Lab # 4: MF and BER via Monte-Carlo Simulations

Preparation

1. Consider a digital communication system when SNR = 5 dB/bit. Find the BER for
   - BPSK
   - OOK
   - QPSK
   - 16 QAM

Compare the answers and make conclusions. What is the impact of the baseband pulse shape?

2. Sketch QPSK constellation which uses the phase values $\pm\pi/4, \pm 3\pi/4$ and indicate bit-to-symbol assignment using Gray mapping (so that 2 adjacent symbols differ by 1 bit only).
3. Find the most general form of the impulse response of a filter matched to a rectangular pulse of duration T and amplitude A.

Laboratory (Parts A and B)

Part A: Matched Filter (MF)

1. Consider binary baseband modulation whereby, for each symbol interval T, the transmitted signal is $s_k(t) = a_k p(t)$, where $a_k = \pm 1$ and $p(t)$ is a rectangular pulse of duration T and unit amplitude,

$$p(t) = \begin{cases} 1, & \text{if } 0 < t \le T \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

We wish to transmit 4 bits using sequential bipolar 2-PAM transmission (baseband BPSK):

$$s(t) = \sum_{k=0}^{3} a_k p(t - kT) \quad (2)$$

where $a_k = (-1)^k$ encodes the transmitted bits. Plot this transmitted waveform for $T = 1$ (this is normalized time to any convenient unit, e.g. μs).

2. In real system, the transmitted signal is corrupted by AWGN noise $\xi(t)$ so that the received signal $x(t)$ is as follows:

$$x(t) = s(t) + \xi(t) \quad (3)$$

To facilitate DSP implementation, we will consider the discrete-time (sampled) implementation:

$$x_i = x(i\Delta t) = s(i\Delta t) + \xi(i\Delta t) = s_i + \xi_i \quad (4)$$

where $\Delta t = T/N$ is sufficiently small, $N = 10^2$ is the number of samples per symbol interval T, and i is the discrete time variable. Plot x_i for 4 symbol intervals as in (2), so that $0 < t = i\Delta t \le 4T$

, assuming that the noise variance $\sigma_0^2 = \mathrm{var}\{\xi_i\} = 1$ (note that, since the noise is AWGN, ξ_i are i.i.d. Gaussian of zero mean) so that the per-sample SNR is 0 dB. Compare this to the plot of step 1 and comment/explain the difference. Now set σ_0^2 so that the per-sample SNR is 3 dB and plot again; compare to the previous plot and comment/explain the difference. Repeat for the per-sample SNR = 6 dB.

3. Now, we consider a filter matched to $p(t)$ and whose input is $x(t)$ in (3). First, we consider only one symbol interval in (2), i.e. $k = 0$ and $0 < t \le T$, and use a running average implementation of the matched filter:

$$y(t) = \frac{1}{t}\int_0^t x(\tau)d\tau \tag{5}$$

Show that, for this implementation, its output $y(T)$ at the sampling time $t = T$ is exactly the same as that of the original MF for the pulse $p(t)$ (as determined in Preparation question 3 with a proper selection of filter parameters). The discrete-time (DSP) implementation of this filter follows from (5):

$$y_i = y(i\Delta t) \approx \frac{1}{i\Delta t}\sum_{n=1}^{i} x(n\Delta t)\Delta t = \frac{1}{i}\sum_{n=1}^{i} x_n \tag{6}$$

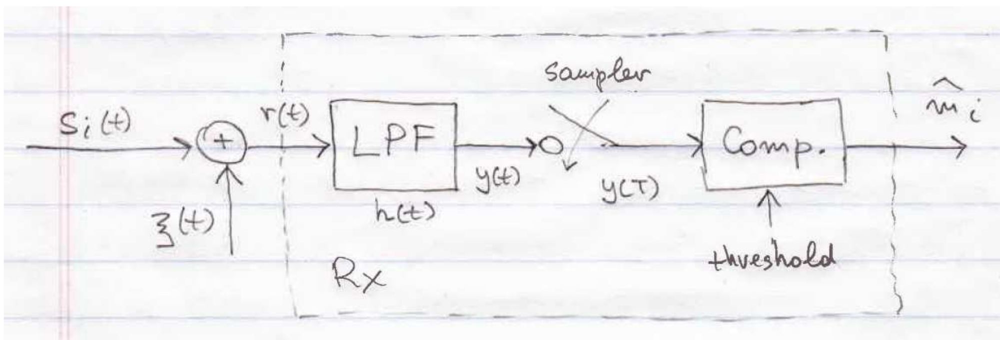
i.e. the discrete-time implementation is also running average. Set the per-sample SNR = 0 dB and plot y_i for first symbol interval only, $0 < t \le T$, and compare it to 1st symbol interval in (4); comment/explain the difference. Now, consider the MF operation for all 4 symbol intervals in (2) using discrete-time implementation as in (4), (6). Assume that, at the beginning of each symbol interval, the MF is set to zero initial state and the computation in (6) begins afresh (i.e. the summation in (6) corresponds only to time in that specific symbol interval and the values from previous interval are not carried over). Plot y_i for all 4 symbol intervals and the per-sample SNR = 0 dB, compare it to the plot of x_i in (4) and comment/explain the difference. Now, change the SNR to 3 dB, repeat and observe the impact of the SNR on y_i and x_i. Do the same for 6 dB SNR.

Part B: BER of the Optimal Receiver

In this part, we will use Monte-Carlo simulations to evaluate numerically error rates for digital communication systems operating in the AWGN channel.

