Assignment #3

Due: Feb. 7, 11:30am, MNO C211 (the tutorial). Hard copies only. Late/electronic/email submissions will **not** be accepted.

1) Can the response of an LTI system to $x(t) = \text{sinc}(t)$ be $y(t) = \text{sinc}^2(t)$? Justify your answer.

2) A band-limited baseband signal has the bandwidth of 6 kHz,

(a) What is the minimum sampling frequency for perfect reconstruction of the signal?

(b) What is the minimum sampling frequency if a guard band of 3 kHz is required?

(c) What is the minimum required sampling frequency and the value of K for the perfect reconstruction if the reconstruction filter has the following frequency response *H(f)*:

$$H(f) = \begin{cases} K, & |f| < 7 \text{ kHz} \\ K - K\dfrac{|f| - 7 \text{ kHz}}{3 \text{ kHz}}, & 7 \text{ kHz} < |f| < 10 \text{ kHz} \\ 0, & otherwise \end{cases}$$

3) Let the signal $x(t) = 10\text{sinc}(1000t)$ be sampled with a sampling frequency of 4000 samples per second. Determine the most general class of reconstruction filters for perfect reconstruction of this signal.

4) A band-limited lowpass signal $x(t)$ with the bandwidth of 50Hz is sampled at the Nyquist rate and the resulting sample values are $x(nT_s)$. (a) Find $x(0.007)$. (b) Is this signal power-type or energy-type? Find its power or energy content. (c) find $x(t)$ and plot it (using e.g. Matlab) over the interval [-80,80] ms.

$$x(nT_s) = \begin{cases} -1 & -4 \le n < 0 \\ 1, & 0 < n \le 4 \\ 0, & otherwise \end{cases}$$

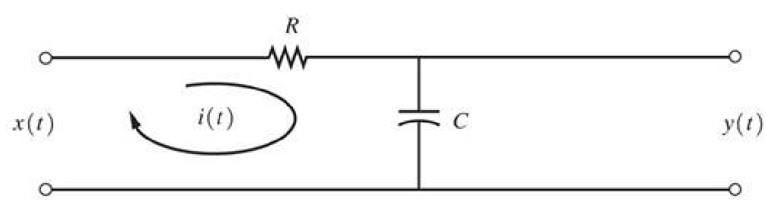
5) The signal $x(t)$ is passed through an RC low-pass filter in the figure below, where $R = 1\,\Omega$ and $C = 1\text{ F}$,

$$x(t) = 0.5 + 1.5\sin\left(2\pi t / 3\right) + 0.5\cos\left(2\pi t\right)$$

(a) What is the input PSD ?

(b) What is the output PSD ?

(c) What is the output power?



(a) RC Low-Pass Filter

6) Let $x(t)$ be a signal with Nyquist rate f_N. Find the Nyquist rate for each of the following signals:

a) $\dfrac{d^2x(t)}{dt^2}$, b) $x(t) \cdot x^2(t-5)$, c) $x(t) * x^2(t-5)$, where * denotes convolution, d) $\int_{-\infty}^{t} x(\tau) d\tau$

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Please include in your solutions all the intermediate results and their numerical values (if applicable). **Detailed solutions with explanations are required**, not just the final answers/equations; **all symbols used must be defined**, including units used, if applicable (e.g. f = frequency [Hz]). Missing explanations, symbol definitions/units will be penalized. Your answers should demonstrate the full extent of your knowledge and the latter will determine your marks.

Plagiarism (i.e. “cut-and-paste” from a student to a student, other forms of “borrowing” the material for the assignment) is absolutely unacceptable and will be penalized. Each student is expected to submit his own solutions. If two (or more) identical or almost identical sets of solutions are found, each student involved receives 0 (zero) for that particular assignment. If this happens twice, the students involved receive 0 (zero) for the entire assignment component of the course in the marking scheme and the case will be send to the Dean’s office for further investigation.

Please read appropriate chapters of the textbook first, study all the examples, attempt to do them with the closed book. Remember the learning efficiency pyramid!

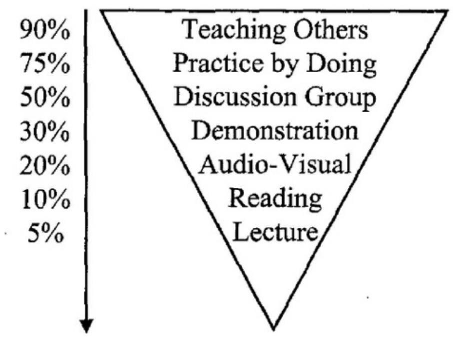

Figure 1. The Learning Pyramid, adapted from David Sousa, *How the Brain Learns*, Reston, VA, The National Association of Secondary School Principals, 1995, ISBN 0-88210-301-6.