The Cellular Concept

Key problems in multi-user wireless system:

- spectrum is limited and expensive
- large # of users to accommodate
- high quality-of-services (QoS) is required
- expandable systems are needed

Therefore,

- efficient use of the spectrum is required

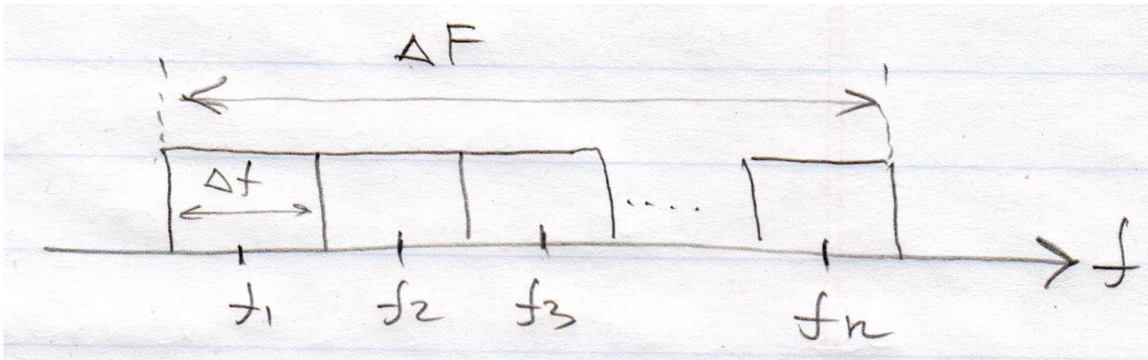
How?

- frequency re-use $\rightarrow$ cellular concept

Re-use the same frequencies in geographically-separated areas; use multiple BSs instead of a single one.

An Example

A single BS to serve large area:



ΔF = total available bandwidth
Δf = channel bandwidth (per user)

Key relationship:

$$\Delta F = n\Delta f \leftrightarrow \boxed{n = \frac{\Delta F}{\Delta f}} \leftrightarrow \Delta f = \frac{\Delta F}{n} \qquad (12.1)$$

Ottawa: $n = 10^6, \ \ \Delta f = 100\text{kHz}$

$\Rightarrow \ \ \Delta F = 10^6 \cdot 100\text{kHz} = 10^5\text{MHz} = 100\text{GHz}$

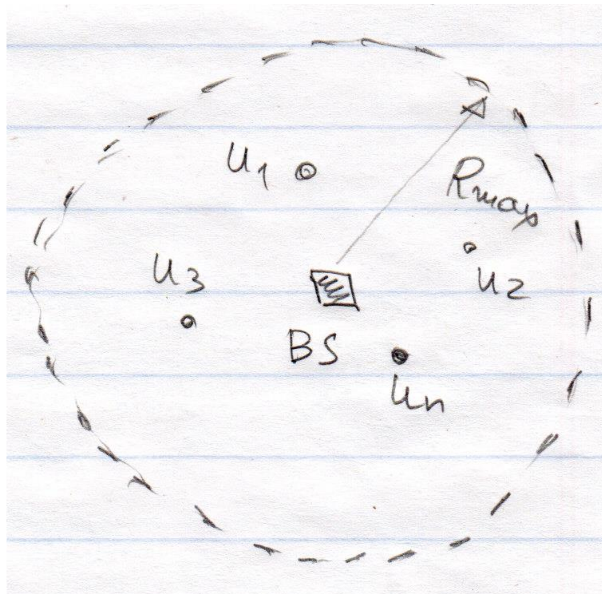
$\rightarrow$ totally unrealistic!

4G/LTE: $\Delta f = 1...20$ MHz

An Example: cont.

Available: $\Delta F = 100\text{MHz}$,

$$\Delta f = \frac{\Delta F}{n} = 100\text{Hz} \rightarrow \text{bad!}$$



<u>Another problem</u>: high P_t.

How much Tx power is needed?

Assume:

$$P_{r,\min} = -90\text{dBm} = 10^{-12}\,\text{W}.$$

$$G_r = G_t = 1;$$

$$h_r = 1\text{m},\ h_t = 100\text{m}$$

$$R_{\max} = 20\text{km}$$

$$P_t = \frac{P_r L_p}{G_t G_r} \qquad (12.2)$$

$$L_p = \frac{R^4}{h_t^2 h_r^2} \approx 10^{13} = 130\text{dB}$$

$$P_t = P_r L_p = 10\text{W/user} \ \ (\text{MU})$$

$$P_T = nP_t = 10^7\,\text{W} = \underline{10\text{MW}}! \ \ (\text{BS})$$

Fading margin 10 ... 20dB $\rightarrow$ $P_t = 100 - 1000\text{W}$ (MU)

$P_T = 100 - 1000\text{MW!}$ (BS)

2 key problems:

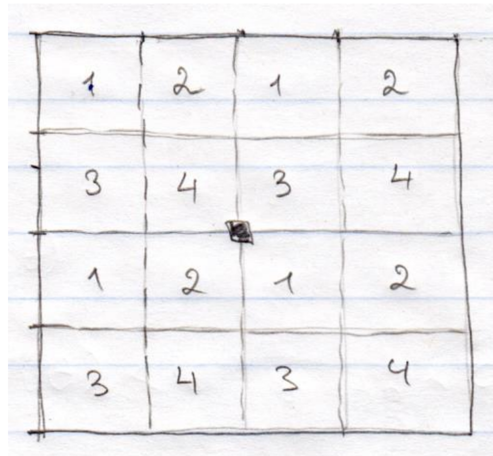
- How to reduce $P_{t,T}$?
- How to improve spectral efficiency?

Frequency Re-Use

Key idea: re-use frequencies in different geographical areas, without creating interference.

$i \rightarrow f_i$

$n = 4$ vs. $n = 16$!



Cell: a small geographical area served by a single BS.

Base Station: collects calls from all users within a given area and sends them to a public telephone network (add also Internet), i.e. serves as an access point for users.

- Adjacent cells use different frequencies (no interf.)
- Distant cells use the same frequencies (frequency re-use).

Key system- level idea: replace single (powerful) BS with many smaller-power BSs distributed over the total coverage area.

How does P_T scale ?

Frequency re-use and cellular architecture

Major break-through: offered high system capacity (# of users) in a limited spectrum.

- Expandable: new users/cells can be added, as the system grows.
- Mobile: users can be moving.
- Various services and QoS.

Each cell is allocated a sub-set of frequencies (not all available).

Frequency planning (allocation): insures that interference is not too high, i.e. SIR is not too low. (interference-limited system).

Recent idea: cognitive radio (CR).

