Multiple-Input Multiple-Output Systems

What is the best way to use antenna arrays? – MIMO!

This is a totally new approach ("paradigm") to wireless communications, which has been discovered in 95-96.

Performance improvement in terms of capacity (spectral efficiency) [bit/s/Hz] is 10-fold and even more (under favorable propagation conditions) as compared to conventional systems.

Since 1948, when Shannon published his now famous paper, "Communication in the presence of noise", this is the most significant single discovery in the field of communications.

The key idea behind MIMO is to use rather than to combat multipath to create multiple parallel (virtual) channels and to use them to send n times more data (n is the number of Tx/Rx antennas).

Thus, multipath becomes an ally rather than enemy.

Space-domain signal processing is fully exploited in this approach.

Achieves the fundamental limits coming from information theory (bit/s).

We begin with a brief historical review.

Wireless system with single antennas

Classical Shannon's limit for channel capacity (spectral efficiency):

$$C = \log_2(1 + SNR) \quad [\text{bit/Hz/s}] \tag{12.1}$$

Increases as log of SNR → very slowly!

Channel capacity is low → few bits/Hz/s

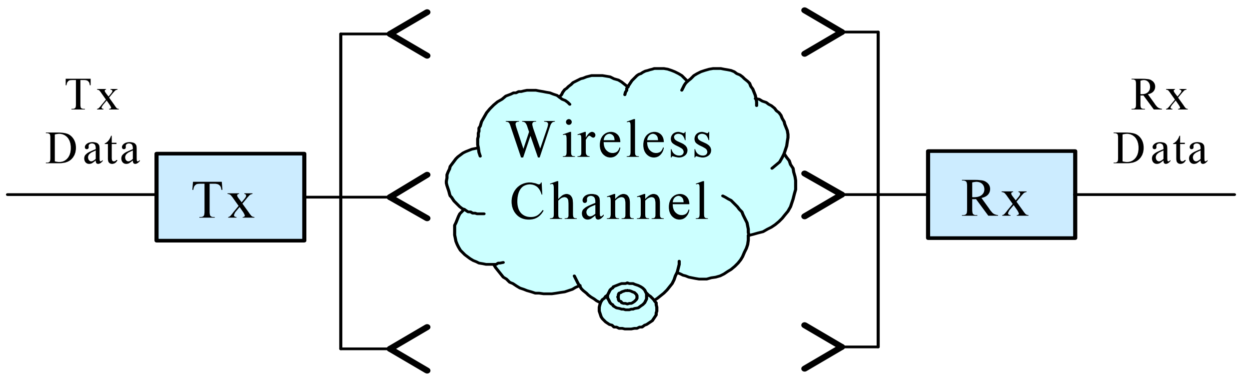
Fading is huge → 20-40 dB

No space domain signal processing.

Design is simple.

Wireless system with multiple antennas

(phased array, diversity combining etc.)



$$C = \log_2\left(1 + SNR \cdot n^2\right) \quad [\text{bit/Hz/s}] \tag{12.2}$$

Increases as the log of n → very slowly!

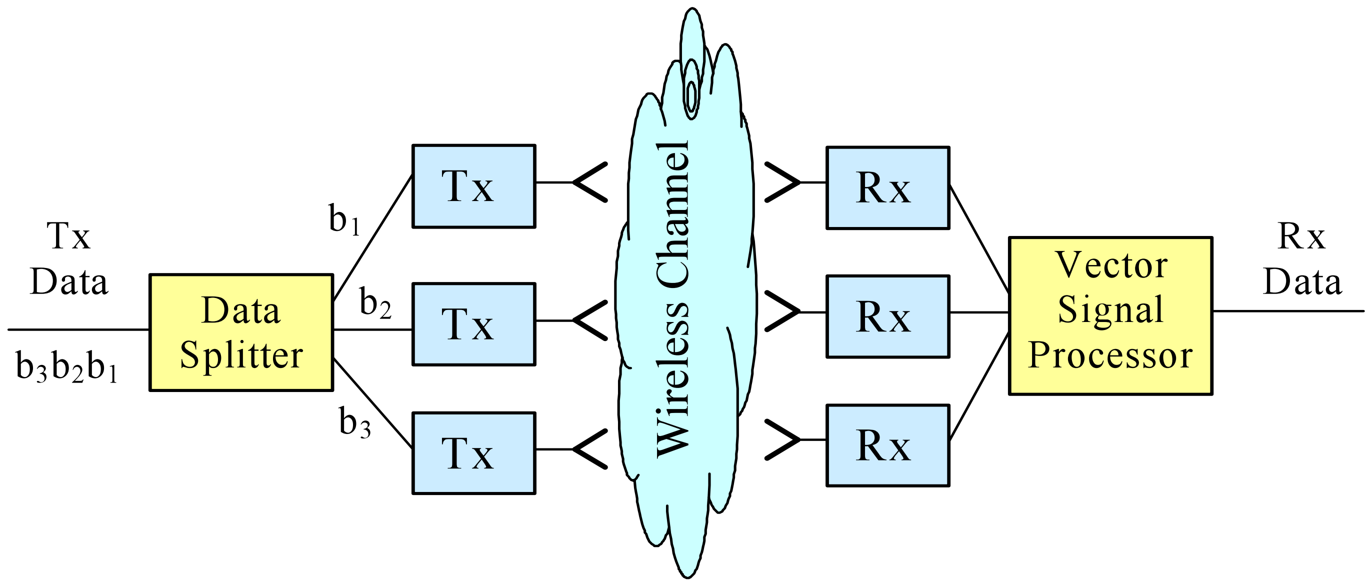
Channel capacity is still low (few bits/Hz/s), additional 1(2) bit/s/Hz for doubling n.