The system block diagram with an optimal receiver is as in Lecture 7, given below, where LPF is in fact the MF. We will use the baseband M-PAM modulation, where $s_i(t) = a_i p(t)$, and a_i takes on any of M possible values (note that i here is the symbol interval index, different from discrete time index of Part A). The impulse response of the MF, optimal sampling time and threshold have been established in Lecture 7 and further developed in Lecture 8 (see also the course textbook and other reference books).

<u>Optimal Rx structure:</u>



The time-domain model of our system is as follows:

$$r(t) = \sum_i a_i p(t - iT) + \xi(t); \quad y(t) = r(t) * h(t), \quad y_i = y(t = iT) \tag{7}$$

which can be reduced, after matched filtering and sampling, to the following discrete-time model:

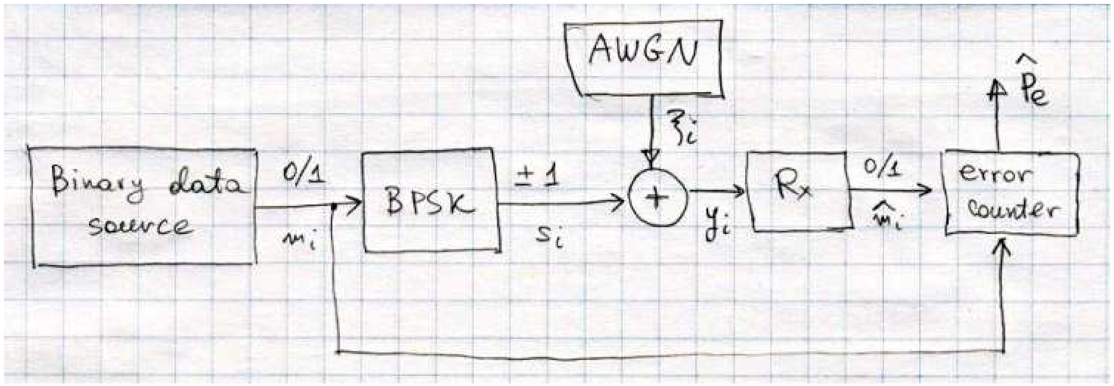
$$y_i = s_i + \xi_i \tag{8}$$

where s_i and ξ_i are the signal and noise samples at time $t = iT$, so that i serves as a discrete-time index (symbol #). ξ_i is Gaussian random variable with zero mean and certain variance σ_0^2, independent from sample to sample.

1. Assume that baseband BPSK is used, so that $a_i = \pm 1$ and, after proper normalization, you can also take $s_i = \pm 1$ and adjust σ_0^2 to obtain desired SNR (here we use the fact that the error rate performance depends on the SNR only, not signal or noise power individually). Generate N samples of the signal s_i (using ± 1 with equal probability) and noise $\xi_i \sim N(0, \sigma_0^2)$. For each sample, compute y_i and the output of the optimal receiver $\hat{m}_i$. The BER can be estimated as follows:

$$P_e \approx \frac{1}{N} \sum_{i=1}^{N} e_i \tag{9}$$

where e_i is an error indicator: $e_i = 0$ if $\hat{m}_i = m_i$ and 1 otherwise. The simulation block diagram is shown below.



Plot BER vs. SNR [dB] graph using log scale for the BER axis over the SNR range 0 to 10 dB (use 1 dB or smaller step). Use appropriate number of samples N for each SNR value so that statistical fluctuations are small (hint: start with moderate value of N and increase it until the impact of fluctuations is small). Use internal Matlab functions (such as cumsum, histc, sum, etc.) and vectorized operations as much as possible since it speeds up simulations significantly; avoid using "for" and "do" loops as they slow down simulations. Compare your graph with that in Lecture 7: do they agree? Also plot the theoretical BER curve (via Q function; you may use the relationship between Q and erfc functions) on the same graph with simulated one for comparison purposes.

2. Do #1 for OOK. Hint: what is the average power of OOK compared to BPSK for the same pulse shape and amplitude provided that 0 and 1 are equiprobable? Use this to set up σ_0^2 appropriately.

3. Do # 1 for QPSK. Use Gray bit-to-symbol mapping (i.e. when two adjacent symbols differ by 1 bit only – see Ref. 2 for more details; this bit mapping minimizes the BER.) Hint: use in-phase/quadrature representation of $y_i = s_i + \xi_i$ on a complex plane; observe that real and imaginary parts of noise are independent of each other and that the decisions on real and imaginary parts of s_i are taken independently so that QPSK is equivalent to 2 independent BPSK transmissions (on I and Q branches). Compare it with the theoretical BER curve on the same graph. Do they agree?

4. Do #1 for 16-QAM assuming Gray bit-to-symbol mapping. Hint: use the fact that

$$M\text{-QAM} = \underbrace{\sqrt{M} - \text{PAM}}_{I} \times \underbrace{\sqrt{M} - \text{PAM}}_{Q} \tag{10}$$

5. Repeat #3 and 4 for the symbol (rather than bit) error rate and compare it to the BER (by plotting on the same graph). Comment on the difference, if any. How does it compare to the bounds in Lecture 8?

References:

1. T.S. Rappaport, Wireless Communications: Principles and Practice, Prentice Hall, New Jersey, 2002. (2nd Edition)
2. J.G. Proakis et al, Contemporary Communication Systems Using MATLAB and Simulink, Thomson & Books/Cole, 2004.
3. W.H. Tranter et al., Principles of Communication Systems Simulation, Prentice Hall, 2004.
4. M.C. Jeruchim et al, Simulation of Communication Systems: Modeling, Methodology, and Techniques, Kluwer, New York, 2000.