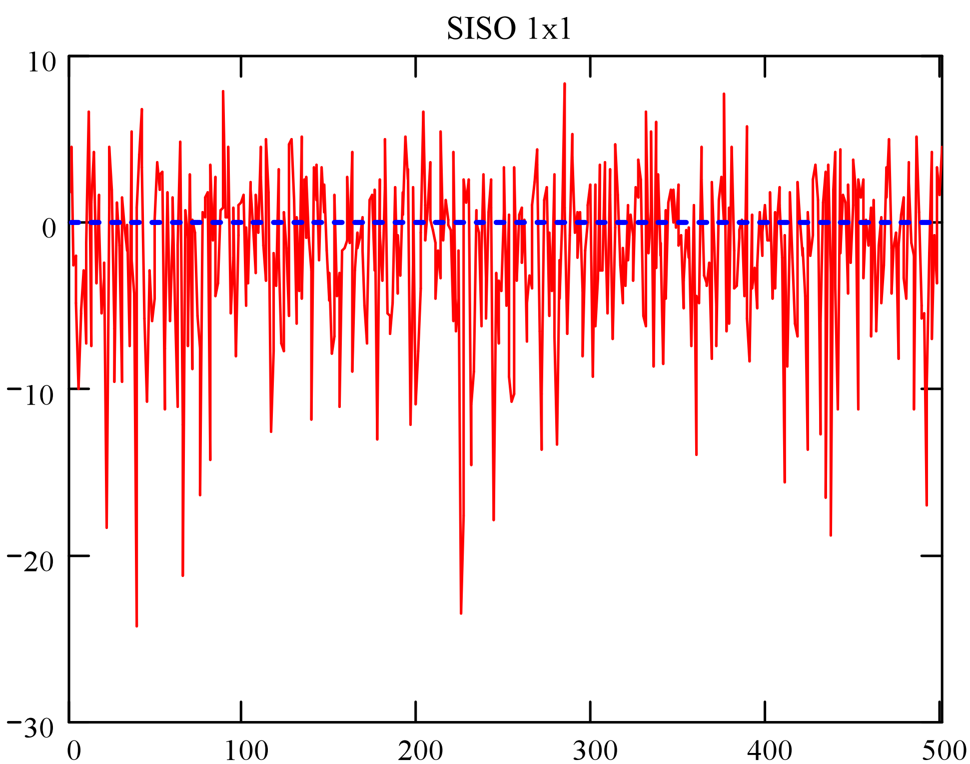
Diversity Combining Techniques

When the required signal is a combination of several waves (i.e, multipath), the total signal amplitude may experience deep fades (i.e, Rayleigh fading), over time or space.

The major problem is to combat these deep fades, which result in system outage.



Most popular and efficient technique for doing so is to use some form of diversity combining.

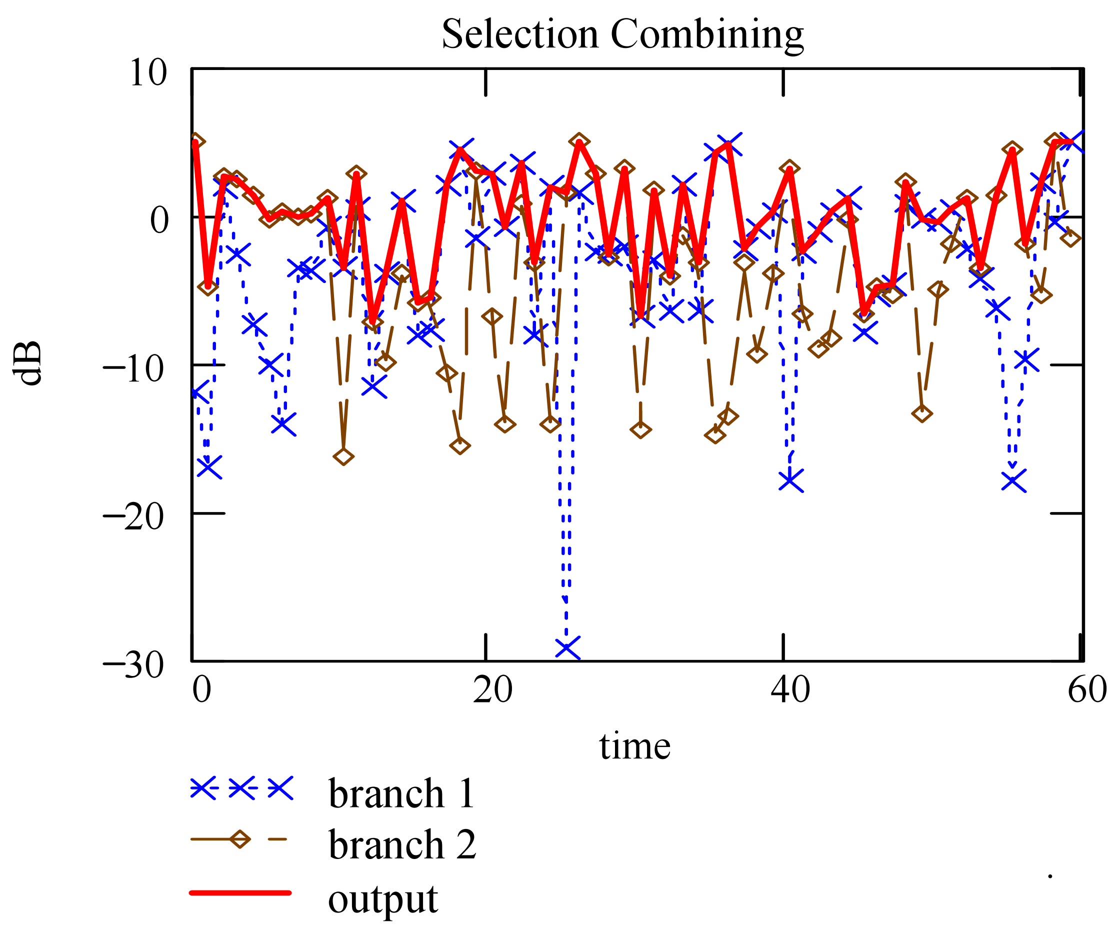
Diversity: multiple copies of the required signal are available, which experience independent fading (or close to that).

Effective way to combat fading.

