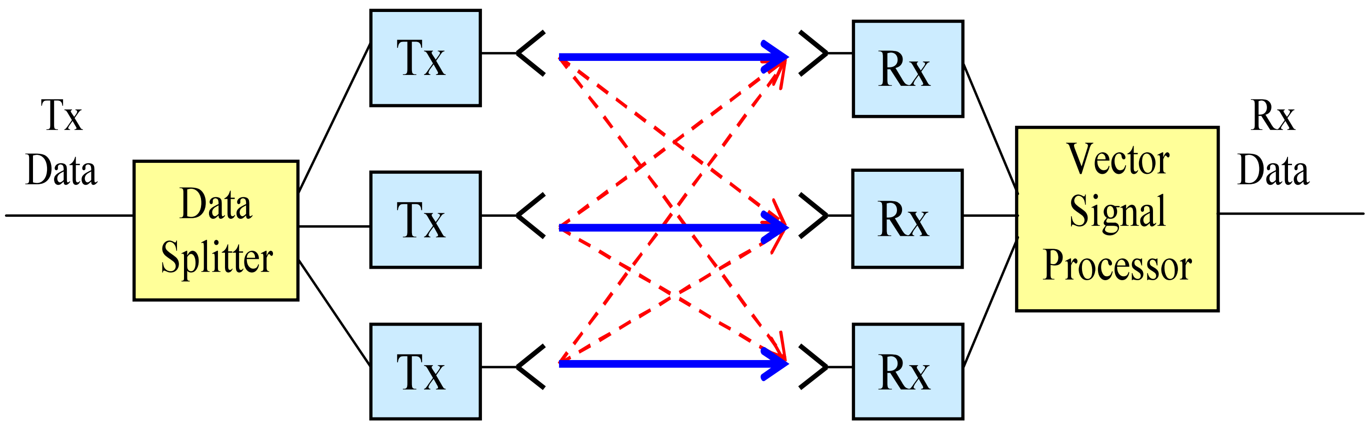
Rx Processing ("demodulation", "decoding"): V-BLAST Algorithm

Recall the basic idea:



$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\xi} \tag{14.1}$$

where $\mathbf{x}$ is the Tx vector, $\mathbf{y}$ is the Rx vector, ξ is the AWGN.

Channel state information (CSI) is available at the Rx only. Given $\mathbf{y}$, how to find $\mathbf{x}$?

Simple solution,

$$\hat{\mathbf{x}} = \mathbf{H}^{-1}\mathbf{y} \tag{14.2}$$

is not efficient, as it requires $O\left(N^3\right)$ operations + possible noise enhancement: for ill-conditioned $\mathbf{H}$ ($\det(\mathbf{H})$ close to 0) it is not optimum.

Q: What is the MMSE solution?

The best solution (min. BER): maximum likelihood (ML) -> too complex (exponential in *N*).

Efficient solution –> V-BLAST detection algorithm (also known as decision-feedback (DF) or successive interference cancellation (SIC)).

Three major steps:

1) Interference cancellation (form already detected symbols)
2) Interference nulling (from yet to be detected symbols)
3) Optimal ordering (max. post-processing SNR)

The Rx (vector) signal can be expressed as:

$$\mathbf{y} = \sum_{i=1}^{n} \mathbf{h}_i x_i + \boldsymbol{\xi} \qquad (14.3)$$

where $\mathbf{h}_i$ is the i-th column of $\mathbf{H}$.

Let's assume that (i-1) symbols, from the Tx 1 to (i-1), have already been detected.

