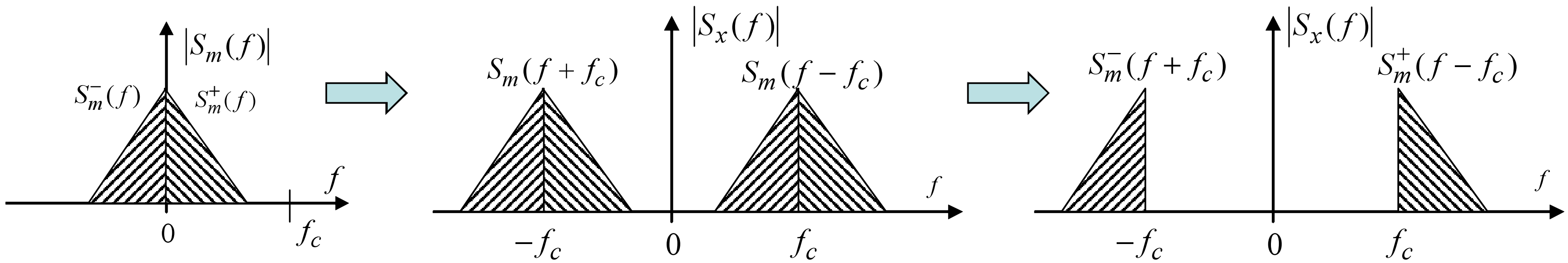
Single Sideband (SSB) AM

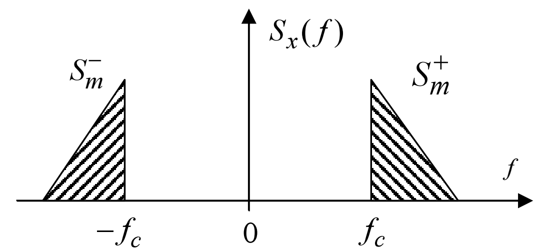
- Why SSB-AM? Spectral efficiency is of great importance.
- Conventional & DSB-SC occupy twice the message bandwidth.
- All the information is contained in either half – the other is redundant.
- Spectral efficiency can be greatly (twice) increased by transmitting one half.



Generation of SSB: Analysis

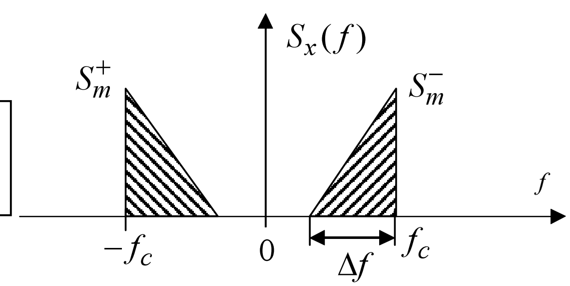
- Hilbert transform can be effectively used.
- Start with the message m(t) and show that USB (upper SSB) is given by

$$x(t) = A_c m(t)\cos 2\pi f_c t - A_c \hat{m}(t)\sin 2\pi f_c t$$



- Similarly, LSB can be expressed as

$$x(t) = A_c m(t)\cos 2\pi f_c t + A_c \hat{m}(t)\sin 2\pi f_c t$$



- In-phase & quadrature channels are required to generate SSB.

USB: Frequency-Domain Viewpoint

- Time-domain signal $x(t) = A_c m(t)\cos 2\pi f_c t - A_c \hat{m}(t)\sin 2\pi f_c t$
- Spectra of individual components:

$$m(t) \leftrightarrow S_m^+(f) + S_m^-(f), \quad \hat{m}(t) \leftrightarrow -jS_m^+(f) + jS_m^-(f)$$

$$\cos(\omega_c t) \leftrightarrow \frac{1}{2}\left(\delta(f - f_c) + \delta(f + f_c)\right), \quad \sin(\omega_c t) = \frac{1}{2j}\left(\delta(f - f_c) - \delta(f + f_c)\right)$$