Basic principle: create multiple independent paths for the signal, and combine them in an optimum or near-optimum way.

Combining techniques

- Selection combining (SC).
- Maximum ratio combining (MRC).
- Equal gain combining (EGC).
- Hybrid combining (two different forms)
- Micro-diversity and macro-diversity.



Types of Diversity

Space diversity: Antennas are separated in space.

Frequency diversity: Multiple copies are transmitted at different frequencies.

Time diversity: Multiple copies are transmitted over different time slots.

Polarization diversity: Multiple copies have different field polarizations.

For all types of diversity, multiple signal copies must be uncorrelated (or weakly correlated, corr. coeff. ≤ 0.5) for diversity combining to be most effective.

Hence different types of diversity are normally efficient in different scenarios.

All the forms have their own advantages and disadvantages. In any particular case, some forms are better than the other.

Example: if the channel is static (or-slow-fading), time diversity is not good.

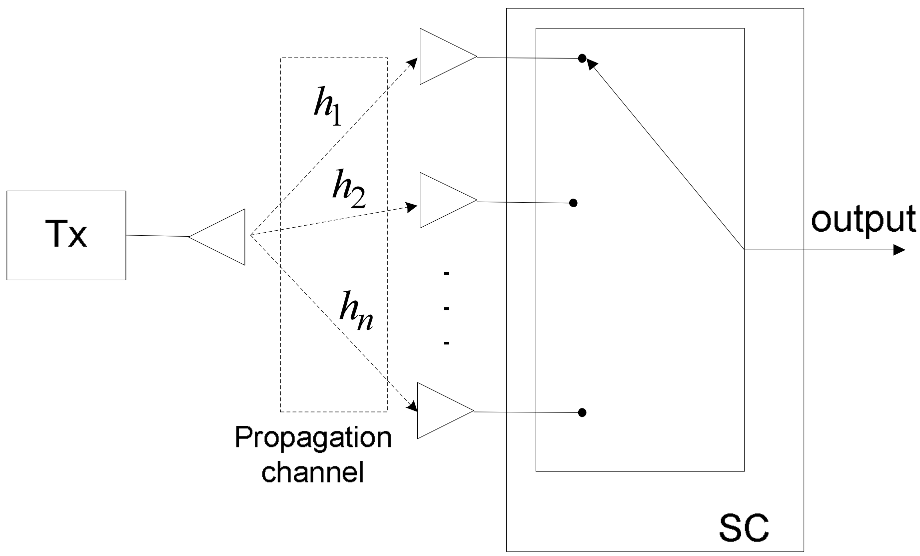
If the channel is frequency-flat, frequency diversity is not good.

A diversity form is efficient when the signals are independent (uncorrelated).

Multiple signals have to be combined in a certain way.

Selection combining (SC)

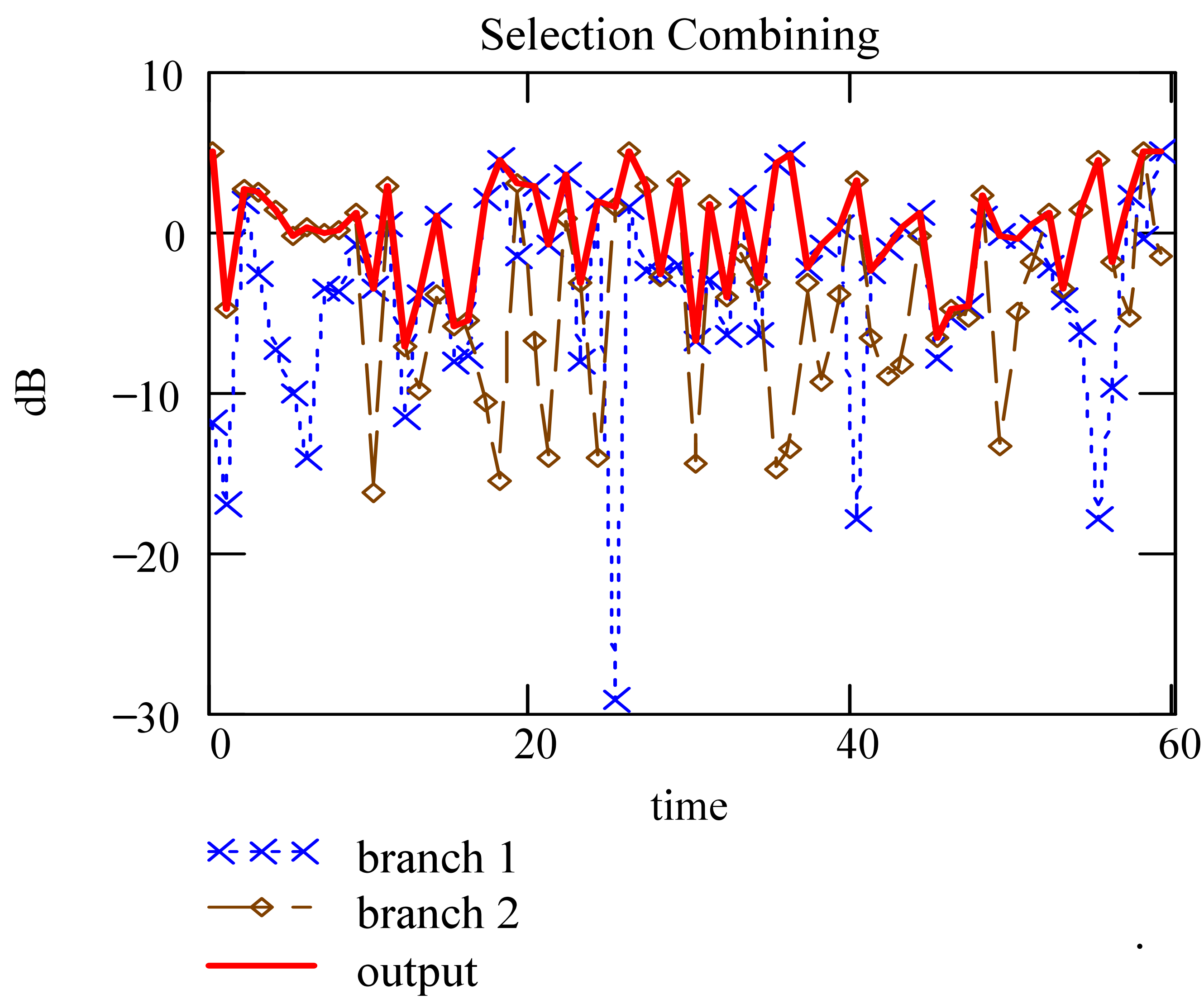
Key idea: at any given moment of time, pick up the branch with the largest SNR.