Fading is smaller but still large (10-20 dB).

Space-domain signal processing – partially.

More complex antennas, beamforming etc.

MIMO: launch multiple bit streams!



$$C = \log_2 \det\left(\mathbf{I} + \frac{\rho}{n}\mathbf{H}\cdot\mathbf{H}^{+}\right) \xrightarrow{\mathbf{H}=\mathbf{I}} n\cdot\log_2\left(1+\frac{SNR}{n}\right) \quad (12.3)$$

Key idea: split the incoming bit stream into N independent sub-streams and launch them independently.

Capacity: linear in n, growth much faster, doubling N doubles the capacity.

Note: this is for uncorrelated channel only.

Enormous channel capacity → 10 fold increase has been demonstrated.

Fading can be reduced significantly.

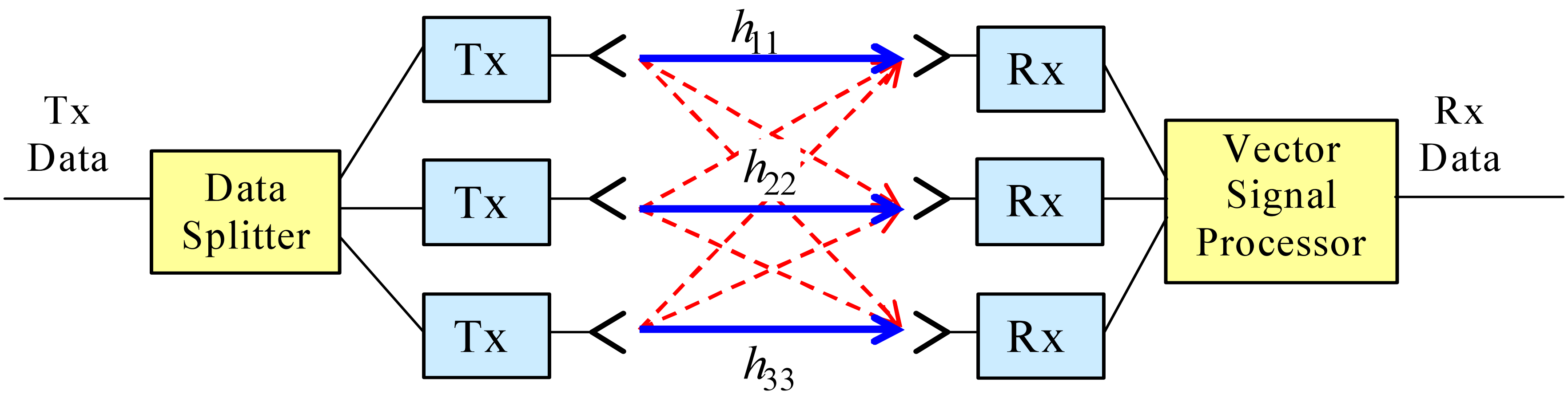
There is a trade-off of "transmission rate-fading reduction".

Full space-domain signal processing.

More complex design is fully compensated by significant advantages.

Why and where it works ?

Uncorrelated sub-channels → parallel independent sub-channels



Mathematically,

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{32} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \tilde{h}_{11} & 0 & 0 \\ 0 & \tilde{h}_{22} & 0 \\ 0 & 0 & \tilde{h}_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Channel matrix diagonalization is a key operation for MIMO.

Signal processing at the receiver must do this job.

Correlated sub-channels → complete diagonalization is not possible → increase in fading and decrease in channel capacity.