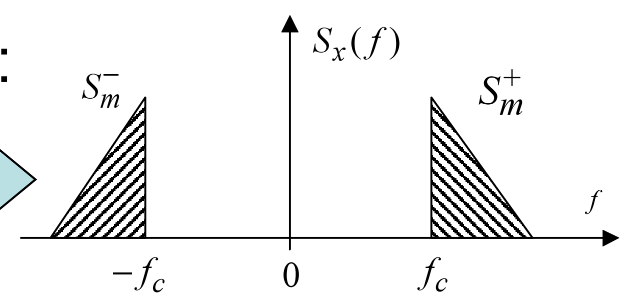
- Use multiplication property of FT:

$$m(t)\cos\omega_c t \leftrightarrow \frac{1}{2}\left(S_m^+(f - f_c) + S_m^-(f - f_c) + S_m^+(f + f_c) + S_m^-(f + f_c)\right)$$

$$\hat{m}(t)\sin\omega_c t \leftrightarrow \frac{1}{2}\left(-S_m^+(f - f_c) + S_m^-(f - f_c) + S_m^+(f + f_c) - S_m^-(f + f_c)\right)$$

- Combine the two expressions above:

$$\boxed{x(t) \leftrightarrow S_m^+(f - f_c) + S_m^-(f + f_c)}$$



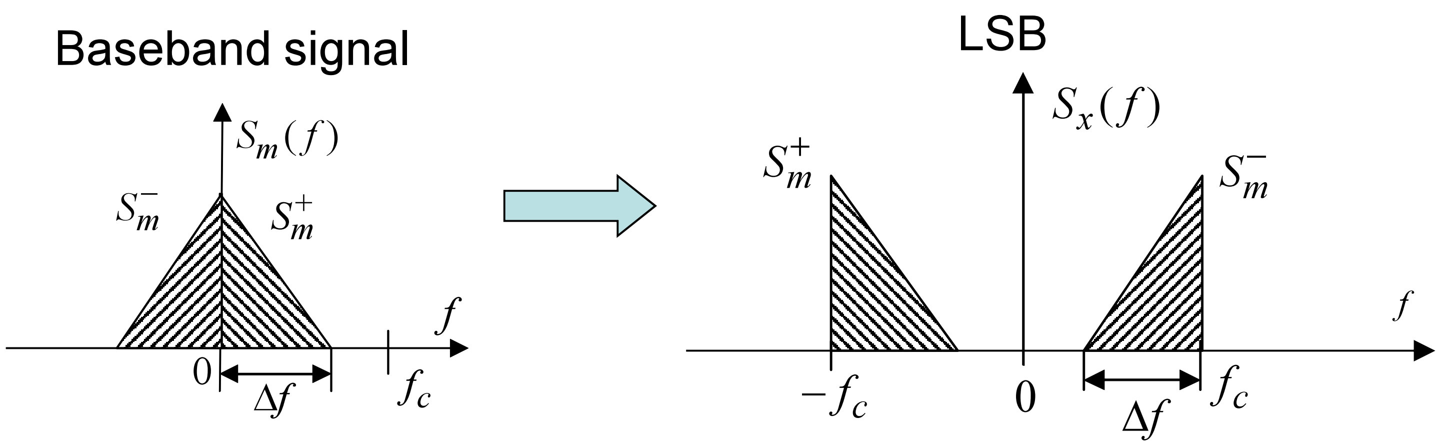
Lower SSB (LSB)

- Analysis method is the same as for USB.
- Time-domain signal is

$$x(t) = A_c m(t)\cos 2\pi f_c t + A_c \hat{m}(t)\sin 2\pi f_c t$$

- Its spectrum is

$$x(t) \leftrightarrow S_m^-(f - f_c) + S_m^+(f + f_c)$$



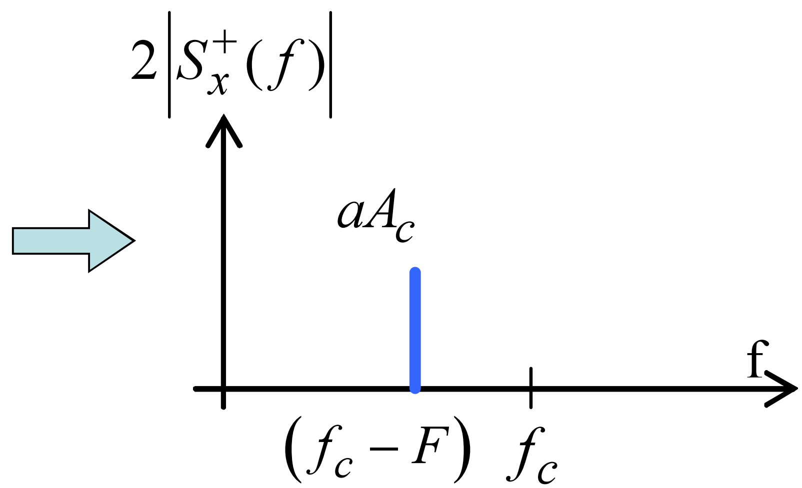
Example: Sinusoidal Modulating Signal

- Assume that $m(t) = a\cos\Omega t$
- Then $x(t) = aA_c \cos\Omega t \cos\omega_c t + aA_c \sin\Omega t \sin\omega_c t =$