System model:

$$\mathbf{y} = \mathbf{h}x + \boldsymbol{\xi} \tag{11.1}$$

Output: select the branch with the largest SNR.


Selection Combining
10
0
−10
−20
−30
dB
0
20
40
60
time
branch 1
branch 2
output

Analysis of a selection combiner

Consider the SC in a frequency-flat slow-fading Rayleigh channel.

Assume that some diversity form provides N independent paths (each is i.i.d. Rayleigh-fading).

Instantaneous SNR in branch i is,

$$\gamma_i = \frac{E}{N_0}|h_i|^2 \tag{11.2}$$

where h_i is the channel (complex) gain, E is symbol energy and N_0 is the noise spectral power density (assumed to be the same in all the branches).

The PDF of γ_i is

$$\rho(\gamma_i) = \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}} \tag{11.3}$$

where $\gamma_0 = \overline{\gamma_i} = \frac{E}{N_0}\overline{|h_i|^2}$ is the average SNR.

The CDF (i.e., the probability that $\gamma_i < \gamma$) is

$$F_{\gamma_i}(\gamma) = \int_0^{\gamma} \rho(\gamma_i) d\gamma_i = 1 - e^{-\gamma/\gamma_0} \tag{11.4}$$

This is also outage probability of i-th branch. The output SNR is

$$\gamma_{out} = \max_i \gamma_i \tag{11.5}$$

The SC outage probability is

$$\begin{aligned} P_{out} &= \Pr\{\gamma_{out} < \gamma\} = \Pr\{\gamma_1, .., \gamma_N < \gamma\} \\ &= F_N(\gamma) = \prod_{i=1}^{N} F_{\gamma_i}(\gamma) = \left(1 - e^{-\gamma/\gamma_0}\right)^N \end{aligned} \tag{11.6}$$

Asymptotically, for $\gamma << \gamma_0$ (low outage prob.),

$$P_{out} = F_N(\gamma) = (\gamma/\gamma_0)^N \ll \gamma/\gamma_0 \tag{11.7}$$

where N is the diversity order. Note significant decrease in outage probability compared to $N = 1$ case.

The average output SNR is

$$\gamma_{out} = \gamma_0 \sum_{i=1}^{N} \frac{1}{i} \tag{11.8}$$

Q.: prove it!

The most important improvement is in terms of outage probability rather than the average SNR!

Example: $N = 2, \gamma/\gamma_0 = 10^{-1}; P_{out} = ?\ \gamma_{out}/\gamma_0 = ?$

PDF of a selection combiner:

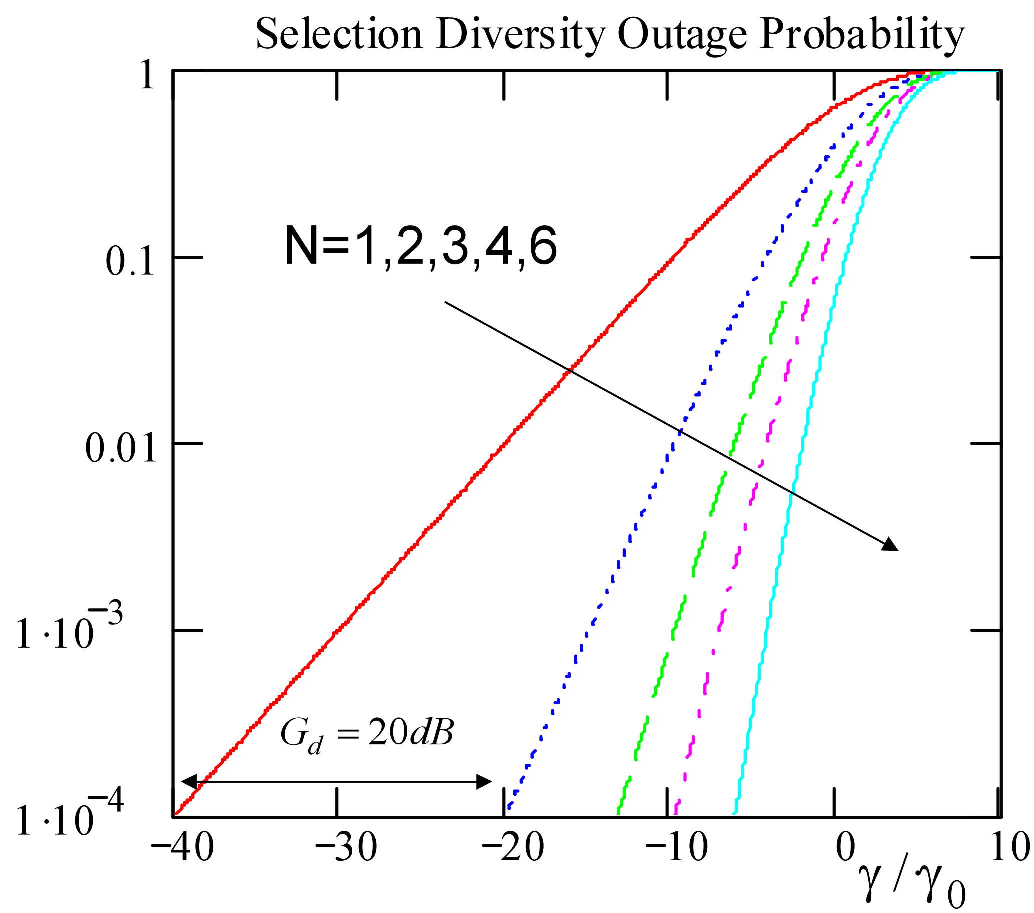
$$f_N(\gamma) = \frac{dF_N(\gamma)}{d\gamma} \tag{11.9}$$

As N increases, the peak of the output PDF moves to the right and the curve becomes more and more closer to Gaussian. The Rayleigh channel becomes a Gaussian one as $N \to \infty$.

Q.: prove it!

