ELG7177: MIMO Comunications

Lecture 7

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MIMO Rx: V-BLAST

- How to recover Tx data at Rx?
- Simple Tx strategy?
- BLAST: Bell Labs Layered Space-Time
- Successive interference cancellation (SIC)

V-BLAST

- MIMO channel:

$$\mathbf{y} = \sum_{k=1}^{m} \mathbf{h}_k x_k + \boldsymbol{\xi} \tag{1}$$

- No Tx CSI, full Rx CSI
- Isotropic signaling at Tx: $\mathbf{R} = \frac{P}{m}\mathbf{I}$
- Its capacity

$$C_{iso} = \log\left|\mathbf{I} + \frac{P}{m}\mathbf{H}\mathbf{H}^{+}\right| = \log\left|\mathbf{I} + \frac{P}{m}\sum_{k}\mathbf{h}_k\mathbf{h}_k^{+}\right| \tag{2}$$

- Optimal Rx to recover Tx data (via x_k)?

Successive Interference Cancellation (SIC)

- Sequential detection, with decision feedback:
  - detect stream 1 treating all others as interference
  - subtract its signal
  - detect stream 2 treating all others as interference, etc.
- Information lossless: achieves capacity, by implementing the chain rule of MI.
- Arbitrary low error rate, via capacity-approaching (scalar) codes for each stream.

SIC Rx: 2 Tx Antennas Case

The $m = 2$ channel:

$$\mathbf{y} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \boldsymbol{\xi},\ P_1 = P_2 = P/2 \tag{3}$$

The SIC receiver:

1. Detect Tx 1 treating Tx 2 as interference:

$$y_{r1} = \mathbf{w}_1^+ \mathbf{y} = y_{s1} + y_{n1} \tag{4}$$

$y_{s1} = \mathbf{w}_1^+ \mathbf{h}_1 x_1 =$ signal part,
$y_{n1} = \mathbf{w}_1^+ (\mathbf{h}_2 x_2 + \boldsymbol{\xi}) =$ "noise" part,
$\mathbf{w}_1 = \mathbf{R}_{n1}^{-1} \mathbf{h}_1 = (\sigma_0^2 \mathbf{I} + P_2 \mathbf{h}_2 \mathbf{h}_2^+)^{-1} \mathbf{h}_1 =$ max. SNR (MMSE) beamformer,

$$R_1 < \log(1 + \gamma_1) = C_1, \tag{5}$$

$$\gamma_1 = \frac{\overline{|y_{s1}|^2}}{\overline{|y_{n1}|^2}} = P_1 \mathbf{h}_1^+ (\sigma_0^2 \mathbf{I} + P_2 \mathbf{h}_2 \mathbf{h}_2^+)^{-1} \mathbf{h}_1 = \text{stream } k \text{ SNR}$$

SIC Rx: 2 Tx Antennas Case

2. Subtract detected signal:

$$\mathbf{y}' = \mathbf{y} - \mathbf{h}_1 x_1 = \mathbf{h}_2 x_2 + \boldsymbol{\xi} \tag{6}$$

3. Detect Tx 2:

$$R_2 < \log\left(1 + \frac{P_2 |\mathbf{h}_2|^2}{\sigma_0^2}\right) = C_2 \tag{7}$$

The sum rate/capacity:

$$R_1 + R_2 < C_1 + C_2 = \log\left|\mathbf{I} + \frac{P}{m\sigma_0^2}\sum_k \mathbf{h}_k \mathbf{h}_k^+\right| = C_{iso} \tag{8}$$

i.e. the isotropic MIMO capacity.

max-SNR/MMSE Beamformer (Filter)

Consider the SIMO channel with correlated noise:

$$\mathbf{y} = \mathbf{h}x + \mathbf{z} \tag{9}$$

where $\mathbf{z}$ = correlated noise (includes interference),

$$\mathbf{z} \sim CN(0, \mathbf{R}_z), \ \mathbf{R}_z = \overline{\mathbf{z}\mathbf{z}^+} \tag{10}$$

Rx beamformer $\mathbf{w}$:

$$y_r = \mathbf{w}^+\mathbf{y} = \mathbf{w}^+\mathbf{h}x + \mathbf{w}^+\mathbf{z} = y_s + y_n \tag{11}$$

and the output SNR:

$$\gamma = \frac{\overline{|y_s|^2}}{\overline{|y_n|^2}} = \frac{P_x|\mathbf{w}^+\mathbf{h}|^2}{\mathbf{w}^+\mathbf{R}_z\mathbf{w}} \tag{12}$$

max-SNR/MMSE Beamformer (Filter)

How to maximize the SNR γ ?

$$\max_{\mathbf{w}} \frac{P_x |\mathbf{w}^+ \mathbf{h}|^2}{\mathbf{w}^+ \mathbf{R}_z \mathbf{w}} = ? \tag{13}$$

Q: prove that

$$\max \gamma = P_x \mathbf{h}^+ \mathbf{R}_z^{-1} \mathbf{h}, \ \mathbf{w}^* = \mathbf{R}_z^{-1} \mathbf{h} \tag{14}$$

which is the max-SNR beamformer.

MMSE beamformer[1]: $\mathbf{w}_{MMSE} = \alpha \mathbf{w}^*$, i.e. scaled max-SNR beamformer.

Q2: Consider the special case of $\mathbf{R}_z = \sigma_0^2 \mathbf{I}$. Do you get an expected result?

[1] A.H. Sayed, Fundamentals of Adaptive Filtering, Wiley-IEEE Press, 2003.