$$= aA_c \cos(\omega_c - \Omega)t$$

- Obviously, this is LSB signal with one spectral component only at $(\omega_c - \Omega)$

▪ Think about it: modulated signal is just a sinusoid !

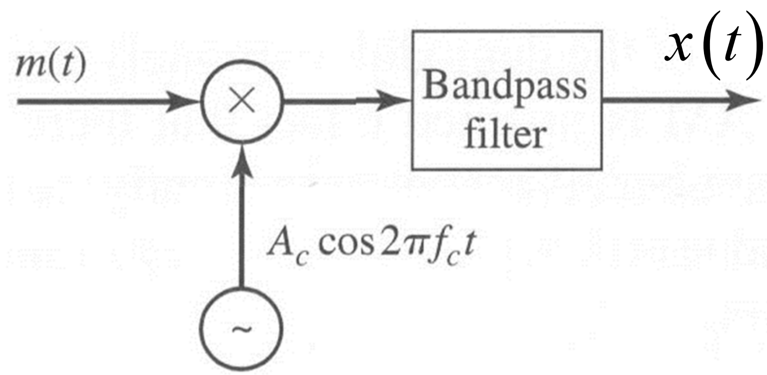


How can one transmit a message using a sinusoid?

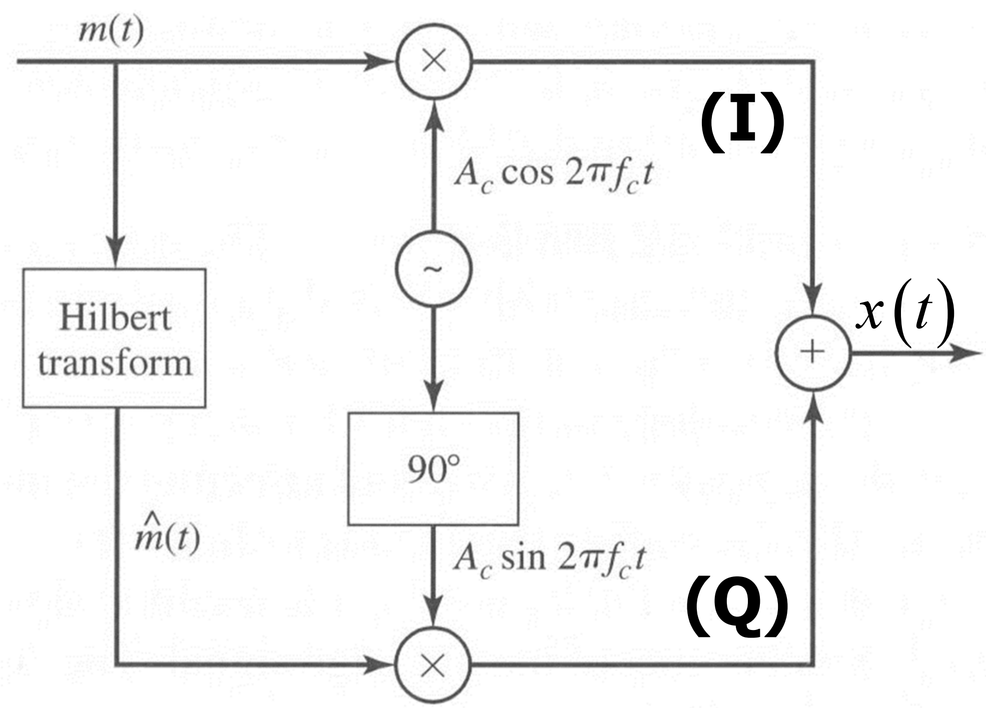
Generation of SSB

- Filtering method:

(not practical in many cases)



- Using balanced modulators:



Hilbert transform is a linear filter (phase shifter):

$$H(f) = \begin{cases} -j, f > 0 \\ j, f < 0 \end{cases}$$

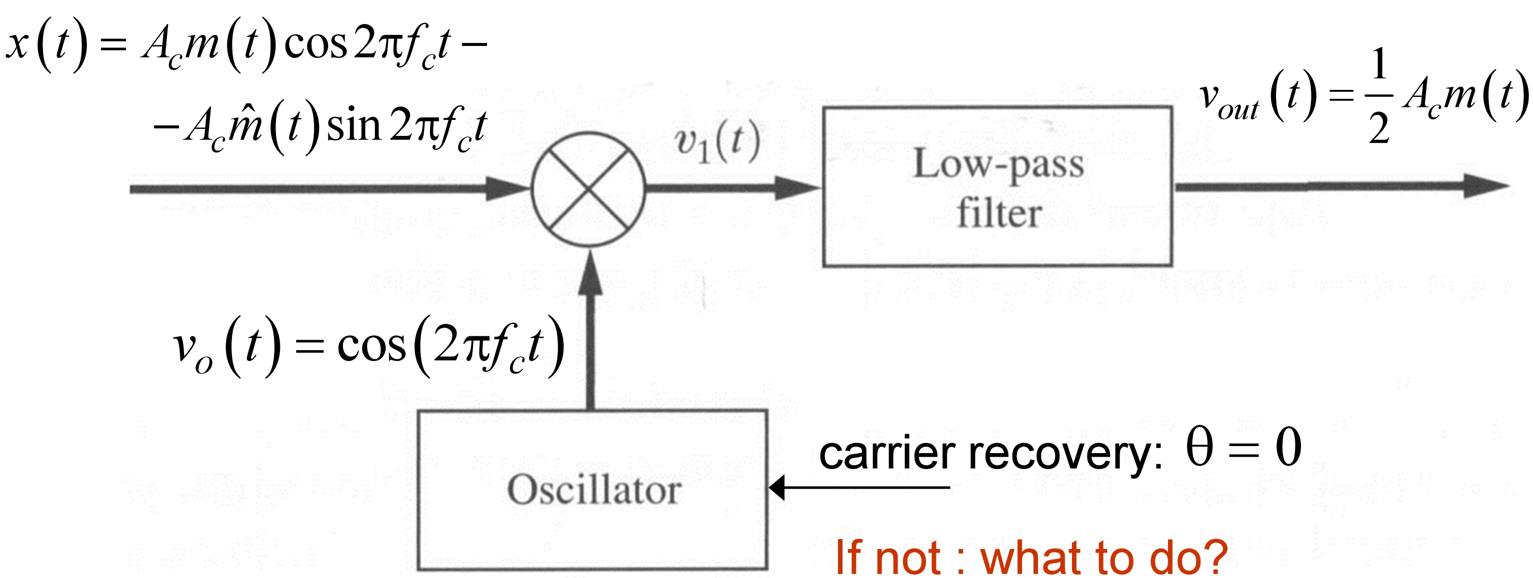
J.Proakis, M.Salehi, Communications Systems Engineering, Prentice Hall, 2002

Demodulation of SSB

- Product detector:

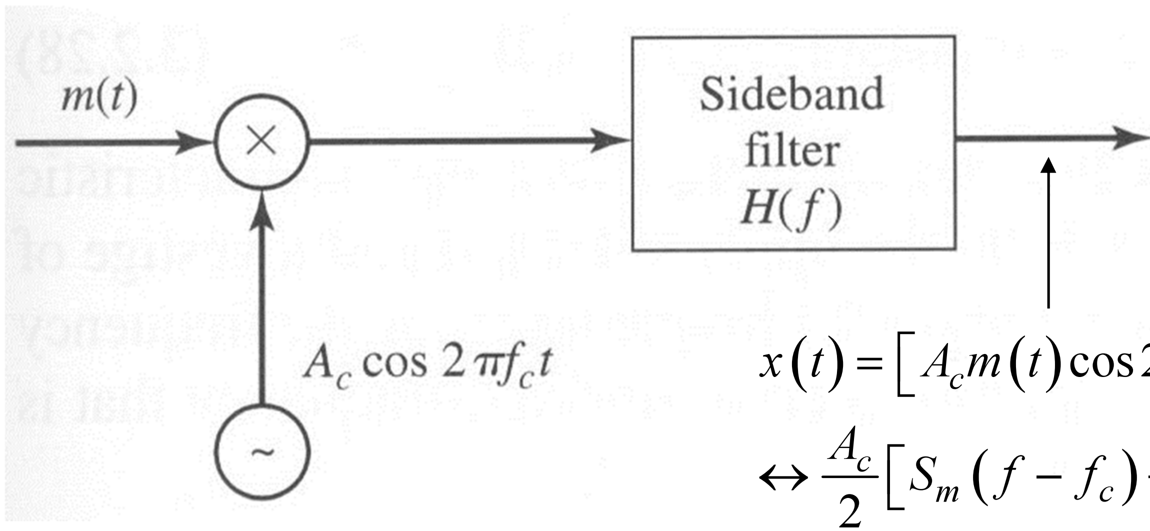
$$x(t)\cos(2\pi f_c t+\theta)=\frac{1}{2}A_c m(t)\cos\theta+\frac{1}{2}A_c\hat{m}(t)\sin\theta \text{ + 2f}_c\text{ terms}$$