Selection Combining

Diversity gain: how much less SNR is required to achieve the same P_{out}



$$P_1(\gamma_1) = P_N(\gamma_N) = P_{out}$$

$$G_d = \frac{\gamma_N}{\gamma_1} \text{ or } G_d = (\gamma_N - \gamma_1)[dB]$$

Diversity order d: number of independent branches, $d = N$,

$$P_{out} \approx \frac{c}{(\gamma_0)^d} \sim \frac{1}{(\gamma_0)^d} \qquad (11.10)$$

which is also the SNR exponent in i.i.d. Rayleigh fading.

Formal definition:

$$d = -\lim_{\gamma_0 \to \infty} \frac{\ln P_{out}}{\ln \gamma_0} \qquad (11.11)$$

Diversity gain: the difference in SNR required to achieve certain outage probability using 1 and N branches.

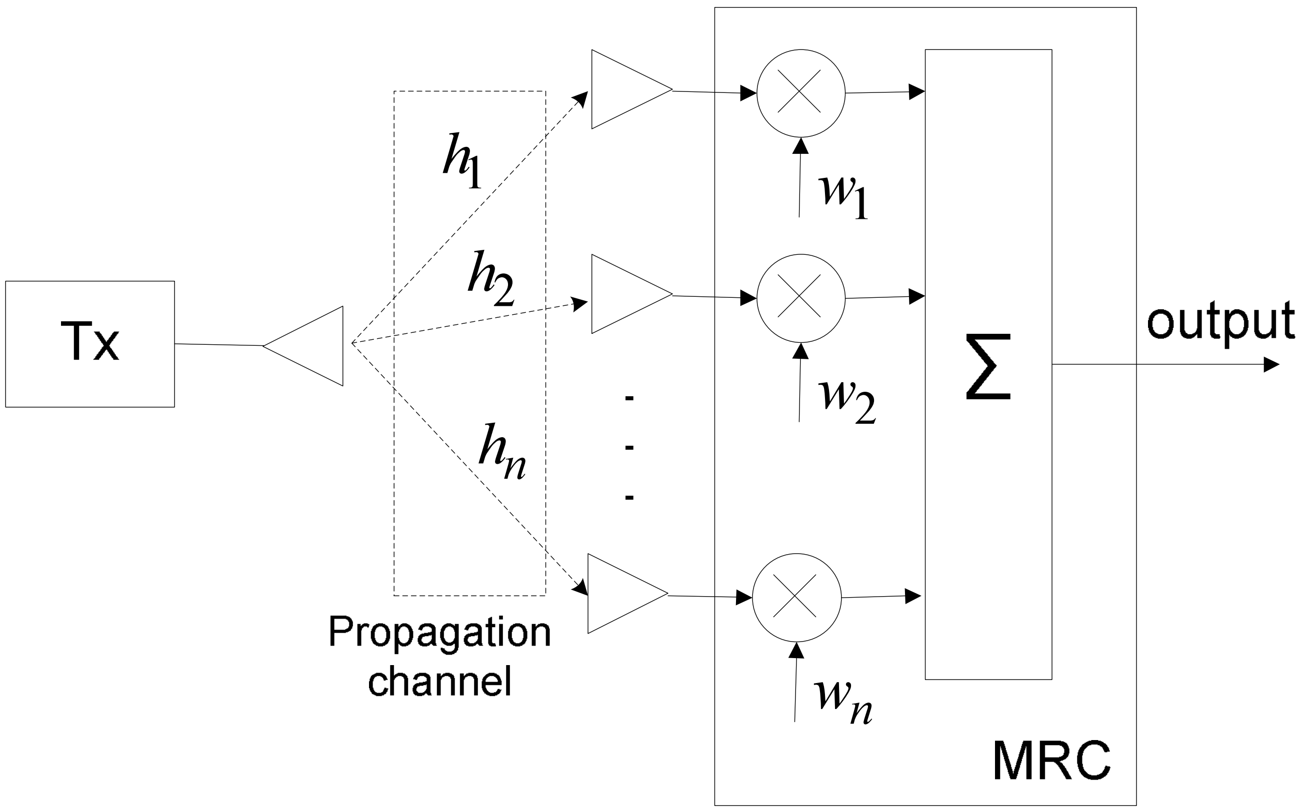
$$P_1(\gamma_1) = P_N(\gamma_N) = P_{out}$$
$$G_d = \frac{\gamma_1}{\gamma_N} \text{ or } G_d = (\gamma_1 - \gamma_N)[dB] \qquad (11.12)$$

Effect of branch correlation: can significantly degrade the performance if too high.

Practical considerations: antenna switch can be used. Then, only one R_x is required —> very simple!

Maximal Ratio Combining (MRC)

Key idea: use linear coherent combining of branch signals so that the output SNR is maximized



The equivalent baseband model is as follows.

The individual branch signals are

$$x_n = A \cdot h_n + \xi_n \tag{11.13}$$

where A - complex envelope (amplitude), h_n - channel (complex) gain , $\xi_n \sim CN\left(0, \sigma_0^2\right)$ is AWGN.

Note that h_n acts as multiplicative noise.

The output of the combiner is

$$x_{out} = \sum_{n=1}^{N} w_n x_n = \underbrace{A\sum_n w_n h_n}_{signal} + \underbrace{\sum_n w_n \xi_n}_{noise} \qquad (11.14)$$

where w_n are the combining weights.

The signal and noise components are given by 1st and 2nd term correspondingly. The signal and noise power at the output are:

$$\begin{aligned} P_s &= \overline{\left| A\sum_n w_n h_n \right|^2} = \frac{1}{2}|A|^2 \left| \sum_n w_n h_n \right|^2 \\ P_\xi &= \overline{\left| \sum_n w_n \xi_n \right|^2} = \sum_n |w_n|^2 \sigma_n^2 \end{aligned} \qquad (11.15)$$

where $\sigma_n^2 = \overline{|\xi_n|^2}$ is the branch noise power. Output SNR is

$$SNR_{out} = \frac{P_s}{P_\xi} = \frac{1}{2}|A|^2 \frac{\left| \sum_n w_n h_n \right|^2}{\sum_n |w_n|^2 \sigma_n^2} \qquad (11.16)$$

We can maximize it using the Lagrange multipliers or better, Cauchy-Schwarz inequality.