<u>The interference cancellation step</u>: the contribution of these symbols (from the Tx 1 to (i-1)) to .. can be cancelled:

$$\mathbf{y}' = \mathbf{y} - \sum_{j=1}^{i-1} \mathbf{h}_j \hat{x}_j \tag{11.4}$$

where $\hat{x}_j$ are the detected symbols, which are assumed to be error free. If this is the case, then (11.4) becomes

$$\mathbf{y}' = \sum_{k=i}^{n} \mathbf{h}_k x_k + \boldsymbol{\xi} \tag{11.5}$$

Our immediate goal now is to detect x_i , which is mixed up with $x_{i+1},\ldots,x_{n_T}$ Hence, the interference nulling stage:

To null out the interference from $\left\{x_{i+1},\ldots,x_{n_T}\right\}$, project $\mathbf{y}'$ to the sub-space orthogonal to the sub-space spanned by $\left\{x_{i+1},\ldots,x_{n_T}\right\}$.

For this, use the projection matrix in (11.9). In fact, this is an interference cancellation problem we discussed before.

This stage of V-BLAST is also called "zero-forcing" (ZF) interference cancellation.

Alternative solution: MMSE interference cancellation.

Q.: which is better? Explain why.

Consider an example of nx2 V-BLAST. At step 1,

$$\mathbf{y} = \underbrace{\mathbf{h}_1 x_1}_{signal} + \underbrace{\mathbf{h}_2 x_2 + \xi}_{ISI+noise} \tag{11.6}$$

At step 2,

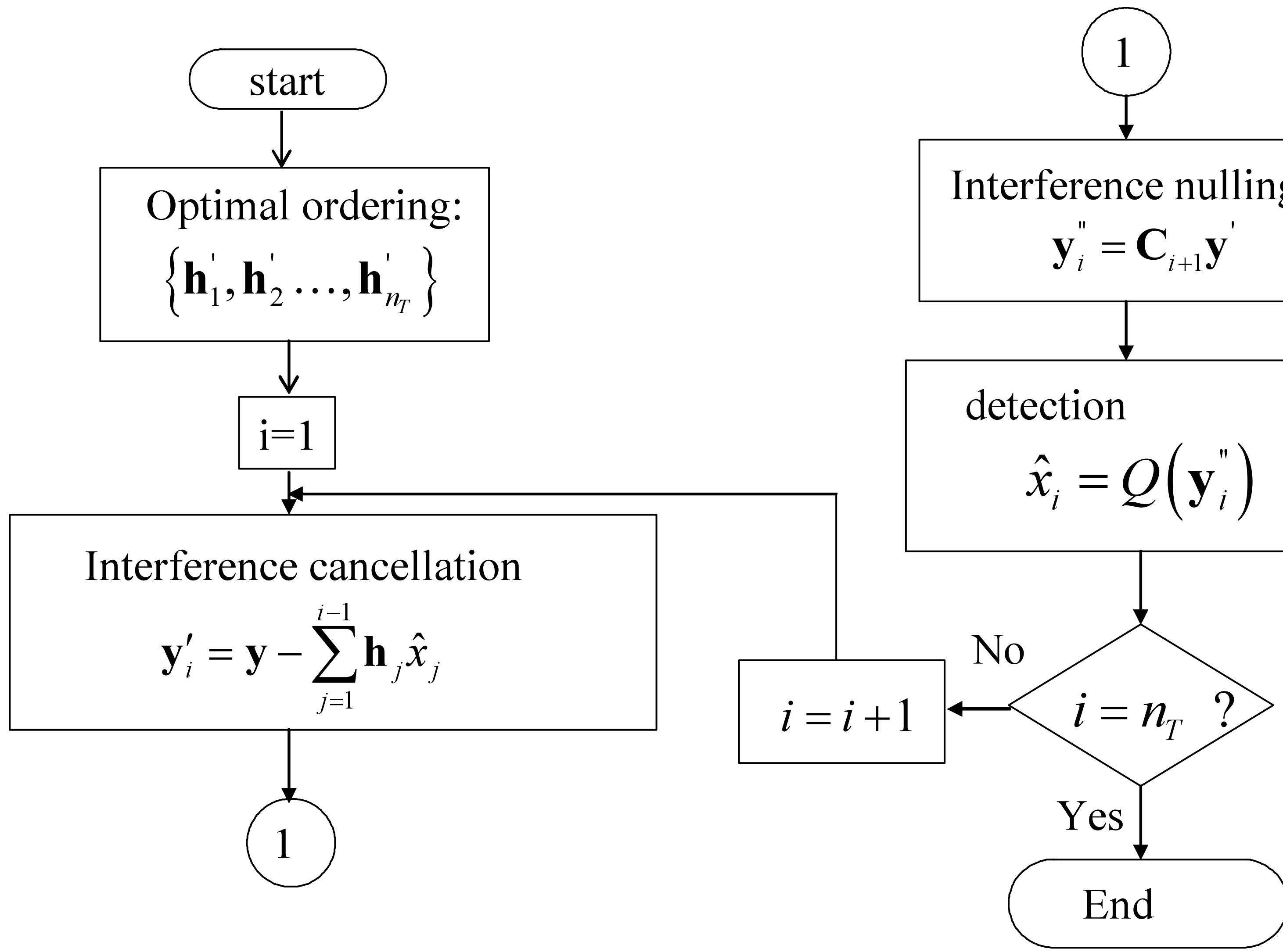
$$\mathbf{y}' = \underbrace{\mathbf{h}_2 x_2}_{signal} + \underbrace{\xi}_{noise} \tag{11.7}$$