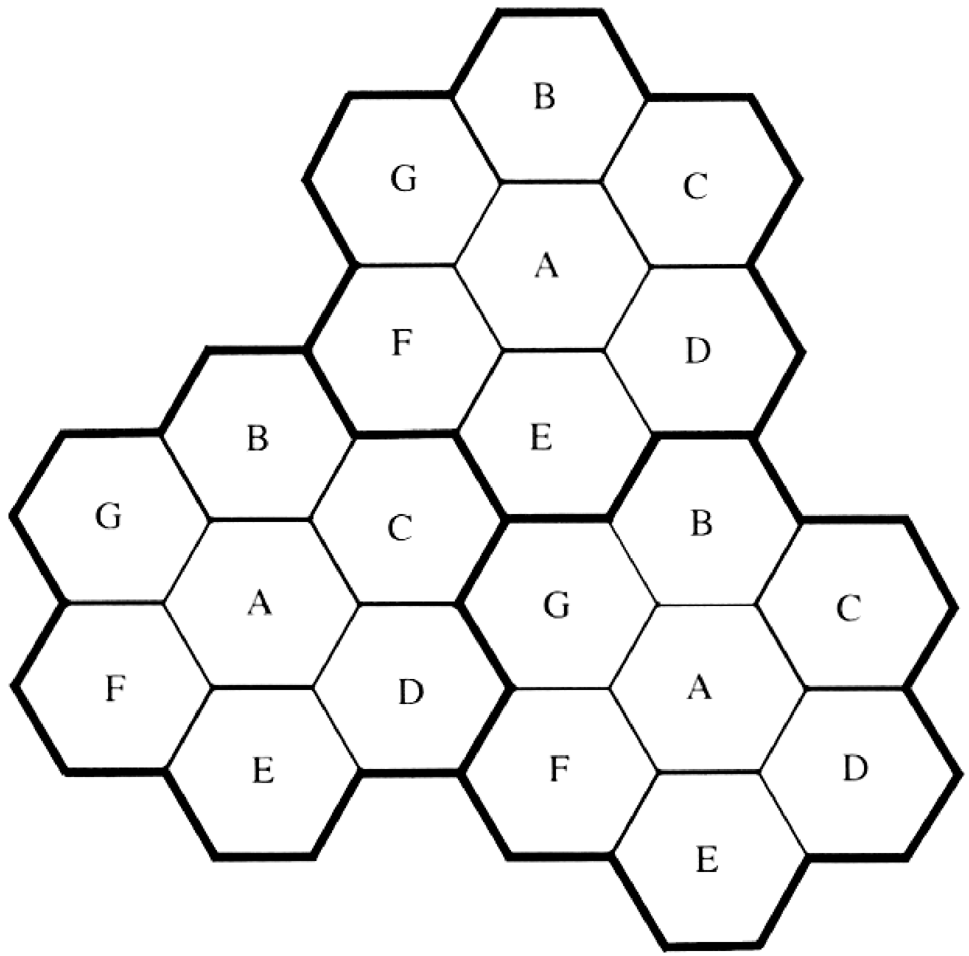
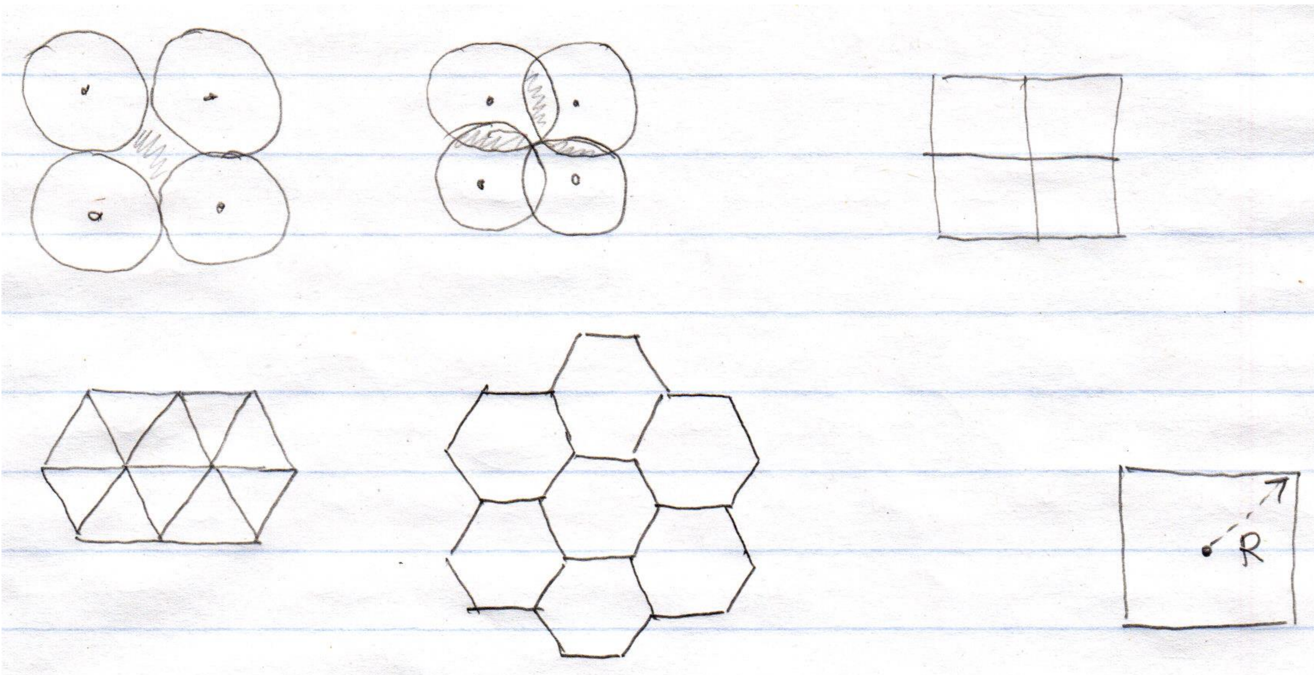


Figure 3.1 Illustration of the cellular frequency reuse concept. Cells with the same letter use the same set of frequencies. A cell cluster is outlined in bold and replicated over the coverage area. In this example, the cluster size, *N*, is equal to seven, and the frequency reuse factor is 1/7 since each cell contains one-seventh of the total number of available channels.

T.S. Rappaport, Wireless Communications, Prentice Hall, 2002

Cell Shape

- Ideally, from the propagation law, circles.
- Cannot cover an area without gaps or overlaps.
- Can use triangles, squares, hexagons (no gaps/overlaps)



Important: $\frac{S}{R^2}$

S = cell area
R =cell radius (distance to furthest point).

Q:. evaluate S/R^2 for circle, square, triangle, hexagon. Which is the best? Absolute best?

Hexagon cells are used in practice (analysis/design).