Cuachy-Schwartz inequality:

$$\left|\sum_n a_n^* b_n\right|^2 \le \sum_n |a_n|^2 \sum_n |b_n|^2 \qquad (11.17)$$

With the equality achieved when $a_n^* = c b_n$

Using (11.17), the best combining weights are

$$w_n = h_n^* / \sigma_n^2 \qquad (11.18)$$

and the output SNR of the best combiner is

$$\gamma_{out} = \sum_n \gamma_n \qquad (11.19)$$

where

$$\gamma_n = \frac{A^2 |h_n|^2}{2\sigma_n^2} \qquad (11.20)$$

is n-th branch SNR.

The resulting combiner is called MRC. It is the best combiner in terms of the SNR.

Q.: prove (11.18) !

Note: $w_n = h_n^* / \sigma_n^2$ is true for any channel, not necessarily Rayleigh.

When all the branch noise powers are equal,

$$\sigma_n = \sigma_0 \to w_n = \frac{h_n^*}{|\mathbf{h}|}, \ \gamma_{out} = N\gamma_0 \qquad (11.21)$$

The branch gain is proportional to the channel gain +coherent combining.

Note N-fold increase in average SNR (but this is not the main gain!).

Q.: explain it!

Outage Probability

Consider a Rayleigh channel and assume there is no correlation between branches (i.i.d. channel gains).

The outage probability can be expressed as

$$P_{out} = 1 - e^{-\frac{\gamma}{\gamma_0}} \sum_{k=1}^{N} \frac{(\gamma/\gamma_0)^{k-1}}{(k-1)!} \qquad (11.22)$$

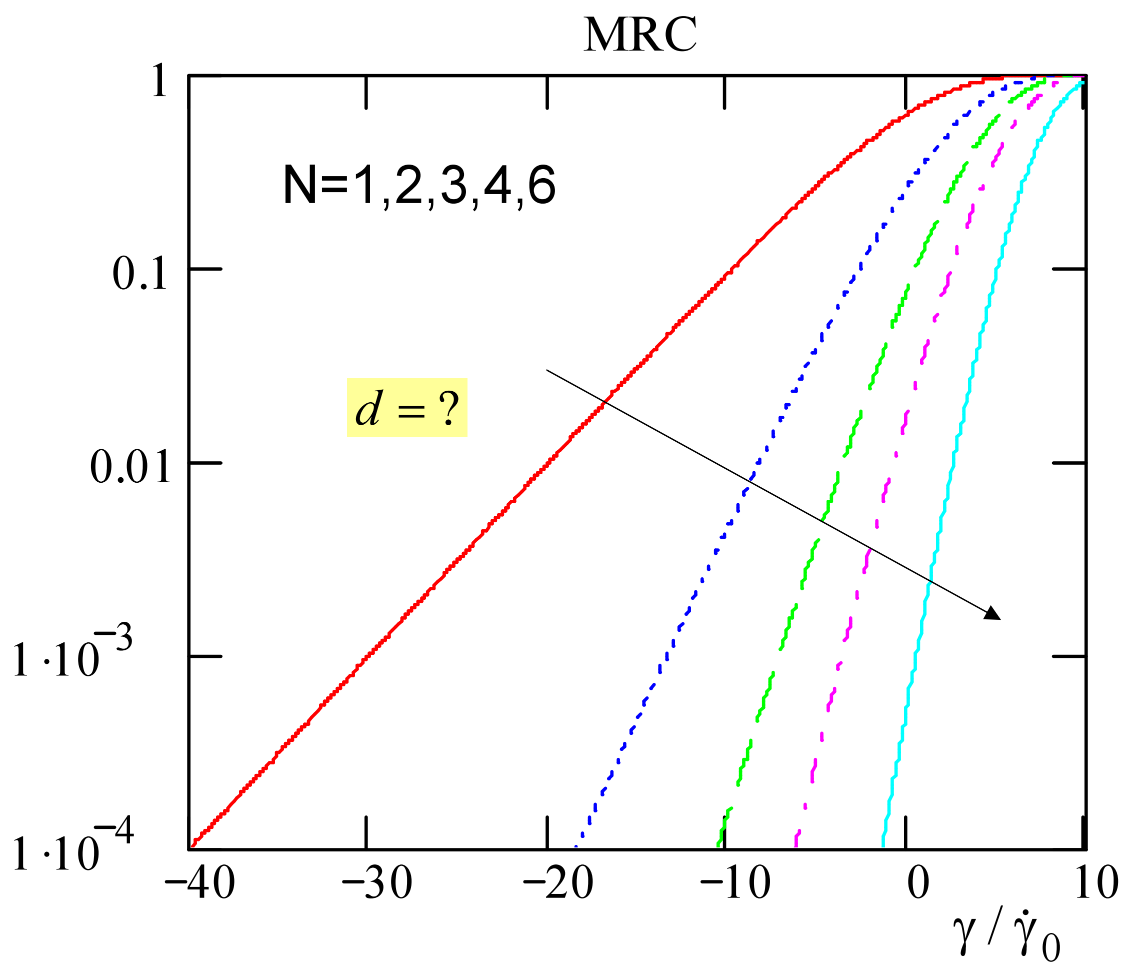
Asymptotically, for $\gamma/\gamma_0 \ll 1$ (small P_{out}),

$$P_{out} \approx \frac{(\gamma/\gamma_0)^N}{N!} = c_N \left(\frac{\gamma}{\gamma_0}\right)^N \sim P_{out,1}^N \sim \frac{1}{(\gamma_0)^N} \qquad (11.23)$$

Diversity order $= N$ (prove it!)

Q.: Diversity gain?

MRC Outage Probability



Q.: compare to SC and EGC!

