Assignment #6

Due: Dec. 6, Wed. 1pm (beginning of the lecture). Hard copy only, no email submissions. *Late entries will not be accepted!*

Before doing the assignment, please read appropriate sections of Chapter 6 and 7 of the course textbook (by Rappaport, 2nd edition) or any other relevant book (e.g. from the reference list).

1) Find the average SNR per bit $\gamma_b = E_b / N_0$ required to detect a DPSK signal in a flat-fading channel so that the average BER does not exceed 10^{-2}. Assume that the fading process is slow (quasi-static) and its distribution is (a) Rayleigh, (b) Ricean, with $K = 0$ dB and 10 dB. Compare your answers and make conclusions. Which is the best case scenario? Why?

2) Repeat Problem 1 with BPSK modulation. Compare the answers, make conclusions.

3) Prove that if α is a Rayleigh distributed RV, then the pdf of α^2 is given by $p(\alpha^2) = \frac{1}{\overline{\alpha^2}} e^{-\alpha^2/\overline{\alpha^2}}$. This is the Chi-square pdf with two degrees of freedom. Using this result, show that the pdf of instantaneous SNR in a Rayleigh fading channel is given by $p(\gamma) = \frac{1}{\gamma_0} \exp(-\gamma/\gamma_0)$, where γ_0 is the average SNR.

4) Consider a single-branch (no diversity) receiver (Rx) operating in a Rayleigh fading channel where the instantaneous Rx SNR has a 20% chance of being 6 dB below some SNR threshold.
   (a) Determine the mean SNR normalized to the threshold.
   (b) Find the probability that the output SNR of a 2-branch (antenna) selection combiner will be 6 dB below the mean SNR threshold, assuming i.i.d. fading.
   (c) Repeat Part (b) for a 3-branch (antenna) selection combiner.
   (d) Repeat Part (b) for a 4-branch (antenna) selection combiner
   (e) Based on your answers above, discuss the advantages and disadvantages of adding more branches/antennas to a selection combiner. How many antennas would you use?

5) In Lecture 4, we defined the outage probability P_{out} as the probability of instantaneous SNR dropping below a given threshold γ_{th} : $P_{out}(\gamma_{th}) = \Pr\{\mathrm{SNR} < \gamma_{th}\}$. In this question, we define outage probability $P_{out,e}$ as the probability of instantaneous BER $P_e(\mathrm{SNR})$ exceeding a given threshold ε : $P_{out,e}(\varepsilon) = \Pr\{P_e(\mathrm{SNR}) > \varepsilon\}$.

   (a) Prove that these two outage probabilities related as follows in Rayleigh fading channel with average SNR γ_0:

$$P_{out,e}(\varepsilon) = \Pr\{P_e(\mathrm{SNR}) > \varepsilon\} = \Pr\left\{\mathrm{SNR} < P_e^{-1}(\varepsilon)\right\} = P_{out}(P_e^{-1}(\varepsilon)) = 1 - e^{-P_e^{-1}(\varepsilon)/\gamma_0} \quad (1)$$

   where $P_e^{-1}(\varepsilon)$ is the inverse function of $P_e(\gamma)$ (which is the BER at a given SNR = γ, so that $P_e(P_e^{-1}(\varepsilon)) = \varepsilon$). Further show that these two definitions give the same outage probability provided that $\gamma_{th} = P_e^{-1}(\varepsilon)$.
   (b) Note that first 3 inequalities in (1) hold for any fading distribution and also when a diversity combining is used, but the last equality does not. Find an equivalent of the right-hand-side of (1) (the last equality) when M antennas are used for selection combining in i.i.d. Rayleigh fading channel.
   (c) Assuming that the design specifications require $P_{out,e} = \varepsilon = 10^{-3}$ for BPSK in a Rayleigh fading channel (no diversity), find the needed average SNR to achieve this objective.
   (d) Repeat (c) when two-branch selection combiner is used, compare your answer to that in (c) and explain the difference, if any.

6) Consider a BPSK receiver operating in a Rayleigh (slow and frequency-flat) fading channel (no diversity) with the average BER = 10^{-1}. Find % of time (or users) when instantaneous error rate exceeds 10^{-1}.

Continued on the next page. ↓

Please include in your solutions all the intermediate results and their numerical values (if applicable). **Detailed solutions with explanations are required**, not just the final answers/equations; **all symbols used must be defined**, including units used (e.g. f = frequency [Hz], L = path loss [dB]). Missing explanations, symbol definitions/units will be penalized. Your answers should demonstrate the full extent of your knowledge and the latter will determine your marks.

Please read appropriate chapters of the textbook first, study all the examples, attempt to do them with the closed book. Remember the learning efficiency pyramid!

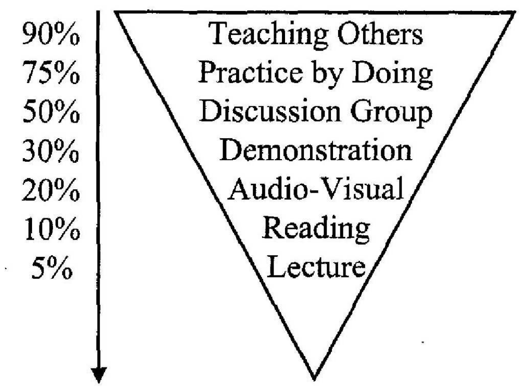

Figure 1. The Learning Pyramid, adapted from David Sousa, *How the Brain Learns*, Reston, VA, The National Association of Secondary School Principals, 1995, ISBN 0-88210-301-6.

Plagiarism (i.e. "cut-and-paste" from a student to a student, other forms of "borrowing" the material for the assignment) is absolutely unacceptable and will be penalized. Each student is expected to submit his own solutions. If two (or more) identical or almost identical sets of solutions are found, each student involved receives 0 (zero) for that particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be send to the Dean's office for further investigation.