Simplified mathematical representation of the key idea

Consider the following MIMO channel model,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\xi} \tag{12.4}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the transmitted and received vectors correspondingly, $\boldsymbol{\xi}$ is AWGN, and $\mathbf{H}$ is the channel matrix (h_{ij} represents complex channel gain between i-th Rx and j-th Tx antenna). Let us ignore the noise (hypothetical case),

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\xi} \xrightarrow{\boldsymbol{\xi}=\mathbf{0}} \mathbf{y} = \mathbf{H}\mathbf{x} \tag{12.5}$$

If one knows $\mathbf{H}$ (this is Rx CSI) and $\mathbf{y}$, and $\mathbf{H}$ is non-singular, one can recover $\mathbf{x}$ is a simple way (channel inversion)[1],

$$\hat{\mathbf{x}} = \mathbf{H}^{-L}\mathbf{y} = \mathbf{x}$$

In a noisy channel,

$$\hat{\mathbf{x}} = \mathbf{H}^{-L}\mathbf{y} = \mathbf{x} + \mathbf{H}^{-L}\boldsymbol{\xi} \tag{12.6}$$

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[1] This is rarely used in practice as many problems are involved, including noise enhancement, poor numerical performance etc.

When the impact of the noise is negligible,

$$\mathbf{H}^{-L}\boldsymbol{\xi} \approx \mathbf{0} \rightarrow \hat{\mathbf{x}} \approx \mathbf{x} \tag{12.7}$$

The job of vector Rx processing is to find a good estimate of $\mathbf{x}$ given $\mathbf{H}$ and $\mathbf{y}$.

Q: find MMSE estimation of **x** (given **H** and **y**)!

Q.: what to do if **H** is singular?

MIMO Key Advantages

Extraordinary high spectral efficiency (from 30-40 bit/s/Hz)

Large fade level reduction (10-30 dB).

Co-channel interference reduction.

Multipath is not enemy, but ally!

Flexible (adaptive) architecture through DSP.

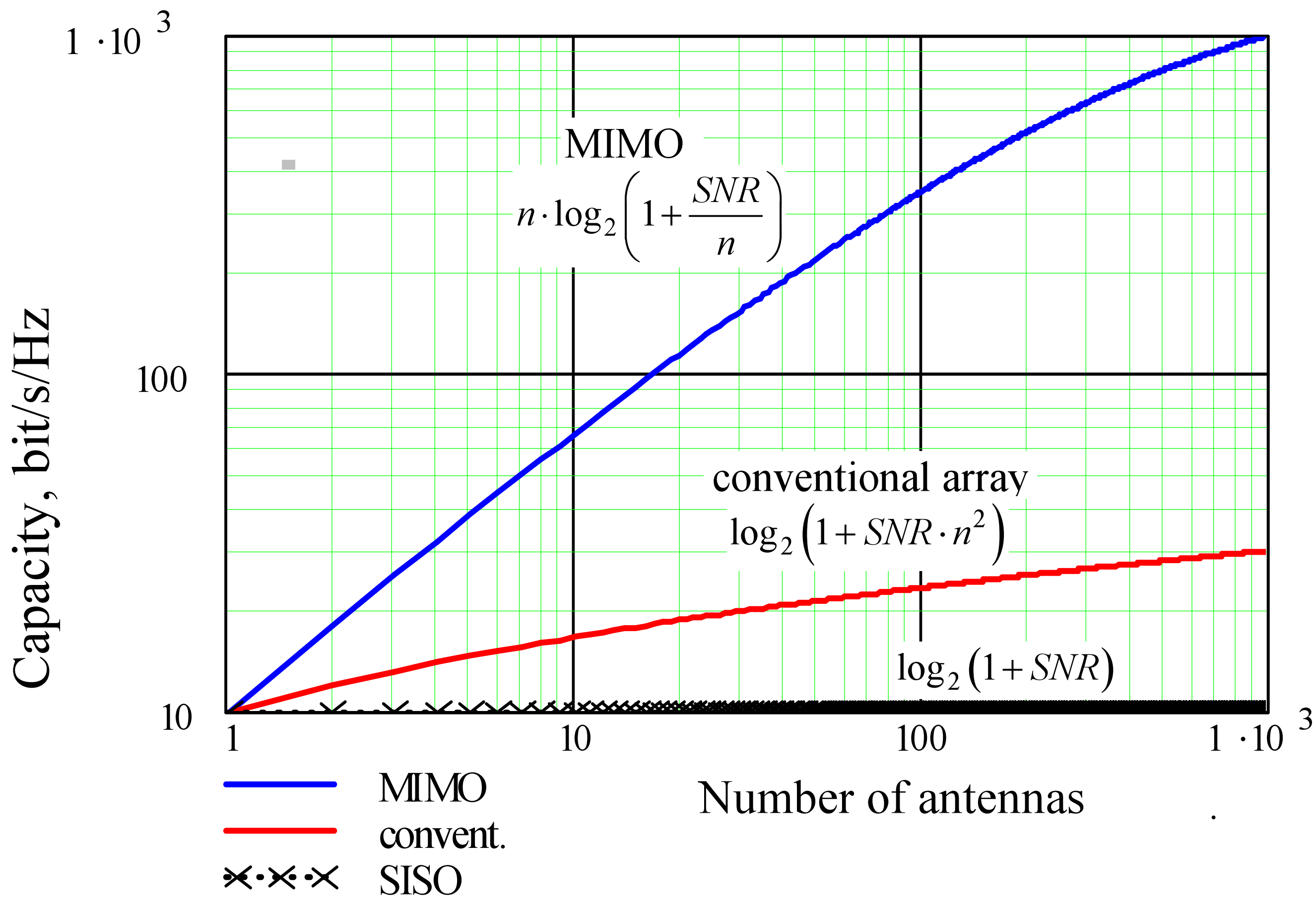
Diversity order (DO) for MIMO $\propto n^2$ and for SIMO $\propto n$