The last component of the V-BLAST is the optimal ordering procedure.

The order of symbol processing is organized according to their after-processing SNR's in decreasing order, i.e, the symbol with highest after-processing SNR is detected first.

Practical way to accomplish this: detect first the symbol whose propagation vector has the lowest correlation with the other vectors.

V-BLAST Block Diagram



V-BLAST Block Diagram

$\mathbf{C}_{i+1}$ is the projection matrix to the sub-space orthogonal to $\left\{\mathbf{h}'_{i+1},\ldots,\mathbf{h}'_{n_T}\right\}$. This can be expressed as

$$\mathbf{C}_{i+1} = \mathbf{I} - \mathbf{H}_i \left(\mathbf{H}_i \mathbf{H}_i^+\right)^{-1} \mathbf{H}_i^+ \qquad (11.8)$$

where $\mathbf{H}_i = \left[\mathbf{h}'_{i+1}, \mathbf{h}'_{i+2} \ldots \mathbf{h}'_{n_T}\right]$.

Q.: what is the equivalent of (11.8) for MMSE V-BLAST?

Optimal Ordering

If the Tx signals are of equal power, then the optimal ordering is equivalent to finding the largest

$$a_i = \left|\mathbf{C}_{i+1}\mathbf{h}_i\right| \tag{11.9}$$

i.e. at step i we detect the symbol

$$j = \arg\max_k \left|\mathbf{C}_i^k \mathbf{h}_k\right|, \quad k \in \left[i \ldots n_T\right] \tag{11.10}$$

where $\mathbf{C}_i^k$ is a projection matrix to the subspace orthogonal to $\left\{\mathbf{h}_i, \ldots, \mathbf{h}_{k-1}, \mathbf{h}_{k+1}, \ldots, \mathbf{h}_{n_T}\right\}$, i.e all the vectors $\mathbf{h}_i \ldots \mathbf{h}_{n_T}$ except for $\mathbf{h}_k$.

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The algorithm described above, i.e. V-BLAST, is also known as ordered ZF SIC (or “decision feedback interference cancellation”).

There are some modifications: unordered one, ordered MMSE SIC, etc.

See the Appendix (at the end) for an extended discussion of those.

It can be proved that the BLAST achieves the full MIMO capacity [8].

Q.: write down explicitly all the steps for nx2 V-BLAST.

Useful references on V-BLAST:

A.U. Toboso, S. Loyka, F. Gagnon, On Optimal Detection Ordering for Coded V-BLAST, IEEE Transactions on Communications, v. 62, N. 1, pp. 100-111, Jan. 2014.