- After low-pass filter, only 1st two terms remain.
- Coherent demodulation: $\theta=0$



Vestigial-Sideband (VSB) AM

- SSB can be simplified by allowing a part of the other sideband to appear.
- A filter implementation is feasible:



$$x(t) = \left[A_c m(t)\cos 2\pi f_c t\right] * h(t) \leftrightarrow$$

$$\leftrightarrow \frac{A_c}{2}\left[S_m(f-f_c) + S_m(f+f_c)\right]H(f)$$

- Filter requirement:

$$H(f-f_c) + H(f+f_c) = \text{constant}, \quad |f| \le \mathrm{W}$$ + linear phase

VSB Spectrum & Filter Response

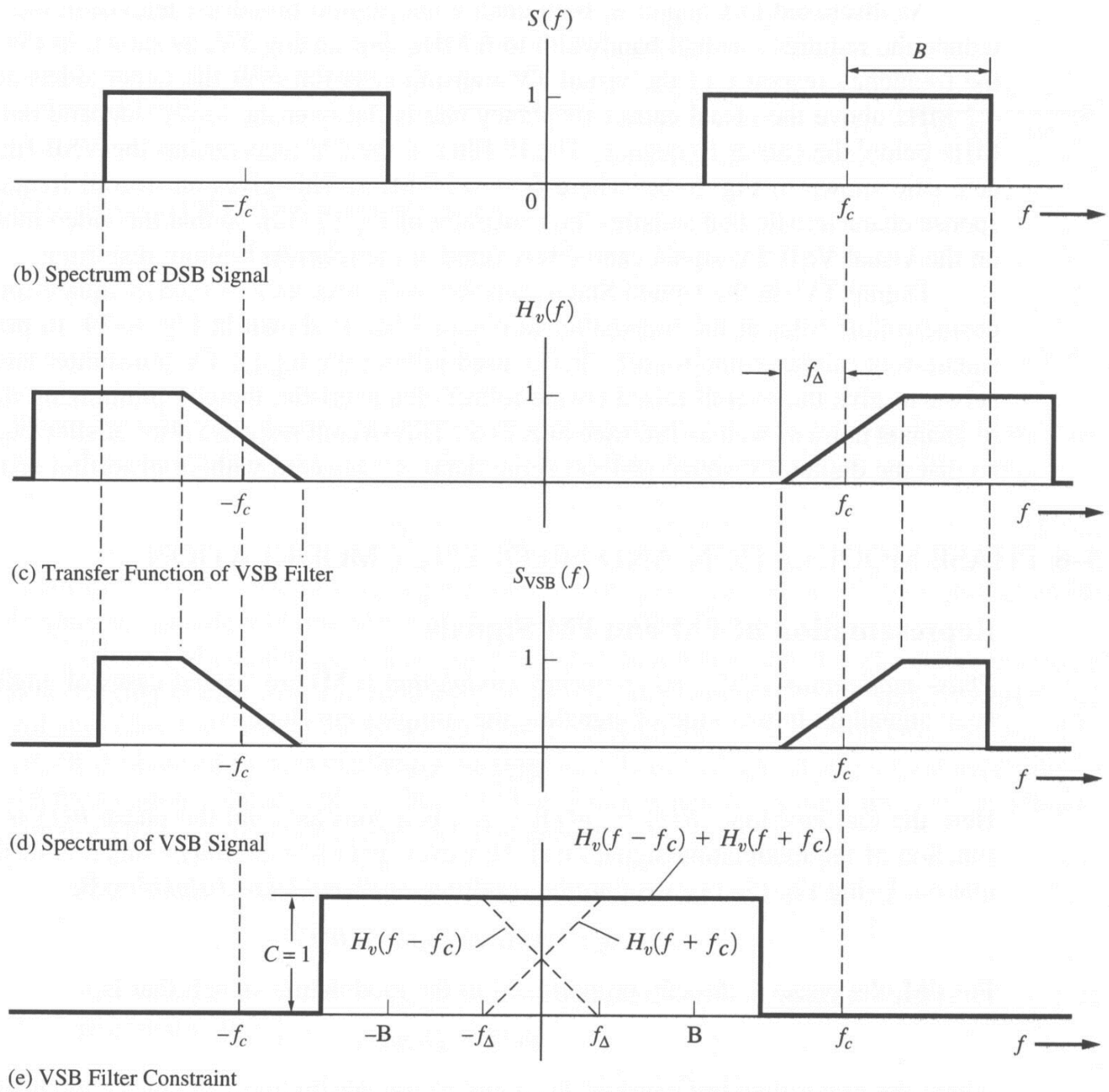
DSB signal

Filter response

VSB signal spectrum

Filter constrain

(real h(t) ?)

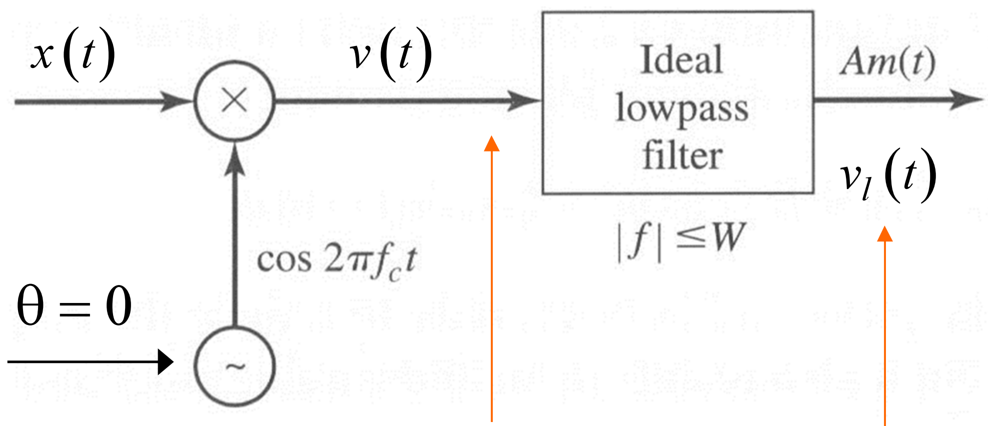


L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Demodulation of VSB

- Multiplier (coherent) demodulator:

carrier recovery: $\theta = 0$



$$S_v(f) = \frac{A_c}{4}\left[S_m(f-2f_c) + S_m(f)\right]H(f-f_c) + \\ + \frac{A_c}{4}\left[S_m(f) + S_m(f+2f_c)\right]H(f+f_c)$$

$$S_l(f) = \frac{A_c}{4}S_m(f)\left[H(f-f_c) + H(f+f_c)\right]$$

Comparison of conventional AM, DSB-SC, SSB and VSB.

- Conventional AM: simple to modulate and to demodulate, but low power efficiency (33-50% max) and double the bandwidth.
- DSB-SC: high power efficiency, but more complex to modulate & demodulate, doubles the bandwidth.
- SSB: high power efficiency, the same (message) bandwidth, but more difficult to modulate & demodulate.
- VSB: lower power efficiency & larger bandwidth but easier to implement.

Summary

- Single sideband (SSB) modulation. USB & LSB.
- Spectra of SSB signals. Generation & demodulation.
- Vestigial sideband (VSB) modulation. Spectra of signals. Generation & demodulation. Filter requirement.
- Comparison of conventional AM, DSB-SC, SSB and VSB.

- **Homework**: Reading: Couch, 5.5. Study carefully all the examples and make sure you understand them. Attempt some end-of-chapter problems.