MRC PDF

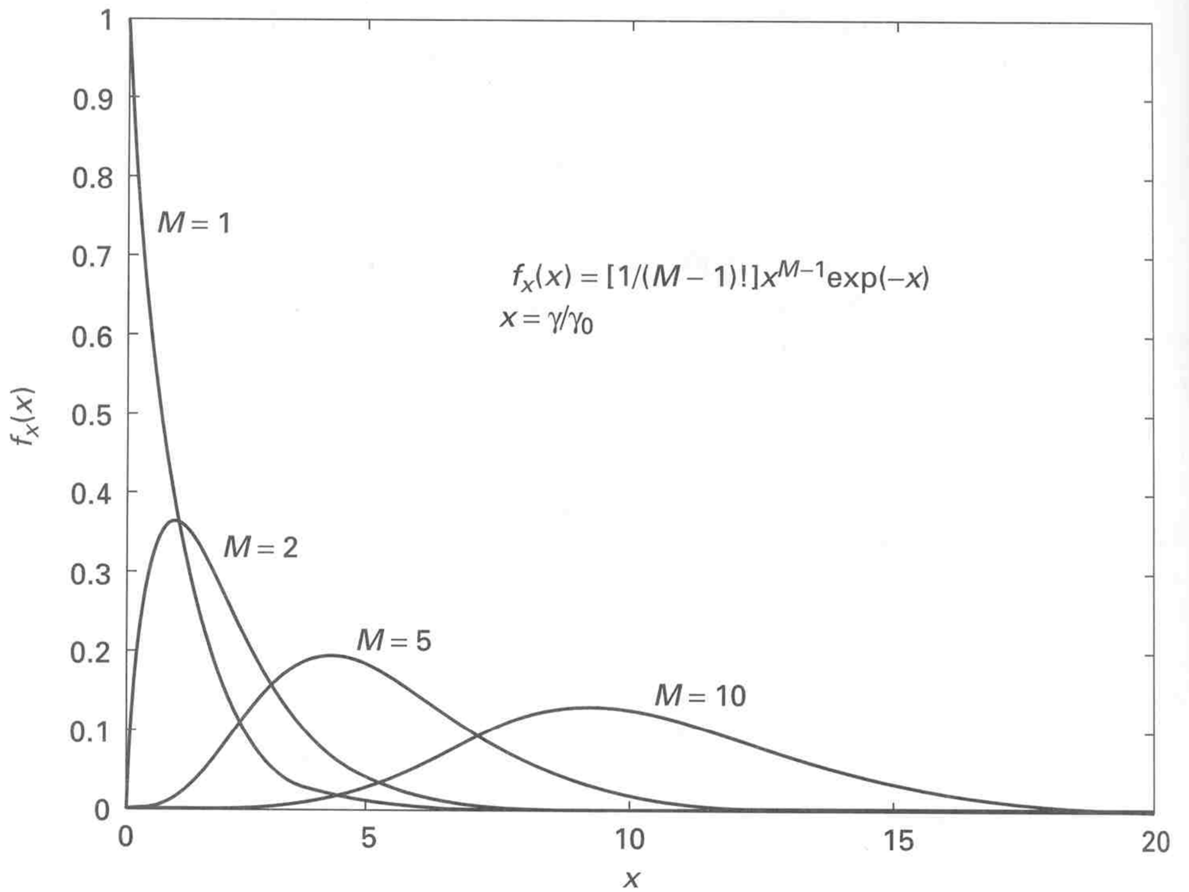


FIGURE 5.16 Probability density function for the maximal ratio combiner.

P.M. Shankar, Introduction to Wireless Systems, Wiley, 2002.

Implementation Issues

Combiner complexity varies significantly from type to type (as well as performance).

Overall, there exists performance/complexity trade-off. High-performance combiner (MRC) is difficult to implement; simpler combiner (SC) does not perform that well as MRC, EGC is in between.

Selection diversity: a branch with the strongest signal is used; in practice - with the largest signal + noise power since it is difficult to measure SNR alone.

This must be done on continuous basis—may be difficult. Can be implemented with some time constant or as scanning (switched) diversity.

SC is the simplest technique.

Maximum ratio combining: coherent technique, i.e., signal's phase has to be estimated. All the signals are co-phased and weighted according to their signal voltage to noise power ratios. It requires for a lot of circuitry (individual receivers)

Provides the best performance. DSP makes it practical. The output may be acceptable even when all the inputs are not.

Equal gain combining: compromise. Less complexity than MRC (all the weights are equal), but co-phasing is till required. Can still produce acceptable output from unacceptable inputs.

If the individual branch signals are not independent (correlated), the performance degrades.

Average BER Improvement Using Diversity Combining

Instantaneous SNR is a R.V. Hence, BER is a RV as well.
Introduce average BER (frequency flat, slow fading),

$$\overline{P_e} = \int_0^\infty P_e(\gamma)\rho(\gamma)\,d\gamma \tag{11.24}$$

where $\rho(\gamma)$ is a pdf of γ at the output of a diversity combiner.

Recall that the BER can be expressed in many cases as

$$P_e(\gamma) = Q\left(\sqrt{\alpha\gamma}\right) \tag{11.25}$$

where $\alpha = 2$ for BPSK and QPSK, $\alpha = 1$ for BFSK (all detected coherently) and sub-optimal DPSK.

Other modulation formats have similar BER expressions (or bounds) . Hence, we evaluate average BER based on this expressions.

Consider N-branch MRC (the best one),

$$F_N(\gamma) = 1 - e^{-\gamma/\gamma_0} \sum_{i=0}^{N-1} \left(\frac{\gamma}{\gamma_0}\right)^i / i!$$
$$f_N(\gamma) = \frac{dF_N(\gamma)}{d\gamma} = \frac{\gamma^{N-1}}{(N-1)!\gamma_0{}^N} e^{-\gamma/\gamma_0} \qquad (11.26)$$

The average BER of a coherent demodulation,

$$\overline{P_e} = \left[\frac{1}{2}(1-\mu)\right]^N \sum_{i=0}^{N-1} C_{N-1+i}^i \left[\frac{1}{2}(1+\mu)\right]^i \qquad (11.27)$$