V. Kostina, S. Loyka, Optimum Power and Rate Allocation for Coded V-BLAST: Instantaneous Optimization, IEEE Transactions on Communications, v. 59, N. 10, Oct. 2011, pp. 2841-2850.

V. Kostina, S. Loyka, Optimum Power and Rate Allocation for Coded V-BLAST: Average Optimization, IEEE Transactions on Communications, v. 59, No. 3, pp. 877-887, Mar. 2011.

S. Loyka, F. Gagnon, On Outage and Error Rate Analysis of the Ordered V-BLAST, IEEE Trans. Wireless Communications, v. 7, N. 10, pp. 3679-3685, Oct. 2008.

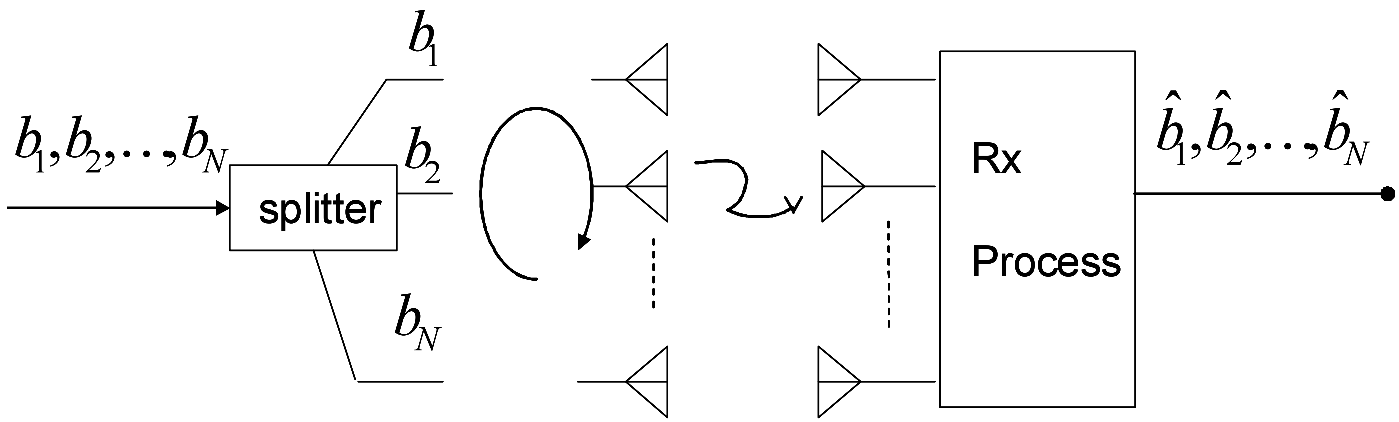
V. Kostina, S. Loyka, On Optimum Power Allocation for the V-BLAST, IEEE Transactions on Communications, v. 56, N. 6, pp. 999-1012, June 2008.

Diagonal BLAST (D-BLAST) Basic idea – cycle Tx antennas periodically over transmitted sub-streams to provide equal conditions for each sub-stream.

Fixed V-BLAST architecture is not optimal because in fixed environment (or slowing varying), one of the sub-streams may be in worst conditions all the time.

Detailed description of the D-BLAST - see Foschini's paper:

G.J. Foschini, Layered Space-Time Architecture for Wireless Communications in a Fading Environment When Using Multi-Element Antennas, Bell Labs Technical Journal, Aug. 1996.



Hint: to facilitate understanding, consider first 2x2 D-BLAST.

Note: the cycling does not affect the system capacity; can be skipped if rates of each stream are properly allocated.

Q.: what is the difference in BER performance of V- and D-BLAST?

Maximum-Likelihood (ML) BLAST

A big disadvantage of V-BLAST is that 1st detected symbol doesn't enjoy any diversity (due to nulling out (n-1) other symbols) when $n_T = n_R = n$, or has the lowest diversity order of all steps, $(n_R - n_T + 1)$, in the general case. This sub-stream will have the worst performance, which will dominate the overall performance due to the error propagation.