BS: located at the center (omni-directional antenna) or in corners (directional antennas)

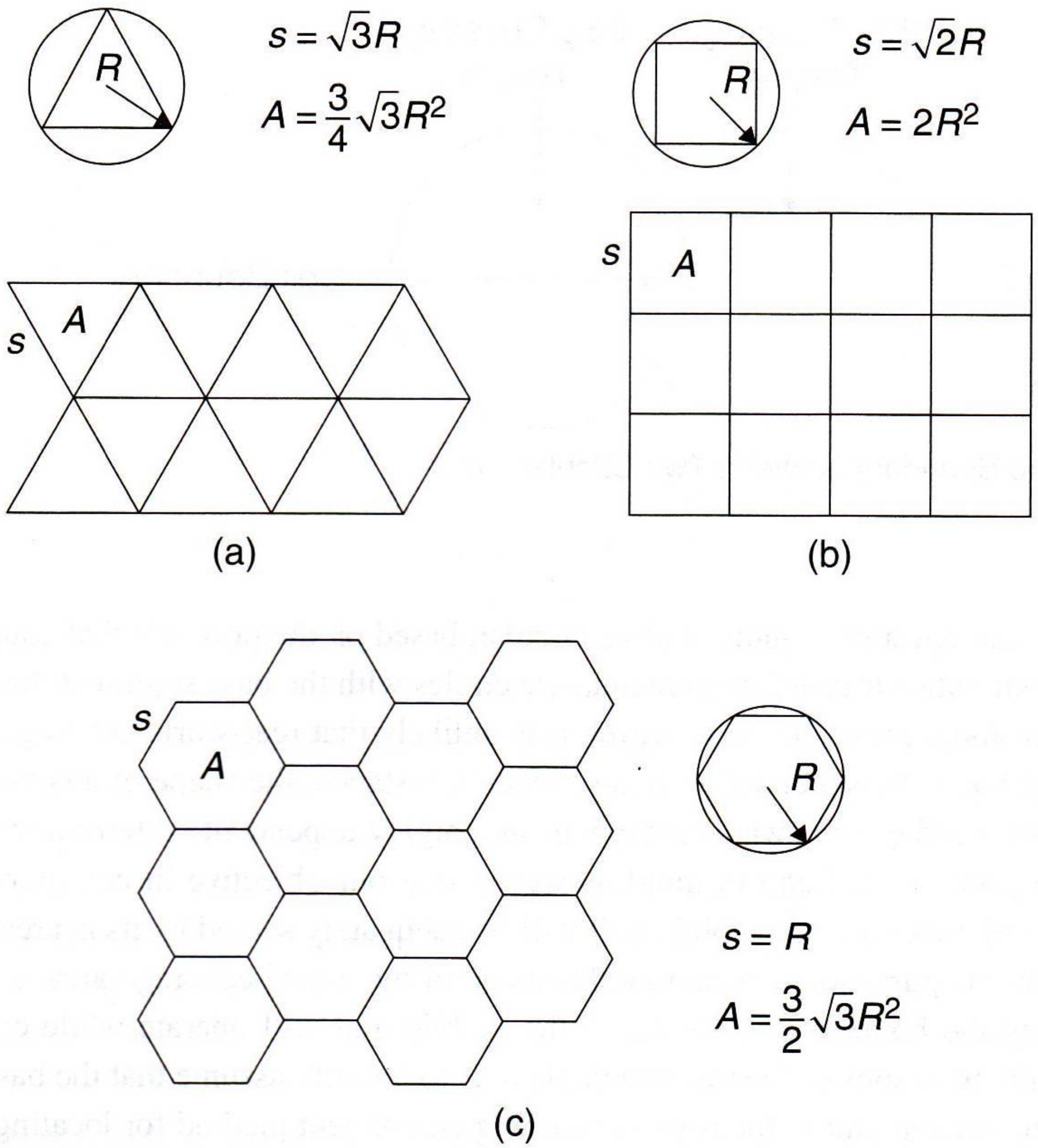


Figure 4.4 Covering a Plane Area with Regular Polygons: (a) Equilateral Triangles; (b) Squares; (c) Hexagons

B.A. Black et al, Introduction to Wireless Systems, Prentice Hall, Boston, 2008.

Basic Analysis

Cluster: a set of adjacent cells using all available spectrum (frequencies).

Important parameters:

N = # of cells in a cluster.
K = total # of channels (frequencies).
$k = \frac{K}{N}$ = # of channels/cell.
M = # of clusters covering all service area.
n = total # of users supported (at the same time), also "system capacity" or total # of channels available.

Key relation:

$$\boxed{n = k \cdot N \cdot M = K \cdot M} \leftarrow C_s = n \tag{12.3}$$

$M = 1 \rightarrow K = n,$ as with 1 BS;

$$M = \frac{S_{tot}}{S_{cl}} = \frac{S_{tot}}{NS_1} \tag{12.4}$$

S_{tot} = service area
S_{cl} = cluster area
S_1 = cell area

Higher $M \Rightarrow$ higher $n \Rightarrow$ higher system capacity (for fixed K).

Example: Ottawa

$$D = 40\text{km} \rightarrow S_a = D^2 = 1600\text{km}^2,$$

$$R = 1\text{km} \rightarrow S_1 = 4\text{km}^2,$$

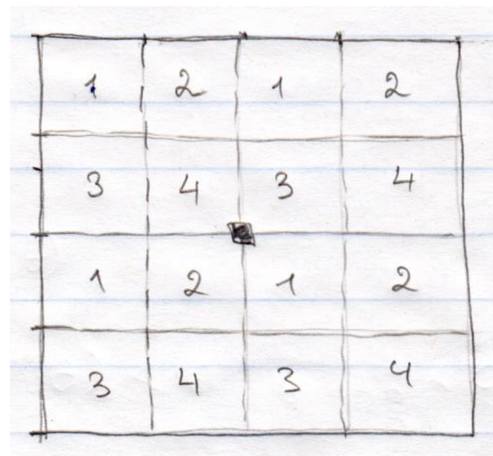
$$N = 4 \rightarrow S_c = 16\text{km}^2,$$

$$M = \frac{S_a}{S_c} = 100$$

$$\Delta f = 100\text{kHz}, \ \Delta F = 100\text{MHz},$$

$$K = \frac{\Delta F}{\Delta f} = 1000$$

$$n = M \cdot K = 10^5 \text{ users!} \rightarrow \text{enough?}$$



User activity:

$$\Delta t = 1 \text{ h/over } 12 \text{ h} \rightarrow P_u = \frac{1}{12} \approx 10^{-1}$$

$$n = P_u n_u \rightarrow n_u = \frac{n}{P_u} \approx 10^6$$

Example: cont.

Tx power:

$$P_{t(2km)} = P_{t(20km)} 10^{-4} = 10^{-3}\text{W} = 1\text{mW/user (no fading)}$$
$$= 10 \text{ ... } 100\text{mW/user (fading)}$$

$$P_T = nP_t = 10^5 \cdot 1\text{mW} = 100\text{W} \text{ (no F) or 1kW…10kW (F)}$$

$$P_{BS} = \frac{P_T}{MN} = \frac{10^3}{4 \cdot 100} = 0.25\text{W/BS (no F.)}, \; = 2.5 - 25\text{W (F)}$$

$\rightarrow$ comp. to 1 BS configuration!

$$n_{BS} \text{(\# of users/BS)} = \frac{10^5}{400} = 250$$

Typical values for current cellular systems:

Cell Type	Typical Cell Radius	PA Power: Range & (Typical Value)
Macro	>1 km	20 W~ 160 W (40 W)
Micro	250 m ~ 1 km	2 W ~ 20 W (5 W)
Pico	100 m ~ 300 m	250 mW ~ >2 W
Femto	10 m ~ 50 m	10 mW~200 mW

Table 1: Different cell radii and Tx power levels

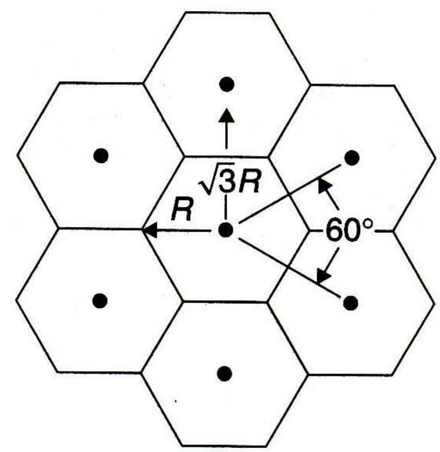
High-Capacity Indoor Wireless Solutions: Picocell or Femtocell? by Fujitsu

Hexagonal Cells

of cells in a cluster:

$$N = i^2 + j^2 + ij \qquad (12.5)$$

Not all integer N are possible.



