ELG7177: MIMO Comunications

Lecture 4

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MISO: Tx antenna array

- multiple Tx antennas
- single Rx antenna
- Tx beamforming

MISO Channel Model

$$y(t) = \mathbf{h}^{+}\mathbf{x}(t) + \xi(t) \qquad (1)$$

$\mathbf{x}(t)$ = Tx signal (vector); $x_i(t)$ = Tx signal at i-th antenna
$y(t)$ = Rx signal (scalar)
$\mathbf{h}^{+}$ = fixed channel vector; h_i^* = channel gain from i-th Tx antenna to the Rx antenna
$\xi(t)$ = Rx noise (scalar)

* Compare to the SIMO channel model.

Tx Beamforming

The Tx signal (vector) $\mathbf{x}(t)$ is

$$\mathbf{x}(t) = \mathbf{w} \cdot x(t) \tag{2}$$

$x(t)$ = scalar Tx signal (complex amplitude, from e.g. QAM constellation), carriers the Tx data
$\mathbf{w}$ = Tx beamforming vector.

Entry-wise,

$$x_i(t) = w_i x(t) \tag{3}$$

Tx Beamforming

How to choose $\mathbf{w}$?

From the channel model,

$$y = \mathbf{h}^{+}\mathbf{x} + \xi = \mathbf{h}^{+}\mathbf{w}x + \xi = y_s + y_n \tag{4}$$

and the Rx SNR γ_r is

$$\gamma_r = \frac{P_s}{P_n} = \frac{\overline{|y_s|^2}}{\overline{|y_n|^2}} = |\mathbf{h}^{+}\mathbf{w}|^2\gamma \tag{5}$$

where $\gamma = \sigma_x^2/\sigma_0^2$ is the Rx SNR with single Tx antenna and $h_1 = 1$;
$|\mathbf{w}| = 1$ to maintain Tx power σ_x^2.

How to maximize γ_r?

Tx Beamforming

- Maximizing γ_r: follow the SIMO approach,

$$\gamma_r = |\mathbf{h}^+\mathbf{w}|^2\gamma \leq |\mathbf{w}|^2|\mathbf{h}|^2\gamma = |\mathbf{h}|^2\gamma \tag{6}$$

with equality iff

$$\mathbf{w} = \mathbf{w}_* = \mathbf{h}/|\mathbf{h}| \tag{7}$$

- (7): max. SNR beamformer (matched Tx filter), also known as MRT (maximum ratio transmission) .

The Capacity of Tx beamforming

Extended channel: the channel + Tx beamforming.

System capacity: the extended channel capacity,

$$C = \log(1 + \gamma_r) = \log(1 + |\mathbf{h}|^2\gamma) \quad (8)$$

This is the largest rate (SE) the Tx beamforming can deliver.

Same as Rx beamforming.

Can we do better ???

The Capacity of MISO Channel

The capacity is

$$C = \max_{p(x)} I(X;Y) \text{ s.t. } \overline{X^+X} \leq P \tag{9}$$

X = the random Tx vector,
$X^+X = \sum_i |X_i|^2$,
$\overline{X^+X} = \overline{\sum_i |X_i|^2} = \mathrm{tr}\,\mathbf{R_x}$ = the total Tx power.

Same as for SIMO?

The Capacity of MISO Channel

The capacity is

$$C = \max_{p(x)} I(X;Y) \text{ s.t. } \operatorname{tr} \mathbf{R}_x \leq P$$

but: **How to find the max???**

Key:

$$H(Y|X) = H(\Xi) = \log \sigma_0^2 + \log(\pi e) \tag{10}$$

$$H(Y) \leq \log \sigma_y^2 + \log(\pi e) = \log(\sigma_x^2 \mathbf{h}^+ \mathbf{R}_x \mathbf{h} + \sigma_0^2) + \log(\pi e) \tag{11}$$

where $\sigma_y^2 = \operatorname{var}\{Y\} = \sigma_x^2 \mathbf{h}^+ \mathbf{R}_x \mathbf{h} + \sigma_0^2$, so that

$$I(X;Y) = H(Y) - H(\Xi) \leq \log \frac{\sigma_y^2}{\sigma_0^2} = \log(1 + \gamma \mathbf{h}^+ \mathbf{R}_x \mathbf{h}) \tag{12}$$

and the UB is achieved by $X \sim CN(0, \mathbf{R}_x)$.

The Capacity of MISO Channel

Thus, $X \sim CN(0, \mathbf{R}_x)$ is optimal and the capacity is

$$C = \max_{\mathbf{R}_x} \log(1 + \gamma_x \mathbf{h}^+ \mathbf{R}_x \mathbf{h}) \text{ s.t. } \operatorname{tr} \mathbf{R}_x \leq P \tag{13}$$

where $\gamma_x = \sigma_x^2/\sigma_0^2$.

But: **How to find the max???**

Note: optimization variable is the Tx covariance matrix $\mathbf{R}_x$.

The Capacity of MISO Channel

How to find the max???

A key is:

$$\mathbf{h}^{+}\mathbf{R}_x\mathbf{h} \leq \lambda_1(\mathbf{R}_x)|\mathbf{h}|^2 \leq P|\mathbf{h}|^2 \tag{14}$$

and both UBs are achieved by $\mathbf{R} = \mathbf{R}_* = P\mathbf{u}\mathbf{u}^{+}$, where $\mathbf{u} = \mathbf{h}/|\mathbf{h}|$.

Hence,

- an optimal Tx strategy is the Tx beamforming along $\mathbf{h}$,

$$\mathbf{R}_* = P\mathbf{u}\mathbf{u}^{+} \leftrightarrow \mathbf{w}_* = \mathbf{h}/|\mathbf{h}| \tag{15}$$

- the MISO capacity is

$$C = \log(1 + \gamma|\mathbf{h}|^2) \tag{16}$$

- an optimal input is $X \sim CN(0, \mathbf{R}_*)$.

The Capacity of MISO Channel

Compare this with the SIMO channel capacity!

An Example: Free-Space Propagation

Repeat the free-space example of the SIMO channel and explain what the optimal Tx is doing precisely. Give the capacity expression in this case.

Observe that, for the MISO channel, the classical phased array is optimal in free space in the information-theoretic sense as well (to maximize the Tx rate).

Example 2: Isotropic Signaling

Consider another Tx strategy, the isotropic signaling:

$$\mathbf{R} = \frac{P}{m}\mathbf{I} \tag{17}$$

which spreads out the total Tx power equally in all directions (why?).

Find the largest achievable rate for this strategy and compare it to the capacity. Quantify the difference (or the advantage offered by the optimal strategy above).

Summary

- MISO channel
- Tx beamforming, its capacity
- the MISO channel capacity
- optimal Tx strategy
- examples