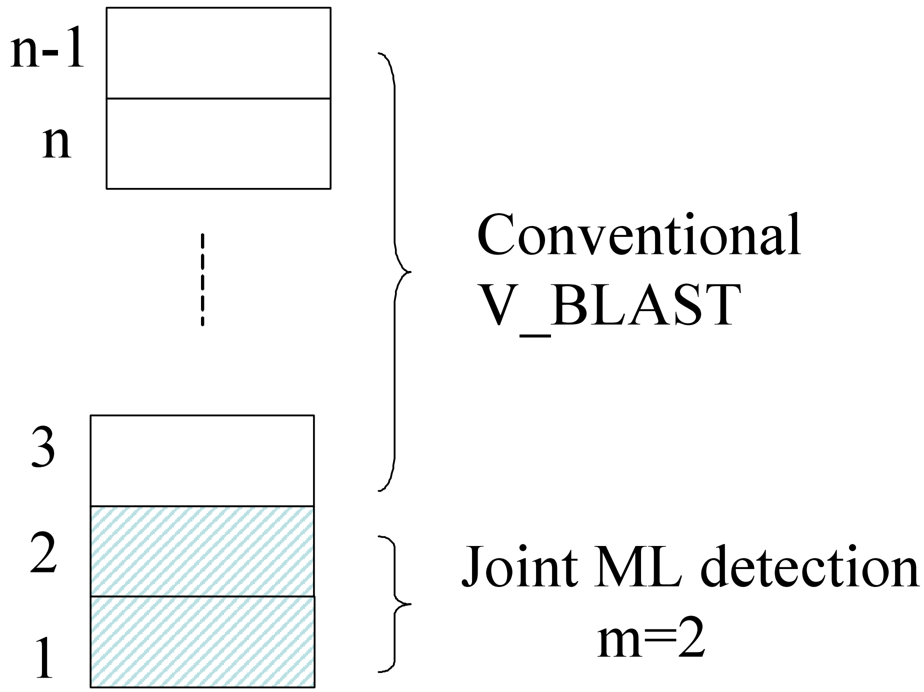
Thus, the algorithm needs an improvement.

The key idea of the ML BLAST: first m symbols, m<n, are jointly detected using the ML approach, and the remaining n-m symbols are detected in a conventional way.

Advantage: diversity order for the first m symbols is m.

Cannot do m=n because ML is exponential in complexity, but it is very feasible for small m (e.g. m=2).

Maximum-Likelihood (ML) BLAST



Summary

- V-BLAST, D-BLAST and ML-BLAST.
- Detailed description of the algorithms.
- Performance analysis.
- Comparison: advantages and disadvantages.
- Links to multiuser systems (MAC).

References

[1] J.R. Barry, E.A. Lee, D.G. Messerschmitt, Digital Communication, 2003 (Third Edition).

[2] D. Tse, P. Viswanath, Fundamentals of Wireless Communications, Cambridge, 2005.

[3] P.P. Vaidyanathan et al, Signal Processing and Optimization for Transceiver Systems, Cambridge University Press, 2010; Ch. 6, 22, App. B, C.

[4] D.W. Bliss, S. Govindasamy, Adaptive Wireless Communications: MIMO Channels and Networks, Cambridge University Press, 2013.

[5] A. Paulraj, R. Nabar, D. Gore, Introduction to Space-Time Wireless Communications, Cambridge University Press, 2003.

[6] G. Larsson, P. Stoica, Space-Time Block Coding for Wireless Communications, Cambridge University Press, 2003.

[7] G.J Foschini et al, Simplified Processing for High Spectral Efficiency Wireless Communication Employing Multi-Element Arrays, *IEEE Journal on Selected Areas in Communications*, v. 17, N. 11, pp. 1841-1852, Nov. 1999.