Frequency re-use factor:

$$\frac{1}{N} = \text{how often a frequency is re-used}$$

i.e. $N = 7 \;\rightarrow \frac{1}{N} = \frac{1}{7}$: every 7th cell is using the same frequency.

Nearest co-channel cell distance:

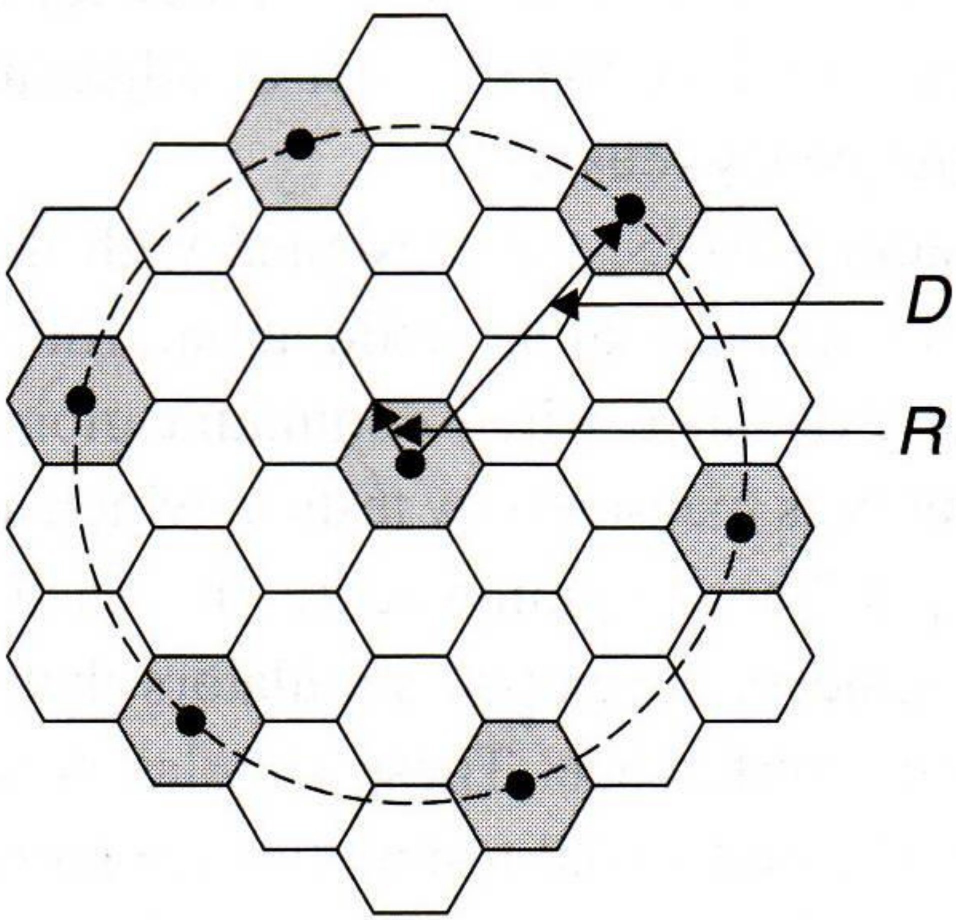
$$D = R\sqrt{3N} = R\sqrt{3(i^2 + j^2 + ij)} \qquad (12.6)$$

Frequency re-use ratio (distance-wise):

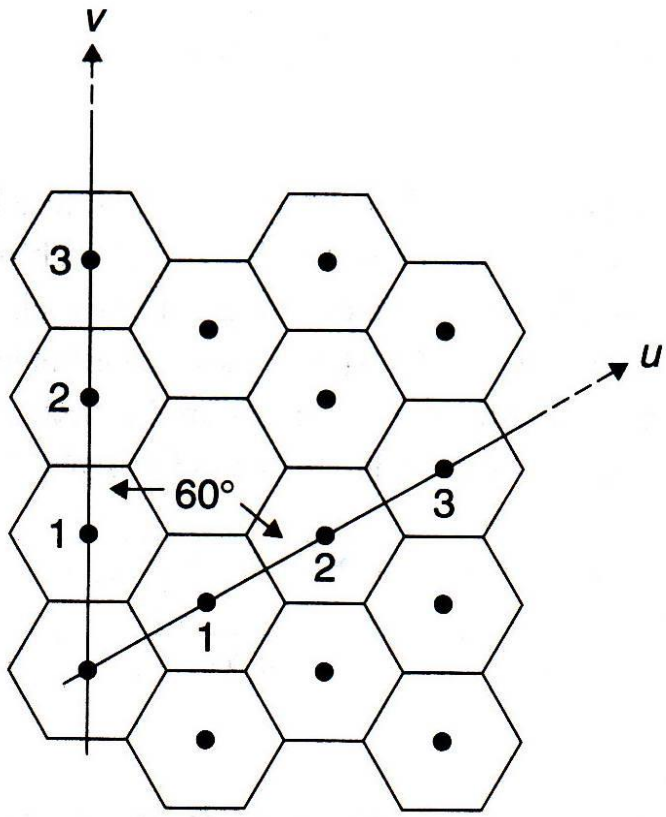
$$Q = \frac{D}{R} = \sqrt{3N} \qquad (12.7)$$