MIMO efficiently exploits diversity at **both** Tx and Rx sites!

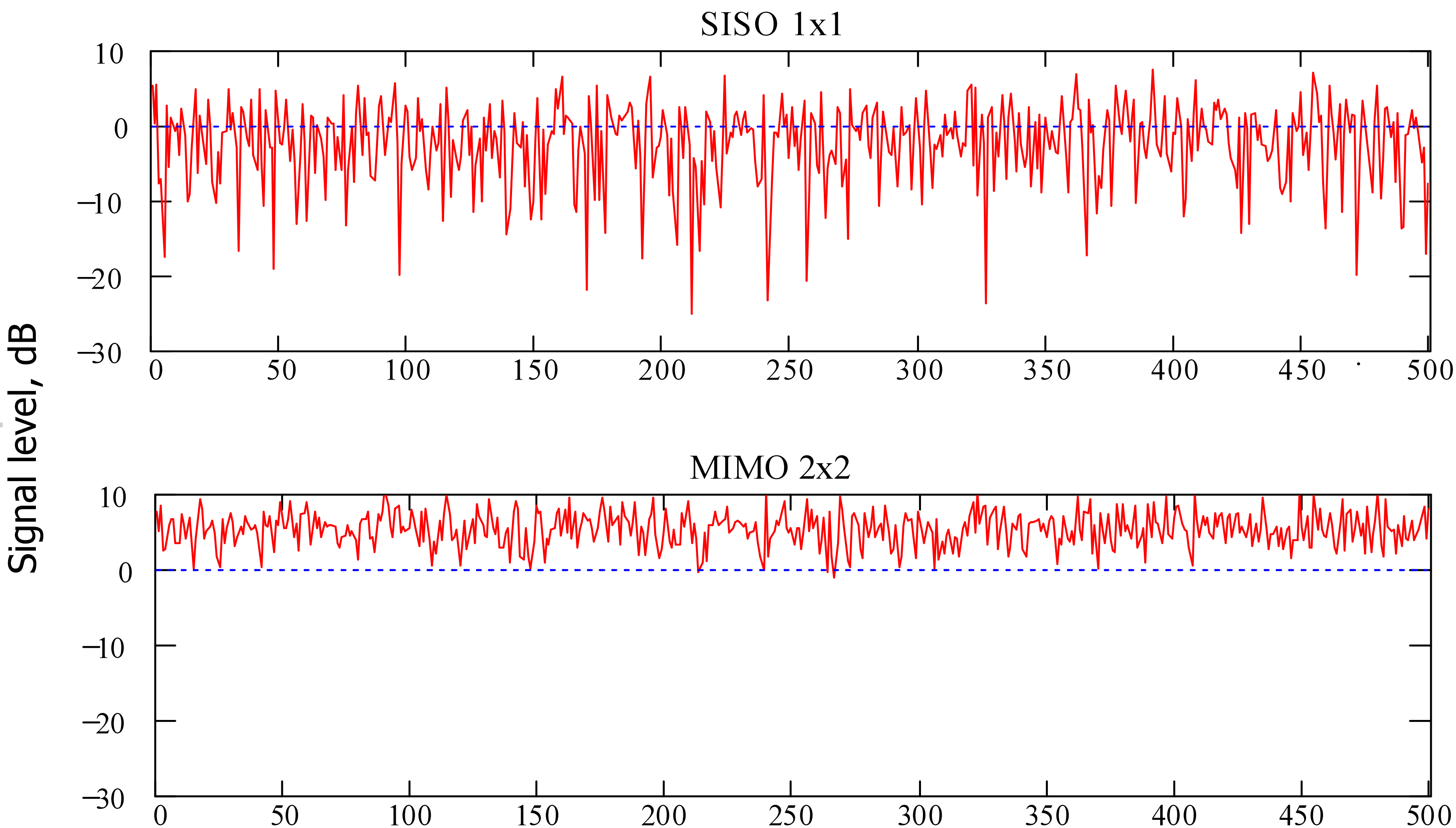
Example: correlated fading at Rx → no SIMO diversity, but MIMO works!