[8] G.J. Foschini et al, Analysis and Performance of Some Basic Space-Time Architectures, *IEEE Journal Selected Areas Comm*., v. 21, N. 3, pp. 281-320, Apr. 2003.

[9] T.K. Moon, W.C. Stirling, Mathematical methods and algorithms for signal processing, Prentice Hall, 2000.

[10] W.Y. Choi, R.Negi, J.M. Cioffi, Combined ML and DFE decoding for the V-BLAST Systems, IEEE ICC'00.

Homework

Fill in the details in the derivations above. Answer the questions. Do the examples yourself.

Appendix: Further Discussion of the V-BLAST and its properties.

Max. SNR/MMSE V-BLAST

Consider regular (ZF) V-BLAST first.
The basic basic channel model:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\xi} = \sum_{i=1}^{m} \mathbf{h}_i x_i + \boldsymbol{\xi} \quad (1)$$

Detection step i: assume the Tx symbols $\left[x_1 ... x_{i-1}\right]$ has been correctly detected,

$$\widehat{x_j} = x_j, \quad j = 1...i-1 \quad (2)$$

Subtract the contribution of already detected symbols from $\mathbf{y}$,

$$\mathbf{y}' = \mathbf{y} - \sum_{j=1}^{i-1} x_j \mathbf{h}_j = \sum_{k=i}^{m} \mathbf{h}_k x_k + \boldsymbol{\xi} = \mathbf{H}_{(i-1)} \mathbf{x}_{(i-1)} + \boldsymbol{\xi} \tag{3}$$

where $\mathbf{H}_{i-1} = \left[\mathbf{h}_i, \mathbf{h}_{i+1}, ..\mathbf{h}_m\right], \quad \mathbf{x}_{(i-1)} = \left[x_i, x_{i+1}, ..x_m\right]^T$. This is the interference cancellation stage.

Next, project out interference from yet to detected symbols $\mathbf{x}_{(i)}$:

$$\mathbf{y}'' = \mathbf{P}_i \mathbf{y}' = \mathbf{P}_i \mathbf{h}_i x_i + \mathbf{P}_i \boldsymbol{\xi}, \tag{4}$$

where $\mathbf{P}_i = \mathbf{I} - \mathbf{H}_i \left(\mathbf{H}_i^+ \mathbf{H}_i\right)^{-1} \mathbf{H}_i^+$ is the projection matrix (orthogonal to span{$h_{i+1}..h_m$}).

Finally, do MRC using $\mathbf{y}''$ to maximize output SNR:

$$\hat{y}_i = \boldsymbol{\alpha}_i^+ \mathbf{y}'' = \mathbf{h}_i^+ \mathbf{P}_i \mathbf{h}_i x_i + \mathbf{h}_i^+ \mathbf{P}_i \boldsymbol{\xi} \quad (5)$$

where $\boldsymbol{\alpha}_i = \mathbf{h}_i$ are MRC weights. (5) can be compactly expressed as:

$$\hat{y}_i = \mathbf{w}_i^+ y', \quad \mathbf{w}_i = \mathbf{P}_i \mathbf{h}_i \quad (6)$$

where we used the fact that $\mathbf{P}_i^+ = \mathbf{P}_i$.

The output SNR is:

$$\gamma_i = \frac{\left\langle \left| \mathbf{h}_i^+ \mathbf{P}_i \mathbf{h}_i \mathbf{x}_i \right|^2 \right\rangle}{\left\langle \left| \mathbf{h}_i^+ \mathbf{P}_i \boldsymbol{\xi} \right|^2 \right\rangle} = \frac{\mathbf{h}_i^+ \mathbf{P}_i \mathbf{h}_i}{\sigma_0^2} \quad (7)$$

assuming $\left\langle |x_i|^2 \right\rangle = 1$ (unit power constellation). $\hat{y}_i$ is the decision variable to find x_i.