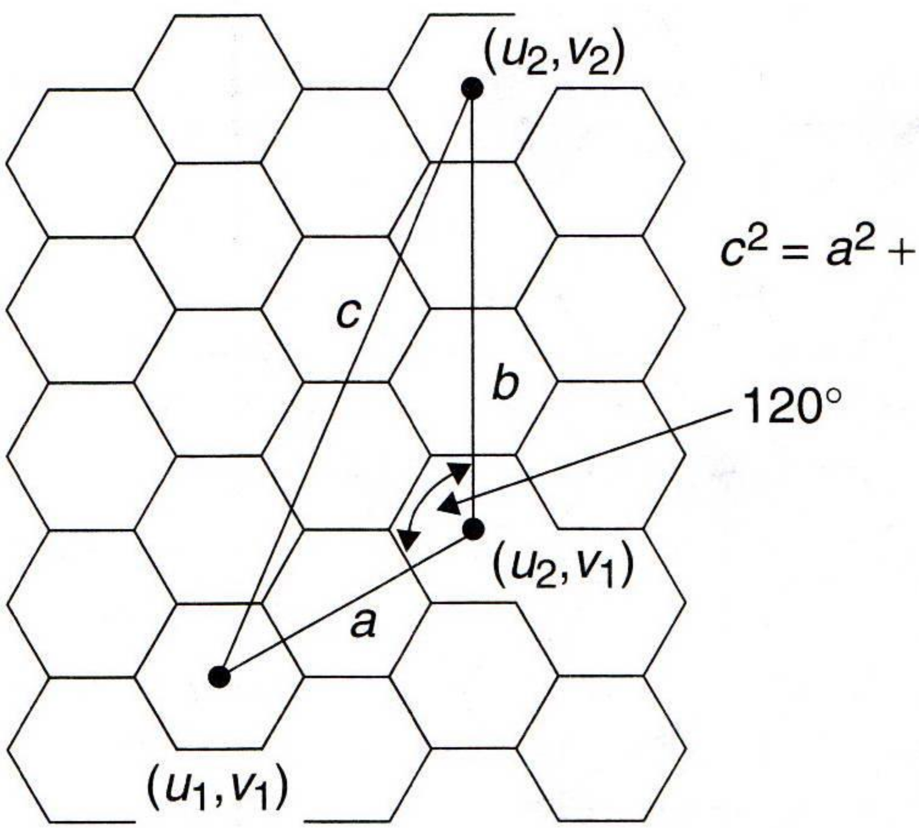
Cell area:

$$S_1 = \frac{3\sqrt{3}}{2} R^2 \qquad (12.8)$$



B.A. Black et al, Introduction to Wireless Systems, Prentice Hall, Boston, 2008.





B.A. Black et al, Introduction to Wireless Systems, Prentice Hall, Boston, 2008.

Co-Channel Interference

Frequency re-use $\rightarrow$ co-channel interference.

Careful analysis/design is required.

Recall the threshold effect:

$$\gamma = SNIR = \frac{S}{I+N} \geq \gamma_{th} \text{ for good performance} \quad (12.9)$$

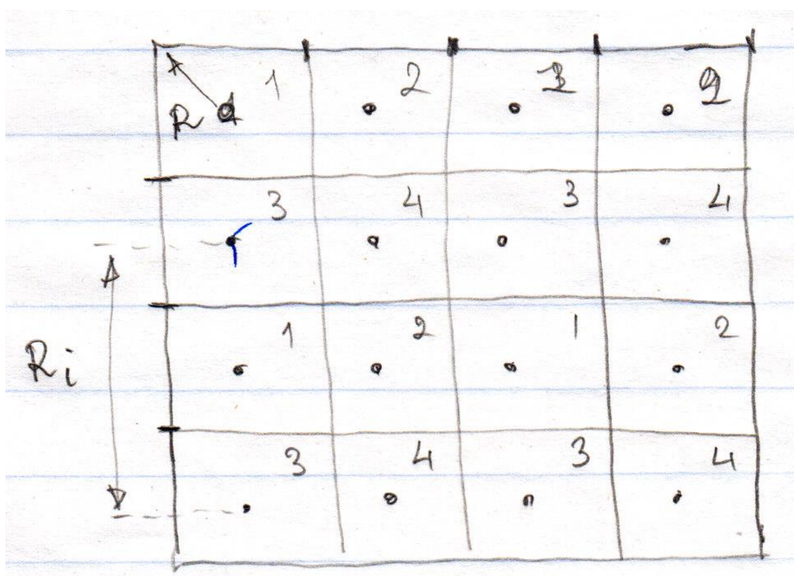
Interference-limited system $\Rightarrow$ noise is negligible:

$$I \gg N \Rightarrow \gamma \approx \frac{S}{I} = SIR \geq \gamma_{th} \quad (12.10)$$

From the path loss (av. power):

$$S = \frac{a_v P_t}{R^v}, \; I_i = \frac{a_v P_t}{R_i^v} \quad (12.11)$$

R = user - BS distance
R_i = interferer - BS distance



Co-Channel Interference: cont.

1st tier interfering cells: the smallest R_i $\rightarrow$ dominates SIR.

For hexagonal cells, $R_i = D$.
N_I = # of 1st tier interferers.

$$\gamma \approx SIR = \frac{S}{\sum_i I_i} \approx \frac{R^{-v}}{N_I D^{-v}} = \frac{(D/R)^v}{N_I} = \frac{Q^v}{N_I} \quad (12.12)$$

assuming P_t is the same for all users.

Hexagonal cells: $Q = \sqrt{3N}$ and

$$\gamma = \frac{(3N)^{v/2}}{N_I} \quad (12.13)$$

Increasing N $\Rightarrow$ decreasing I $\Rightarrow$ increasing SIR
but decreasing $n = C_s$!

QoS: $\gamma \geq \gamma_{th}$ = 18dB for AMPS (2G), variable for 4G,

$$N \geq \frac{(\gamma_{th} N_I)^{2/v}}{3} \Rightarrow N = \left\lceil \frac{(\gamma_{th} N_I)^{2/v}}{3} \right\rceil \quad (12.14)$$

where $\lceil x \rceil$ =ceiling (smallest integer $\geq x$).

SNIR Thresholds for AMC:

Table 7.1 Lookup table for mapping SINR estimate to modulation scheme and coding rate

CQI index	Modulation	Coding rate	Spectral efficiency (bps/Hz)	SINR estimate (dB)
1	QPSK	0.0762	0.1523	−6.7
2	QPSK	0.1172	0.2344	−4.7
3	QPSK	0.1885	0.3770	−2.3
4	QPSK	0.3008	0.6016	0.2
5	QPSK	0.4385	0.8770	2.4
6	QPSK	0.5879	1.1758	4.3
7	16QAM	0.3691	1.4766	5.9
8	16QAM	0.4785	1.9141	8.1
9	16QAM	0.6016	2.4063	10.3
10	64QAM	0.4551	2.7305	11.7
11	64QAM	0.5537	3.3223	14.1
12	64QAM	0.6504	3.9023	16.3
13	64QAM	0.7539	4.5234	18.7
14	64QAM	0.8525	5.1152	21.0
15	64QAM	0.9258	5.5547	22.7

Note the rate-SE relationship:

$$R_b = \mathrm{SE} \cdot \Delta f \quad \text{[b/s]}$$

An Example

$\gamma_{th} = 18\text{dB}\ (\approx 63),\ \ v = 4.$

$$N = 7 \rightarrow N_I = 6 \rightarrow \gamma = \frac{(3N)^{v/2}}{N_I} \approx 74 > 63 \rightarrow \text{O.K.}$$

But $\gamma_{th} = 20\text{dB}$ would require larger N.