SIC Rx (V-BLAST): General Case

1. Step k: subtract already detected signals:

$$\mathbf{y}'_k = \mathbf{y} - \sum_{i=1}^{k-1} \mathbf{h}_i x_i = \mathbf{h}_k x_k + \sum_{i=k+1}^{m} \mathbf{h}_i x_i + \boldsymbol{\xi} \qquad (15)$$

2. Detect signal x_k treating yet-to-be detected signals as interference:

$$y_{rk} = \mathbf{w}_k^+ \mathbf{y}'_k, \ \mathbf{w}_k = \mathbf{R}_{\boldsymbol{\xi}'_k}^{-1} \mathbf{h}_k = \text{max-SNR beamformer} \qquad (16)$$

$$\mathbf{R}_{\boldsymbol{\xi}'_k} = \sigma_0^2 \mathbf{I} + \sum_{i=k+1}^{m} P_i \mathbf{h}_i \mathbf{h}_i^+ = \text{noise covariance} \qquad (17)$$

$$R_k < \log(1 + \gamma_k) = C_k, \ \gamma_k = P_k \mathbf{h}_k^+ \mathbf{R}_{\boldsymbol{\xi}'_k}^{-1} \mathbf{h}_k = \text{SNR} \qquad (18)$$

SIC Rx (V-BLAST): General Case

The sum capacity:

$$\sum_{k=1}^{m} C_k = \sum_{k=1}^{m} \log\left(1+\gamma_k\right)$$
$$= \sum_{k=1}^{m} \log\left(1 + P_k \mathbf{h}_k^+ \left(\sigma_0^2 \mathbf{I} + \sum_{i=k+1}^{m} P_i \mathbf{h}_i \mathbf{h}_i^+\right)^{-1} \mathbf{h}_k\right)$$
$$= \log\left|\mathbf{I} + \sum_{k=1}^{m} P_k \mathbf{h}_k \mathbf{h}_k^+\right| = C_{iso} \tag{19}$$

i.e. the isotropic MIMO capacity, so that

SIC (V-BLAST) Rx is optimal (information lossless)

SIC Rx & Chain Rule of MI

Why is the SIC Rx optimal, e.i. achieves the capacity?

It is *information-lossless*: implements the chain rule of MI,

$$I(\mathbf{y}; x_1, x_2) = I(\mathbf{y}; x_1) + I(\mathbf{y}; x_2 | x_1) \tag{20}$$

$I(\mathbf{y}; x_1) = I(\mathbf{w}_1^+ \mathbf{y}; x_1) = \log(1 + P_1 \mathbf{h}_1^+ (\sigma_0^2 \mathbf{I} + P_2 \mathbf{h}_2 \mathbf{h}_2^+)^{-1} \mathbf{h}_1)$
$I(\mathbf{y}; x_2 | x_1) = \log\left(1 + |\mathbf{h}_2|^2 P_2 / \sigma_0^2\right)$

Max-SNR (MMSE) beamformer: also information-lossless (optimal), since $\mathbf{w}_1^+ \mathbf{y}$ is sufficient statistics for x_1:

$$\mathbf{y} \perp x_1 \mid \mathbf{w}_1^+ \mathbf{y} \tag{21}$$

SIC Rx & Chain Rule of MI

In the general case, the chain rule is

$$I(\mathbf{y}; x_1, .., x_m) = \sum_{k=1}^{m} I(\mathbf{y}; x_k | x_1, .., x_{k-1}) \quad (22)$$

$$\begin{aligned} I(\mathbf{y}; x_k | x_1, .., x_{k-1}) &= I(\mathbf{w}_k^+ \mathbf{y}'_k; x_k | x_1, .., x_{k-1}) \\ &= \log\left(1 + P_k \mathbf{h}_k^+ \mathbf{R}_{\xi'_k} \mathbf{h}_k\right) \end{aligned} \quad (23)$$

Sufficient statistics of stream k is $\mathbf{w}_k^+ \mathbf{y}'_k$:

$$\mathbf{y}'_k \perp x_k \mid \mathbf{w}_k^+ \mathbf{y}'_k \quad (24)$$

SIC Rx (V-BLAST)

Q1: assume that the channel is orthogonal, i.e. $\mathbf{h}_i^+ \mathbf{h}_j = 0$ for any $i \neq j$. Show that the optimal Rx cancels the interference from all yet-to-be detected signals, i.e.

$$\mathbf{w}_k^+ \mathbf{h}_j = 0, \ \forall \ j > k \tag{25}$$

This is called "Zero-Forcing" (ZF). Find the per-stream SNR γ_k and capacity C_k in this case. How can you explain these expressions?

Q2: show that the same results hold for the general full-rank channel at high SNR, $\sigma_0 \to 0$.

Q3: what about low SNR regime, i.e. $\sigma_0 \to \infty$? What is the optimal Rx in this case?

Summary

- V-BLAST architecture
- SIC/V-BLAST Rx:
  - subtract interference from already detected symbols
  - filter out interference from yet-to-be detected symbols
  - detect current symbol
- max-SNR(MMSE) beamformer
- Special cases

Reading

- D. Tse, P. Viswanath, Fundamentals of Wireless Communications: Ch. 7.1-2, 8.1-8.3, Appendix A and B.
- A.H. Sayed, Fundamentals of Adaptive Filtering, Wiley-IEEE Press, 2003, Ch. 1.3.2, 2.4, 2.6.
- J.R. Barry, E.A. Lee, D.G. Messerschmitt, Digital Communications (3rd Ed.), Kluwer, Boston, 2004. - Ch. 10.3.