Consequence: 2-fold higher system availability for MIMO than for SIMO.

Spectral Efficiency



Fading Reduction



Current R&D[2]

Matrix channel modeling, simulation, characterization & measurement.

Basic system architecture development.

Space-time coding/decoding & modulation/demodulation, and performance evaluation.

Elements of system-level simulation. Prototyping.

Application areas (indoor, cellular, LMDS (WLAN) etc.).

Future R&D[2]

Matrix channel will be still a problem.

Space-time codes into design! Adaptive MIMO architecture.

Nonlinear effects in Tx/Rx branches. Full-scale system-level simulation.

First products on the market.

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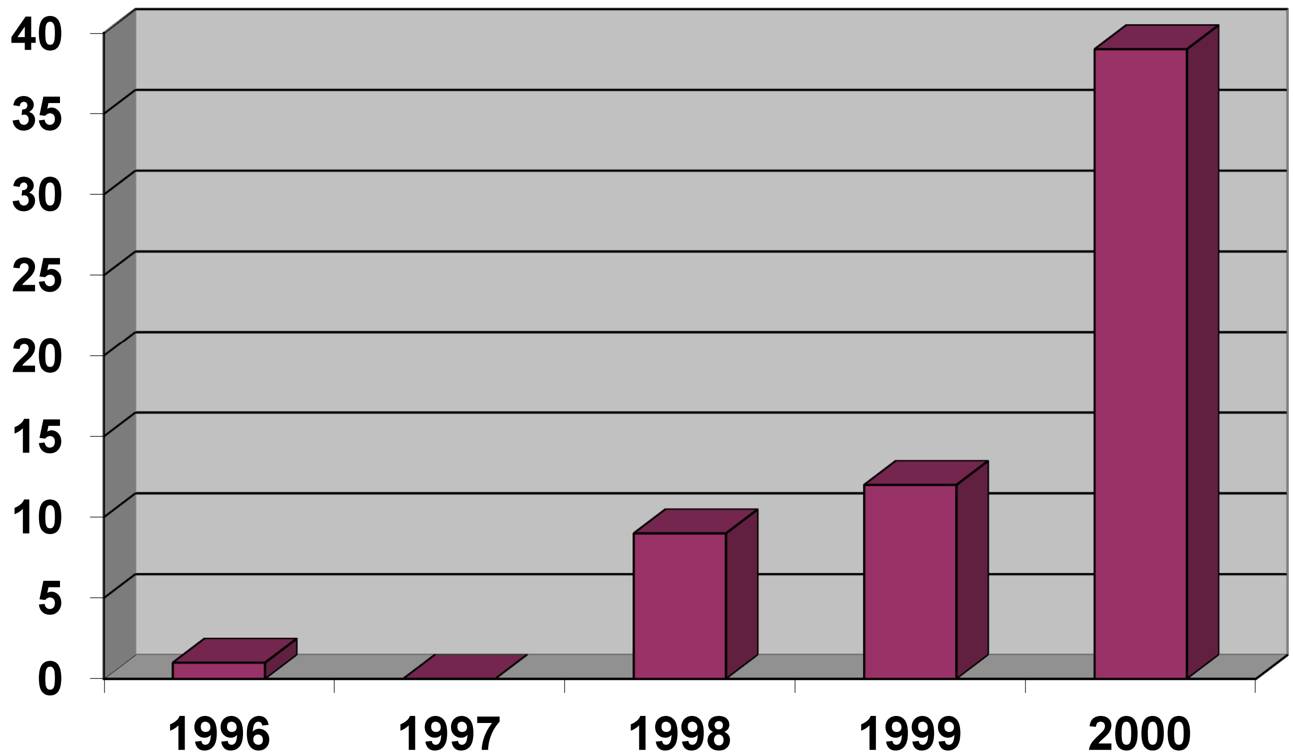
[2] This topic list was originally created in 2001. Which topics would you add to the list today? What is still current and what is not?