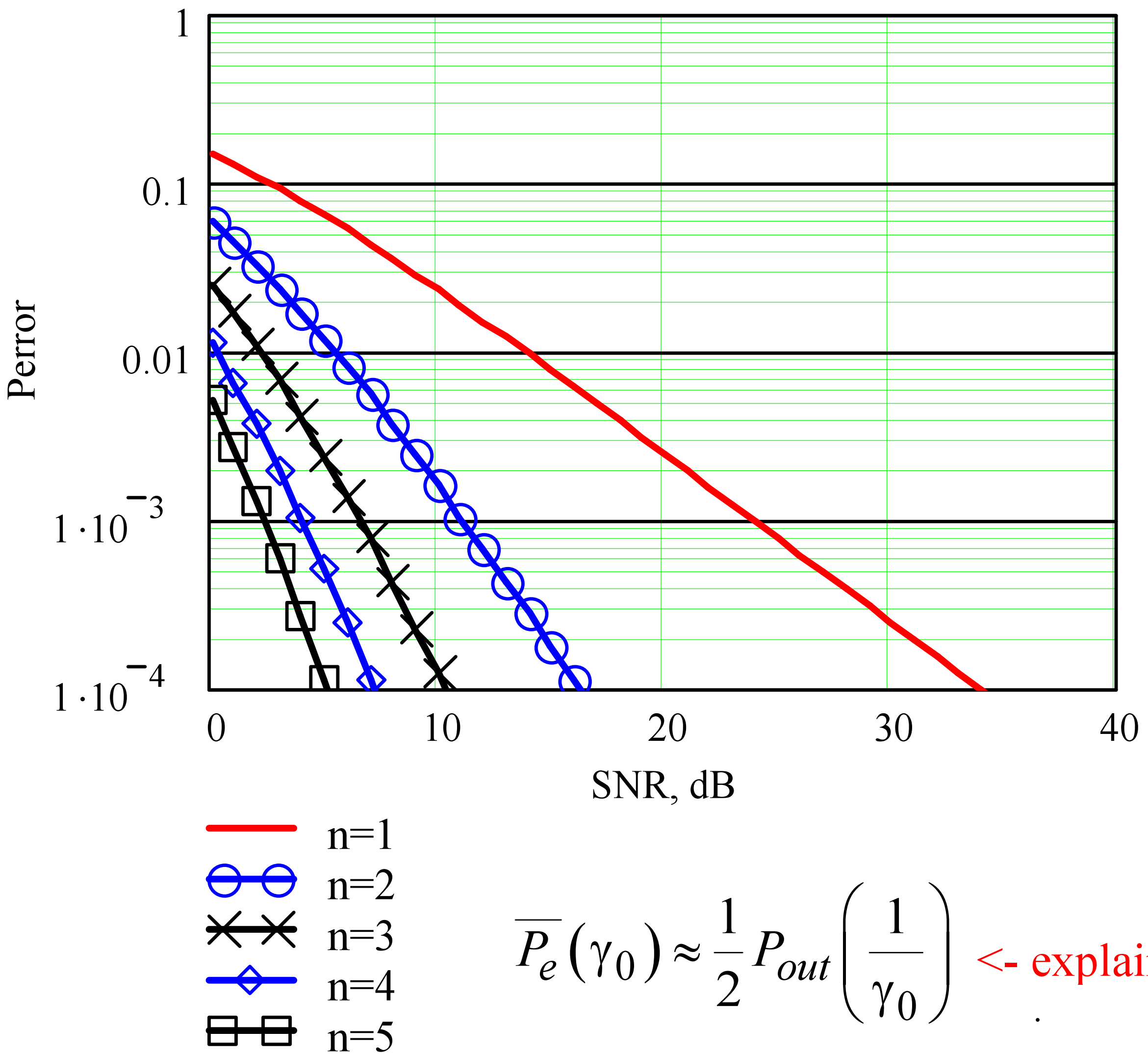
where $\mu = \sqrt{\alpha\gamma_0 / (2+\alpha\gamma_0)}$, $C_n^k = n!/(k!(n-k)!)$.

Asymptotically, $\gamma_0 \gg 1$,

$$\overline{P_e} \approx \left[\frac{1}{2\alpha\gamma_0}\right]^N C_{2N-1}^N \ll \frac{1}{2\alpha\gamma_0} \qquad (11.28)$$

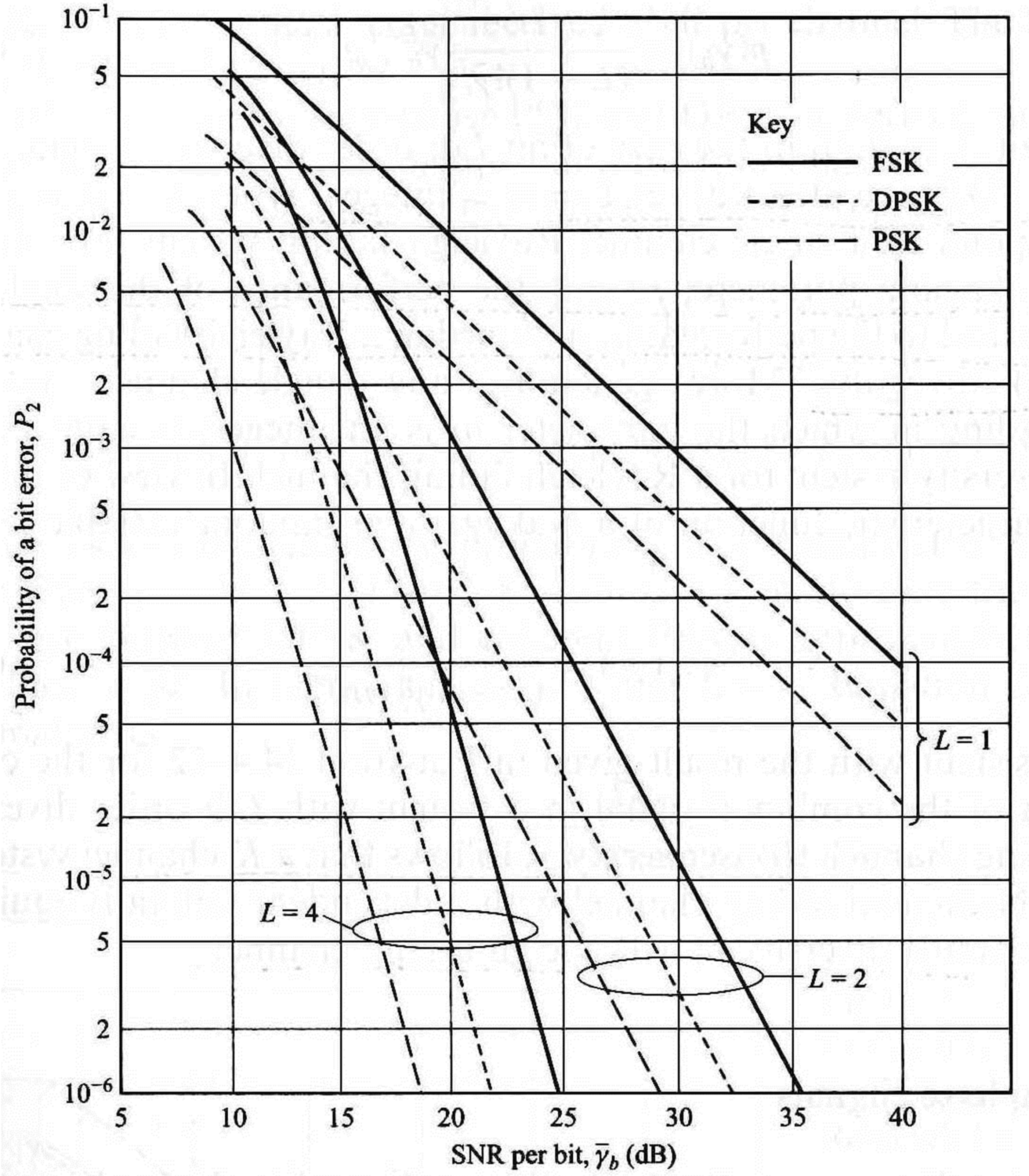
Diversity order $= N$.