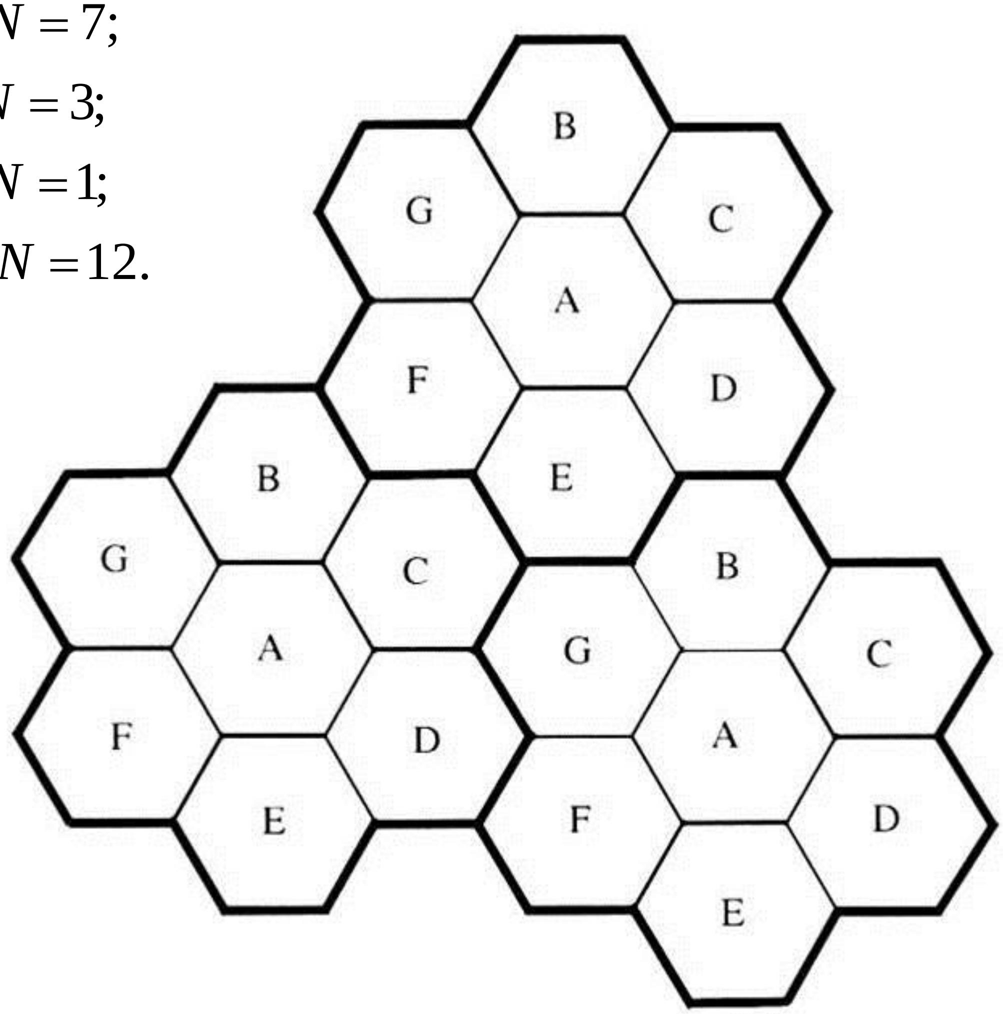
Recall: $N = i^2 + j^2 + ij$, so that:

$i = 1,\ j = 2:\ N = 7;$

$i = 1,\ j = 1:\ N = 3;$

$i = 1,\ j = 0:\ N = 1;$

$i = 2,\ j = 2:\ N = 12.$



System Design Trade-offs

Parameters:

- Cell radius R, cluster size N,
- # of clusters M, # of users n $(= C_s)$,
- K = total # of channels

Key equations:

$$\boxed{M = \frac{S_{tot}}{S_{cl}} = \frac{S_{tot}}{NS_1}} \quad \boxed{n = C_s = KM} \qquad (12.15)$$

K = fixed: $\boxed{M \uparrow \Rightarrow n \uparrow}$ $\quad \boxed{\gamma = \frac{(3N)^{\nu/2}}{N_I}}$

How to increase M ?

S_{tot} = fixed: $S_{cl} \downarrow \Rightarrow M \uparrow$

$S_{cl} \downarrow$: via $N \downarrow$, fixed S_1 or N fixed, $S_1 \downarrow$

$$\left.\begin{array}{l} N \downarrow \Rightarrow \gamma \downarrow: \text{ keep } \gamma \geq \gamma_{th}! \\ \quad \Rightarrow n \uparrow \end{array}\right\} \text{trade-off 1.}$$

$$\left.\begin{array}{l} S_1 \downarrow \Rightarrow n \uparrow: \ \gamma \text{ fixed, but:} \\ \quad \Rightarrow n_{BS} = \dfrac{S_{tot}}{S_1} \uparrow \text{ (complexity cost)} \end{array}\right\} \text{trade-off 2.}$$

Cell Splitting

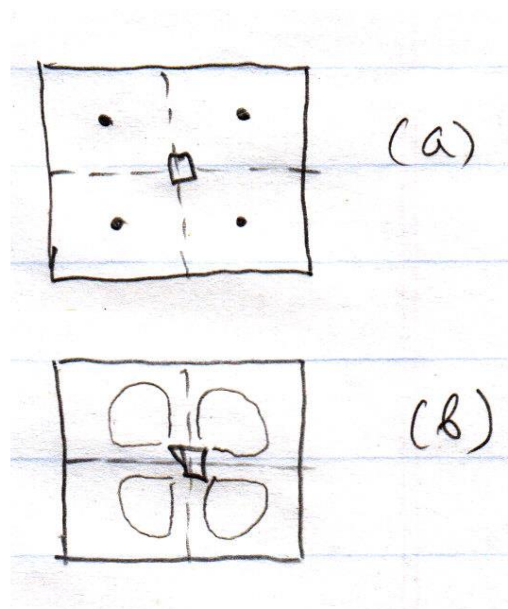
System expansion:

- in the area (add more cells/clusters)
- in user density (splitting/sectoring)

Cell Splitting: split a big (macro) cell into # of smaller (micro) cells.

Can accommodate:

- growing demand (# of users)
- non-uniform user density



Congested cells (downtown): split into micro-cells.

Microcells:

- lower P_t and BS antenna height
- maintains the same $Q = D/R$ (same SIR)
- preserves frequency re-use plan
- Tx power scaling

$$\frac{P_{t1}}{R^{v}} = \frac{P_{t2}}{(R/2)^{v}} \Rightarrow \boxed{P_{t2} = P_{t1}/2^{v}} \qquad (12.16)$$

i.e. $P_{t2}/P_{t1} = 1/16$ for $v = 4$.

- BS antennas: center vs. corners.