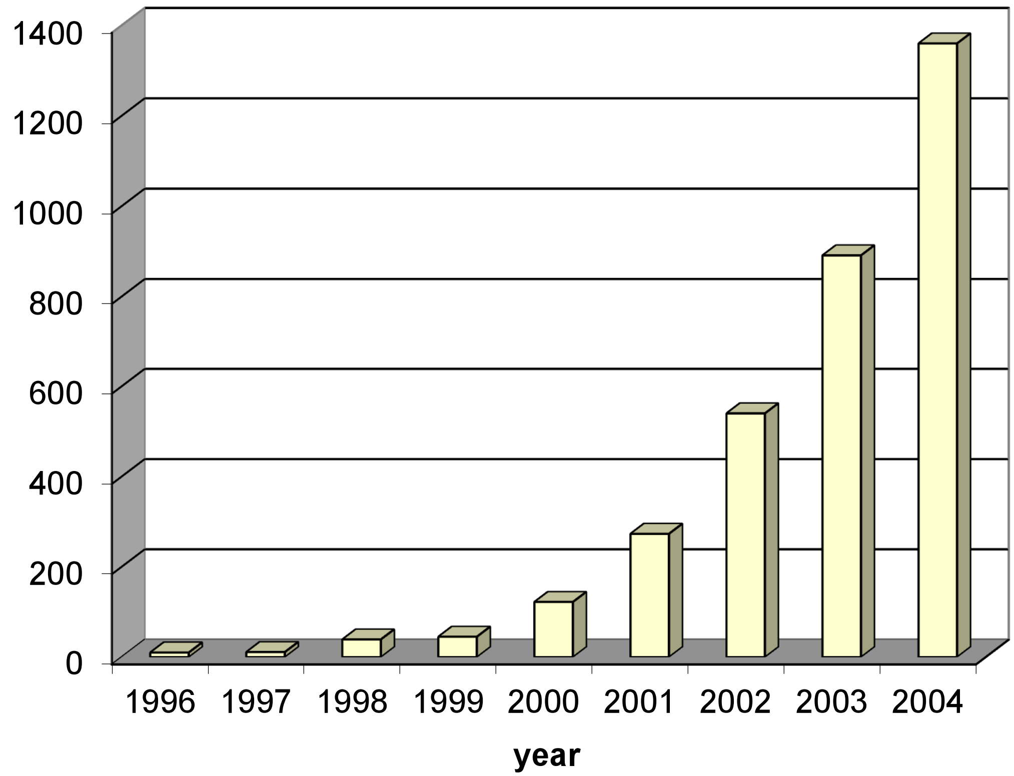
Number of MIMO publications

(up to 2000)



Number of MIMO publications

(up to 2004, by V. Kostina)



This data includes any published paper containing all of the keywords "MIMO", "wireless", "channel", "space-time", "communications" returned by the Google Scholar search engine.

Q.: continue this graph to the current year.

Review of information theory and channel capacity

Intuitive notion of information must be substituted by precise definition.

Since all processes are essentially band-limited and hence, can be sampled, we consider discrete random variables.

Information is related to some new knowledge, conveyed to us by the signal. If signal is totally deterministic (i.e. known), it does not carry any information.

Entropy of discrete R.V.

$$H(X) = -\sum_{i=1}^{N} p_i \log p_i \qquad (12.8)$$

where RV X can assume any of N values $\{x_1, x_2, \ldots, x_N\}$ with corresponding probabilities $\{p_1, p_2, \ldots, p_N\}$, $p_i = p(x_i)$

Joint entropy

$$H(X,Y) = -\sum_{i,j} p(x_i, y_i) \log p(x_i, y_i) \qquad (12.9)$$

where $p(x_i, y_i)$ is the joint probability.

For vector RV $\mathbf{x} = (x_1, x_2, \ldots, x_M)$,

$$H\left(X\right)=-\sum_{x_1,x_2,\ldots,x_M} p\left(x_1,x_2,\ldots,x_M\right)\log p\left(x_1,x_2,\ldots,x_M\right) \qquad (12.10)$$

Intuitively, $H\left(X\right)$ is the amount of uncertainty in our knowledge about RV X, i.e., when we know it exactly, $H\left(X\right)=0$.

Q.: prove it!

Conditional entropy

$$H\left(X\,|\,Y\right)=-\sum_{i,j} p\left(x_i,y_i\right)\log p\left(x_i\,|\,y_i\right) \qquad (12.11)$$

This is a measure of uncertainty in our knowledge about X provide that Y is known.