The average BER for MRC and the coherent BPSK receiver.



$$\overline{P_e}(\gamma_0) \approx \frac{1}{2} P_{out}\left(\frac{1}{\gamma_0}\right)$$ <- explain this!

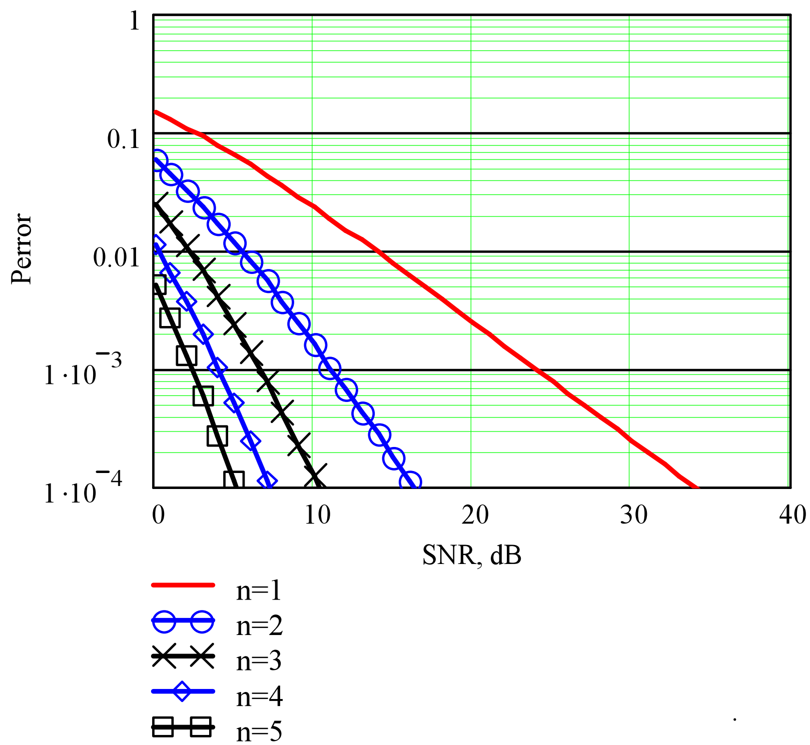
Average BER for various modulation formats

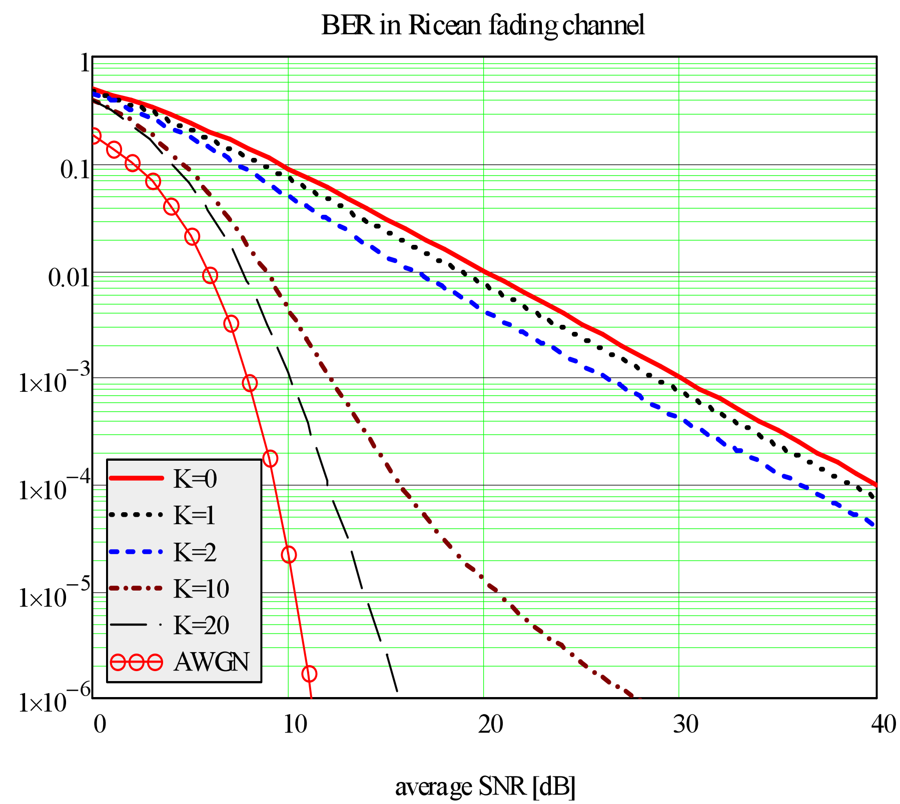


J. Proakis, Digital Communications, McGraw Hill, Boston, 2001.

How to reduce $\overline{P}_e, P_{out}$ in a fading channel?

- Diversity combining techniques
- Strong LOS (large K-factor) via proper location of antennas





Summary

- Diversity combining techniques.
- Forms of diversity and types of combining
- Performance improvement due to diversity combining
- Average BER and outage probability

Reading:

- Rappaport, sec. 6.12, 7.10
- Stuber, Ch. 6
- Any other text that covers the topics above.

Note: Do not forget to do end-of-chapter problems. Remember the learning efficiency pyramid!