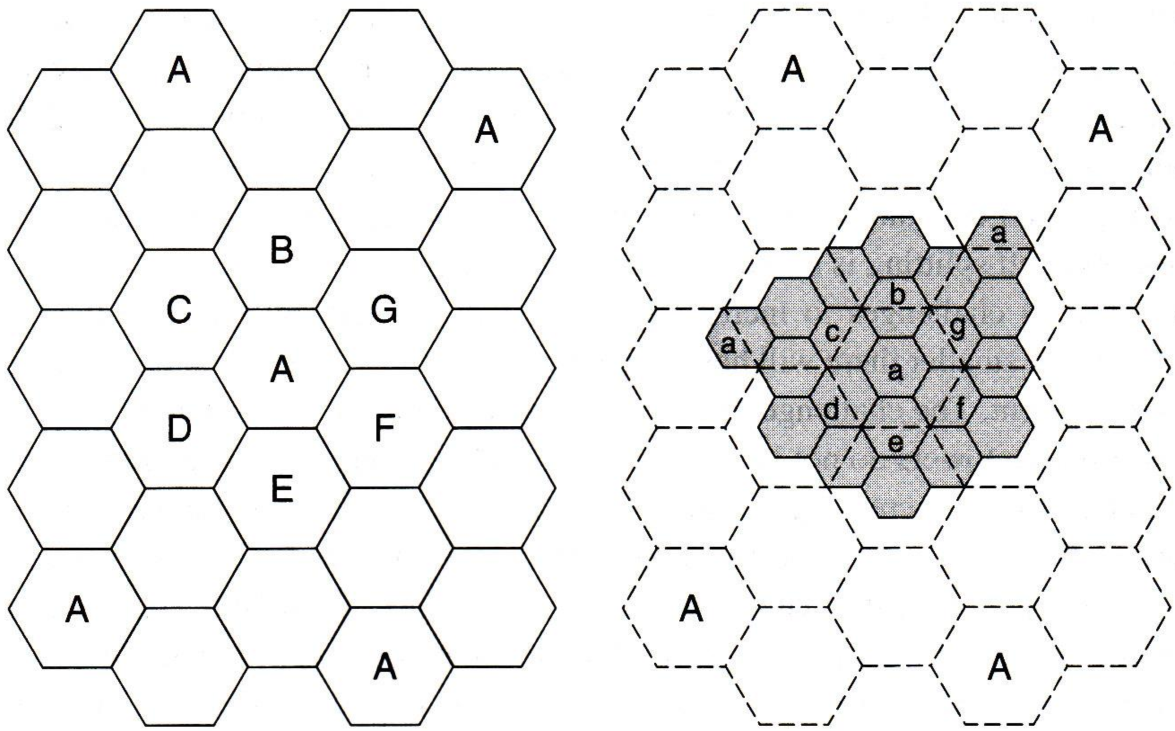
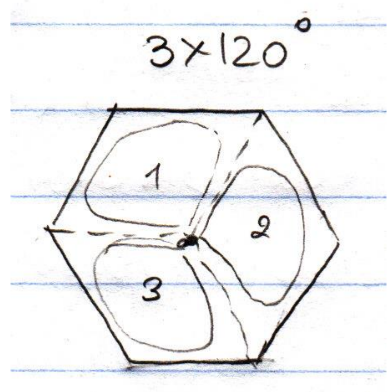


Figure 4.19 (a) Seven-Cell-Cluster Layout before Splitting; (b) Cell Splitting: Overlay of Half-Radius Cells

B.A. Black et al, Introduction to Wireless Systems, Prentice Hall, Boston, 2008.

Sectoring

Key idea: using directional antennas focuses radiation and thus decreases interference.



No sectoring: 360°

3 sectors: $3 \text{ x } 120^\circ$

6 sectors: $6 \text{ x } 60^\circ$

# of 1st tier interfering cells decreases:

$$360^\circ \rightarrow N_I = 6$$

$$3 \text{ x } 120^\circ \rightarrow N_I = 2$$

$$6 \text{ x } 60^\circ \rightarrow N_I = 1$$

Recall that:

$$\gamma = \frac{(3N)^{\nu/2}}{N_I} \Rightarrow N_I \downarrow \Rightarrow \gamma \uparrow \overset{\text{fixed } \gamma_{th}}{\Rightarrow} N \downarrow \Rightarrow C_s \uparrow \quad (12.17)$$

Smart antennas: much better improvement is possible, via SDMA.

MIMO: much larger link capacity (Mb/s).

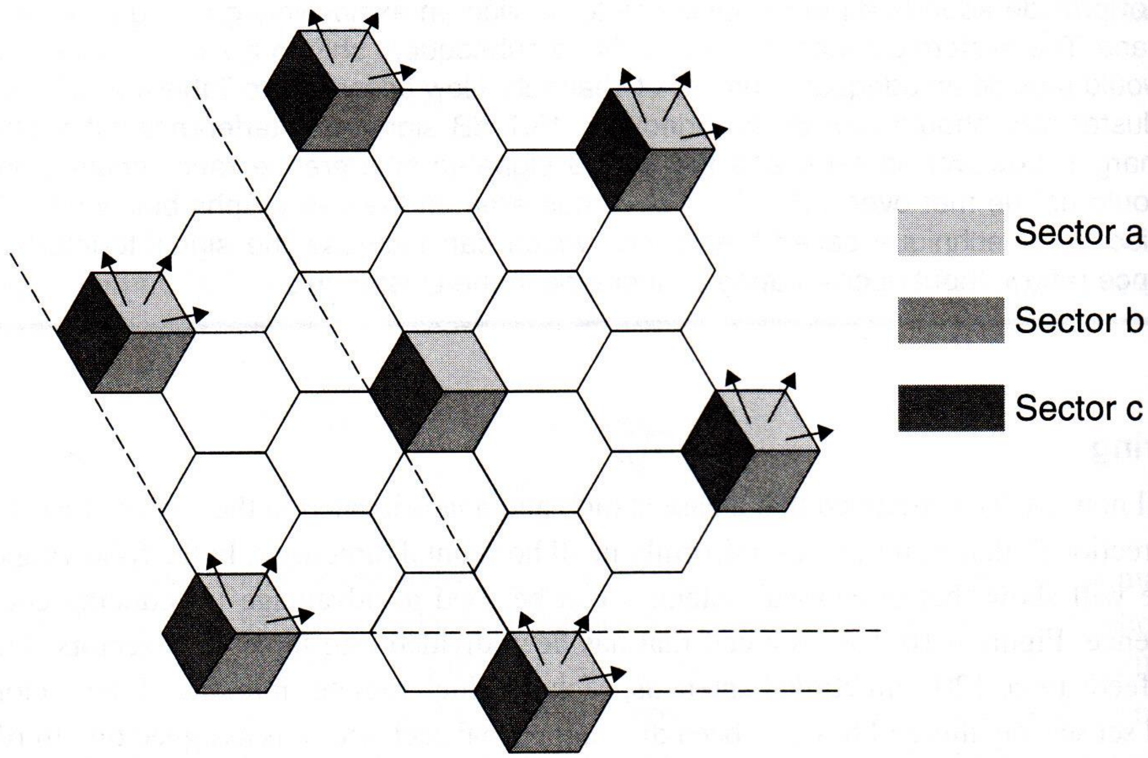


Figure 4.17 A Seven-Cell Cluster with 120° Sectors; the Arrows Suggest Base Station Radiation in Sector a

B.A. Black et al, Introduction to Wireless Systems, Prentice Hall, Boston, 2008.