For vector RV

$$H\left(x_M\,|\,x_1,x_2,..,x_{M-1}\right)=-\sum_{x_1,x_2,..,x_M} p\left(x_1,x_2,..,x_{M-1}x_M\right)\log p\left(x_M\,|\,x_1,x_2,..,x_{M-1}\right)$$

(12.12)

The chain rule:

$$H\left(X,Y\right)=H\left(X\right)+H\left(Y\,|\,X\right)=H\left(Y\right)+H\left(X\,|\,Y\right) \qquad (12.13)$$

i.e. the information transmitted by X and Y is the information transmitted by X plus the info transmitted by Y provided that X is known.

Q.: prove (12.13).

If M RV's $\{x_1, x_2, \ldots, x_M\}$ are independent, then

$$H(X) = \sum_{i=1}^{M} H(x_i) \tag{12.14}$$

$H(X|Y)$ is uncertainty of X after Y is known. If we start with $H(X)$, then $H(X) - H(X|Y)$ is the amount of uncertainty that has been removed by Y

Mutual information:

$$I(X,Y) = H(X) - H(X|Y) \tag{12.15}$$

This is the amount of information about X that is transmitted by Y.

Some <u>important properties of mutual information:</u>

$$a)\ I(X,Y) \ge 0$$

$$b)\ I(X,Y) \le \min\left[H(X), H(Y)\right]$$

$$c)\ I(X,Y) = \sum_{X,Y} p(X,Y) \log \frac{p(X,Y)}{p(X)p(Y)} \Rightarrow\ I(X,Y) = I(Y,X)$$

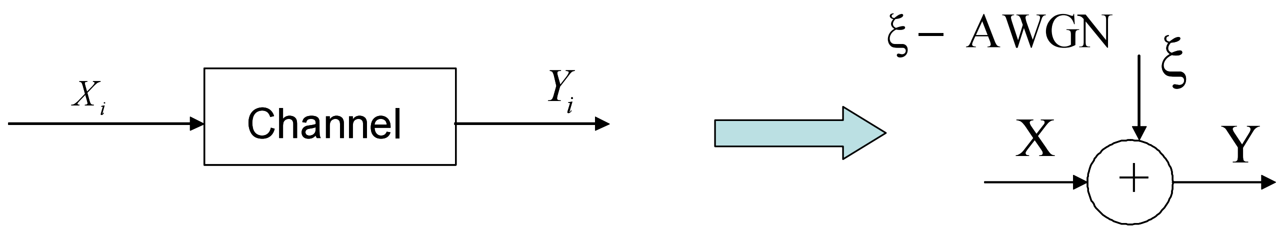
$$d)\ I(X,Y) = H(X) + H(Y) - H(X,Y)$$

$$e)\ I(XY,Z) = I(X,Z) + I(Y,Z \mid X) \rightarrow \text{ Chain rule}$$

$$\text{where } I(Y,Z \mid X) = \sum_{x} p(X) I(Y,Z \mid X = x)$$

Mutual information defined above quantifies the amount of information about one R.V. transmitted by the other RV.

Discrete memoryless channel



Channel capacity

Channel capacity as the maximum mutual information:

$$C = \max_{\substack{p(X) \\ \langle |X|^2 \rangle \le P_T}} \left[I(X,Y) \right] \tag{12.16}$$

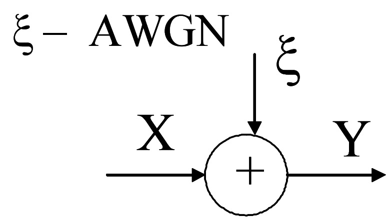
Operational significance: If the transmission rate $R < C$, then there exists such a code that BER can be made arbitrarily low.

But if $R > C$, BER is bounded away from 0 and cannot be made arbitrarily small.

This is the most fundamental result in communication and information theory. It gives a fundamental limit on reliable communication over a noisy channel.

Any channel has its own capacity, regardless the system we use and of any other properties.

AWGN channel capacity



$$y = x + \xi, \quad \xi \sim \mathbb{N}\left(0, \sigma_0^2\right) \tag{12.17}$$

Power constraint:

$$\left\langle |x|^2 \right\rangle \le \sigma_x^2 \tag{12.18}$$

Then, the capacity is

$$C = \Delta f \log\left(1 + \frac{\sigma_x^2}{\sigma_0^2}\right) = \Delta f \log\left(1 + SNR\right) \text{ [bit/s]} \tag{12.19}$$

where σ_x^2 and σ_0^2 are the signal and noise powers.

For thermal noise, $\sigma_0^2 = \Delta f \cdot N_0$, N_0 is the noise spectral power density.

Then,

$$C = \Delta f \log\left(1 + \frac{P}{\Delta f N_0}\right) \tag{12.20}$$

Geometrical illustration — sphere packing argument.