This algorithm is sometimes called ZF (zero-forcing) V-BLAST as $\mathbf{P}_i$ cancels completely ISI (inter-stream interference) from $\{x_{i+1} \dots x_m\}$. It does not minimize BER, however.

Max. SNR V-BLAST

Consider step i and find such weights w_i that the output SNR is maximized,

$$\hat{y}_i = \mathbf{w}_i^+ \mathbf{y}' = r_{si} + r_{\xi i}$$
$$r_{si} = \mathbf{w}_i^+ \mathbf{h}_i x_i, \quad r_{\xi i} = \sum_{k=i+1}^{m} \mathbf{w}_i^+ \mathbf{h}_k x_k + \mathbf{w}_i^+ \boldsymbol{\xi} \tag{8}$$

Output signal and noise/interference powers:

$$P_s = \left\langle \left| r_{si} \right|^2 \right\rangle = \left| \mathbf{w}_i^+ \mathbf{h}_i \right|^2$$

$$P_\xi = \left\langle \left| r_{\xi i} \right|^2 \right\rangle = \left\langle \mathbf{w}_i^+ \mathbf{H}_i \mathbf{x} \mathbf{x}^+ \mathbf{H}_i^+ \mathbf{w}_i \right\rangle + \sigma_0^2 \mathbf{w}_i^+ \mathbf{w}_i = \mathbf{w}_i^+ \left(\sigma_0^2 \mathbf{I} + \mathbf{H}_i \mathbf{H}_i^+ \right) \mathbf{w}_i \tag{9}$$

where we have used $\left\langle \mathbf{x}_{(i+1)}\mathbf{x}_{(i+1)}^{+} \right\rangle = \mathbf{I}, \quad \left\langle \boldsymbol{\xi}\boldsymbol{\xi}^{+} \right\rangle = \sigma_0^2\mathbf{I}$ (i.e. i.i.d. signals and noise).

Finally, the output SNR is

$$\gamma_i = \frac{P_s}{P_\xi} = \frac{\mathbf{w}_i^{+}\mathbf{h}_i\mathbf{h}_i^{+}\mathbf{w}_i}{\mathbf{w}_i^{+}R_\xi\mathbf{w}_i} \tag{10}$$

where $\mathbf{R}_\xi = \sigma_0^2\mathbf{I} + \mathbf{H}_i\mathbf{H}_i^{+}$ is noise and ISI correlation matrix.

Optimization problem:

$$\max_{\mathbf{w}_i} \gamma_i \tag{11}$$

The solution is

$$\mathbf{w}_i = \mathbf{R}_\xi^{-1}\mathbf{h}_i \tag{12}$$

and the max SNR is

$$\gamma_{i,\max} = \mathbf{h}_i^{+}\mathbf{R}_\xi^{-1}\mathbf{h}_i \tag{13}$$

Compare to ZF solution (7); for large average SNR, $\sigma_0^2 \to 0$,

$$\mathbf{R}_\xi^{-1} = \left(\sigma_0^2 \mathbf{I} + \mathbf{H}_i \mathbf{H}_i^{+}\right)^{-1} \approx \frac{1}{\sigma_0^2} \mathbf{P}_i \qquad (14)$$

and max SNR solution is very close to ZF solution (7),

$$\gamma_{i,\max} \approx \gamma_{iZF} \qquad (15)$$

Max SNR solution (12)-(13) has very important property.

Theorem: Max SNR (MMSE) V-BLAST achieves the full MIMO capacity (no Tx CSI, isotropic signaling).