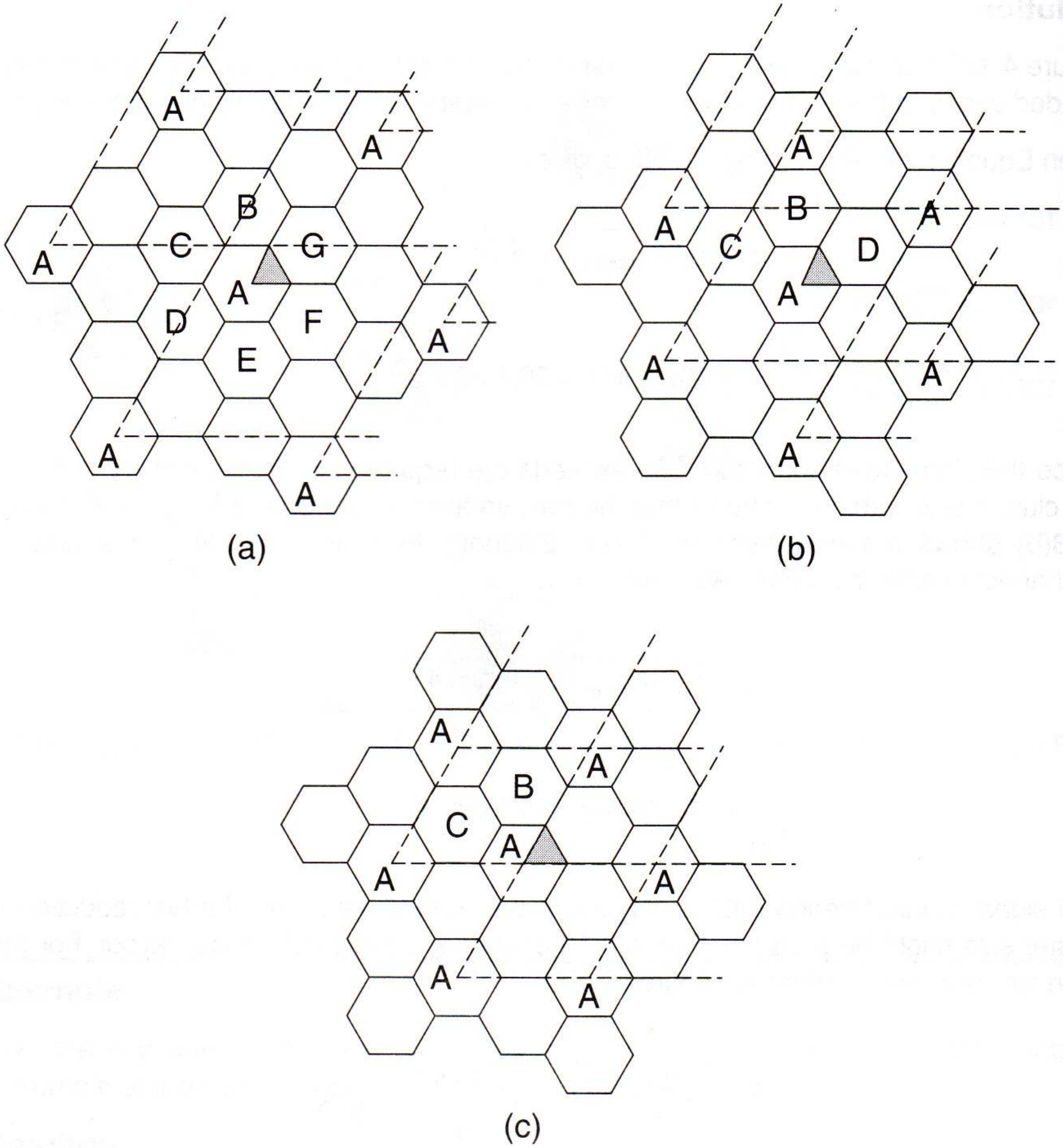


Figure 4.18 Cochannel Interference from First-Tier Sources with 60° Sectors: (a) Seven-Cell Clusters; (b) Four-Cell Clusters; (c) Three-Cell Clusters

B.A. Black et al, Introduction to Wireless Systems, Prentice Hall, Boston, 2008.

Traffic Engineering

Grade of service(GOS) = quality of service (QoS) = P_b (blockage prob.) = outage probability.

Given # of channels K, how many users can be supported?

Key observation: not all users are active at the same time, $n_a \neq n$.

$$n_a \leq K, \quad n_a = \text{ \# of active users} \\ n = \text{ total \# of users} \tag{12.18}$$

Average # of active users:

$$\overline{n_a} = p_a n, \quad p_a = \text{ prob. of a user being active} \tag{12.19}$$

Using $\overline{n_a} = K$,

$$\boxed{n = \frac{\overline{n_a}}{p_a} = \frac{K}{p_a}} \tag{12.20}$$

Blocking probability:

$$P_b = \frac{\lambda_n^K / K!}{\sum_{i=0}^{K} \lambda_n^i / i!} = \frac{\overline{n_a}^K / K!}{\sum_{i=0}^{K} \overline{n_a}^i / i!} \tag{12.21}$$

where: $\lambda_n = n\lambda = \overline{n_a}$

$\lambda_n =$ av. # of calls/unit time/user

$\lambda = T\lambda_n =$ traffic intensity $= p_a$

$T =$ av. holding time/call

Q.: assuming that $P_b = \varepsilon \ll 1$ is required, prove using (12.21) that

$$\overline{n_a} \approx \frac{K}{1-\varepsilon} \Rightarrow n \approx \frac{K}{(1-\varepsilon)\, p_a} = \frac{K}{p_a} \qquad (12.22)$$

hint: assume that

$$\sum_{i=0}^{K} \frac{\overline{n_a}^{\,i}}{i!} \approx \frac{\overline{n_a}^{\,K}}{K!} + \frac{\overline{n_a}^{\,K-1}}{(K-1)!},$$

justify this assumption.

Q.: how many users can be supported if $\varepsilon = 1/2$, 0.1? 0.01? Assume $K = 100$.

Recent Activities: 5G & related

1. J. G. Andrews et al, Are We Approaching The Fundamental Limits of Wireless Network Densification?, IEEE Comm. Mag., vol. 54, no. 10, pp. 184–190, Oct. 2016.

2. K. Briggs, A. Shojaeifard, Coverage Regions Under Multi-Slope Pathloss Propagation, IEEE Trans. Veh. Tech., vol. 69, no. 10, pp. 11786-11789, Oct. 2020.

3. A. AlAmmouri et al, A Unified Asymptotic Analysis of Area Spectral Efficiency in Ultradense Cellular Networks, IEEE Trans. Info. Theory, vol. 65, no. 2, pp. 1236–1248, Feb. 2019.

4. Study on Evaluation Methodology of New Vehicle-to-Everything(V2X) Use Cases for LTE and NR (Release 15), TR 37.885, 3GPP, Sophia Antipolis, France, 2019.

Summary

- The cellular concept. Frequency re-use.
- Cell shape. Hexagonal cells.
- Basic analysis.
- Co-channel interference.
- System design trade-offs.
- Cell splitting. Sectoring.
- Traffic engineering.

Reading:

- Rappaport, Ch. 3.
- B.A. Black et al
- Other books (see the reference list).

Note: Do <u>not</u> forget to do end-of-chapter problems. Remember the learning efficiency pyramid!