Note: for AWGN channel, the capacity is achieved when p(x) is Gaussian.

Important open problem: capacity of non-Gaussian channels.

Summary

- Historical development of wireless systems.
- Basic MIMO architecture. Its main advantages.
- Basic principles of MIMO operation.
- Review of information theory.
- Entropy. Mutual information. Channel capacity.
- Capacity of AWGN channel.

Reading:

Review of basic information theory concepts: any text on basic communication/ information theory, e.g.

1. T.M. Cover, J.A. Thomas, Elements of Information Theory, Wiley, 2006.

Modern communications & MIMO systems textbooks:

2. D. Tse, P. Viswanath, Fundamentals of Wireless Communications, Cambridge, 2005. Ch. 7, App. B.
3. J.R. Barry, E.A. Lee, D.G. Messerschmitt, Digital Communication, 2003 (Third Edition). Ch. 10, 4, 6.

MIMO systems: books

4. D.W. Bliss, S. Govindasamy, Adaptive Wireless Communications: MIMO Channels and Networks, Cambridge University Press, 2013.
5. A. Paulraj, R. Nabar, D. Gore, Introduction to Space-Time Wireless Communications, Cambridge University Press, 2003.
6. G. Larsson, P. Stoica, Space-Time Block Coding for Wireless Communications, Cambridge University Press, 2003.
7. E. Biglieri et al, MIMO Wireless Communications. Cambridge University Press, 2007.
8. H. Bolcskei et al (Eds.), Space-TimeWireless Systems: From Array Processing to MIMO Communications, Cambridge Univ. Press, 2006.
9. T.L. Marzetta, Massive MIMO: An Introduction, Bell Labs Technical Journal, v. 20, 2015. +book

Journal papers (special isues):

10. S.M. Alamouti, A simple transmit diversity technique for wireless communications, IEEE Journal on Selected Areas in Communications, v.16, N.8, pp. 1451 -1458, Oct. 1998
11. I.E. Telatar, "Capacity of Multi-Antenna Gaussian Channels," AT&T Bell Lab. Internal Tech. Memo., June 1995 (European Trans. Telecom., v.10, N.6, Dec.1999).
12. Foschini, G.J., Gans M.J.: 'On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas', Wireless Personal Communications, vol. 6, No. 3, pp. 311-335, March 1998. (available for download)
13. V. Tarokh, N. Seshadri, A.R. Calderbank, Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction, IEEE Trans. Information Theory, v. 44, N. 2, pp. 744-765, Mar. 1998.
14. Special Issue on MIMO Systems, IEEE Transactions on Signal Processing, v. 50, N. 10, Oct. 2002.
15. Special Issue on MIMO Systems, IEEE Journal Selected Areas Comm, v. 21, N. 3 and 5, April and June 2003.
16. Special Issue on Space-Time Transmission, Reception, Coding and Signal Processing, IEEE Trans. Information Theory, v. 49, N. 10, Oct. 2003.
17. Special Issue on Gigabit Wireless, Proceedings of the IEEE, v. 92, N.2, Feb. 2004.
18. Special Issue on Large-Scale Multiple Antenna Wireless Systems, IEEE Journal on Selected Areas in Communications (JSAC), vol. 31, no. 2, Feb. 2013.
19. Special Issue on Massive MIMO, Journal of Communications and Networks (JCN), vol. 15, no. 4, Aug. 2013.
20. Special Issue on Signal Processing for Large-Scale MIMO, IEEE Journal of Selected Topics in Signal Processing (JSTSP), Vol. 8, No. 5, Oct. 2014.

21. E. G. Larsson et al, Massive MIMO for Next Generation Wireless Systems, IEEE Communications Magazine, vol. 52, no. 2, pp. 186-195, Feb. 2014.
22. Special Issue on Signal Processing for Millimeter Wave Wireless Communications, IEEE Journal of Selected Topics in Signal Processing, vol. 10, no. 3, pp. 433-435, April 2016.
23. L. Lu et al, An Overview of Massive MIMO: Benefits and Challenges, IEEE JSTSP, Vol. 8, No. 5, Oct. 2014.
24. F. Rusek et al, Scaling up MIMO: Opportunities and Challenges with Very Large Arrays, IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 40-46, Jan. 2013.
25. H. Q. Ngo, E. G. Larsson, T. L. Marzetta, Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems, IEEE Trans. Comm., vol. 61, no. 4, pp. 1436-1449, Apr. 2013.
26. R. W. Heath et al, An Overview of Signal Processing Techniques for Millimeter Wave MIMO Systems, IEEE Journal of Selected Topics in Signal Processing, vol. 10, no. 3, pp. 436-453, April 2016.