Proof:

$$C = \log\left| I + \frac{\rho}{m}\mathbf{H}\mathbf{H}^{+} \right| = \log\left| I + \frac{\rho}{m}\mathbf{H}_1\mathbf{H}_1^{+} + \frac{\rho}{n}\mathbf{h}_1\mathbf{h}_1^{+} \right|$$

$$= \log\left| I + \frac{\rho}{m}\mathbf{H}_1\mathbf{H}_1^{+} \right| + \log\left| I + \frac{\rho}{m}\left(I + \frac{\rho}{n}\mathbf{H}_1\mathbf{H}_1^{+} \right)^{-1}\mathbf{h}_1\mathbf{h}_1^{+} \right| \quad (16)$$

$$= \log\left| I + \frac{\rho}{m}\mathbf{H}_1\mathbf{H}_1^{+} \right| + \Delta_1$$

$$\Delta_1 = \log\left(1 + \frac{\rho}{m}\mathbf{h}_1^{+}\left(I + \frac{\rho}{m}\mathbf{H}_1\mathbf{H}_1^{+} \right)^{-1}\mathbf{h}_1 \right) \quad (17)$$

Note that with our normalization, $\left\langle \left| x_i \right|^2 \right\rangle = 1$,

$$\frac{\rho}{m} = \frac{1}{\sigma_0^2} \tag{18}$$

and

$$\Delta_1 = \log\left(1+\gamma_1\right), \quad \gamma_1 = \mathbf{h}_1^+\left(\sigma_0^2\mathbf{I} + \mathbf{H}_1\mathbf{H}_1^+\right)^{-1}\mathbf{h}_1 \tag{19}$$

Comparing (19) to (13), we conclude that γ_1 is the output SNR of the max SNR processing at step1 (considering T_x 2…m as sources of interference, ISI). Hence, Δ_1 is the capacity at step 1.

Applying the same expansion to $\log\left|\mathbf{I} + \frac{\rho}{m}\mathbf{H}_1\mathbf{H}_1^+\right|$, one obtains:

$$C = \sum_{i=1}^{m} \Delta_i; \qquad \begin{aligned} \Delta_i &= \log\left(1+\gamma_i\right) \\ \gamma_i &= \mathbf{h}_i^+\left(\sigma_0^2\mathbf{I} + \mathbf{H}_i\mathbf{H}_i^+\right)\mathbf{h}_i \end{aligned} \tag{20}$$

where Δ_i is the capacity of i-th stream, and γ_i is the SNR with max SNR processing. Q.E.D.

Note: from (14), one may conclude that asymptotically, $\sigma_0^2 \ll 1$, ZF V-BLAST also achieves MIMO capacity.

MMSE BLAST

In a similar way, one may consider MMSE solution to the stream separation problem in (1),

$$\min_{\mathbf{w}_i} \varepsilon_i^2 \quad , \quad \varepsilon_i^2 = \left\langle \left| x_i - \mathbf{w}_i^+ \mathbf{y} \right|^2 \right\rangle \tag{21}$$

Using

$$\frac{d\varepsilon_i^2}{d\mathbf{w}_i} = 0 \tag{22}$$

one finds MMSE weights as

$$\mathbf{w}_i = \left(\sigma_0^2 \mathbf{I} + \mathbf{H}_{(i-1)} \mathbf{H}_{(i-1)}^+\right)^{-1} \mathbf{h}_i \tag{23}$$

and

$$\varepsilon_{i,\min}^2 = 1 - \mathbf{h}_i^+ \left(\sigma_0^2 \mathbf{I} + \mathbf{H}_{(i-1)} \mathbf{H}_{(i-1)}^+\right)^{-1} \mathbf{h}_i \tag{24}$$

After some manipulations, it can be shown that max SNR and MMSE weights are related as

$$\mathbf{w}_{MMSE} = \frac{\mathbf{w}_{SNR}}{1+\gamma_i}, \quad \gamma_i = \mathbf{h}_i^+ \mathbf{R}_\xi^{-1} \mathbf{h}_i \tag{25}$$

and, hence, MMSE solution also provides max SNR.

Important relationship between min MMSE and max SNR:

$$\frac{1}{\varepsilon_{\min,i}^2} = 1 + \gamma_i \tag{26}$$

Exercise: prove (14), (23), (25), (26).

Ref. [1] has an especially good chapter on MIMO systems. Highly recommended, as well as [2].