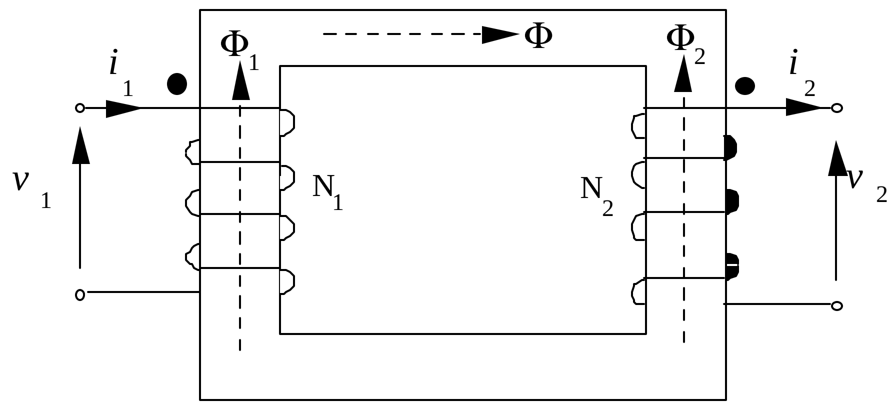
# CHAPTER 2
# TRANSFORMERS

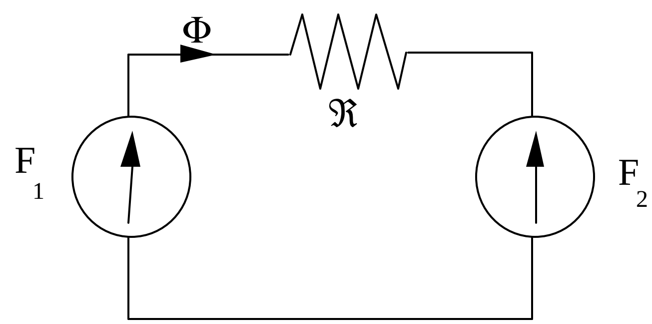
## 2.1 Ideal Transformer Equations

### 2.1.1 Voltage and Current Equations

A basic transformer consists of two coils wound around a common magnetic core. A simplified representation of a transformer is shown in **Figure 2.1a** and the corresponding magnetic circuit is shown in **Figure 2.1b**.




a) Winding Configuration




b) Magnetic Equivalent Circuit

**Figure 2.1 Basic Transformer**

Coil 1 has $N_1$ turns and a current of $i_1$ and a voltage of $v_1$. Similarly, coil 2 has $N_2$ turns and a current of $i_2$ and a voltage of $v_2$. The coils are connected in opposing polarity as shown in Figure 2.1a, such that the magnetic flux produced by each coil will be in opposite polarities. By convention a dot is used to indicate one end of each coil such that if current goes into the dotted end of one coil it will come out the dotted end of the other coil. As a first approximation we assume that there is no leakage flux, and thus;

$$\phi_1 = -\phi_2 = \phi$$

The voltage induced in each winding can be determined;

$$v_1 = N_1 \frac{\partial \Phi_1}{\partial t} = N_1 \frac{\partial \Phi}{\partial t}$$

$$v_2 = -N_2 \frac{\partial \Phi_2}{\partial t} = N_2 \frac{\partial \Phi}{\partial t}$$

The above equations can be combined to show that;

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = N$$

Where N is defined as the turns ratio;

$$N \equiv \frac{N_1}{N_2}$$

Also, the magnetic equivalent circuit shown in Figure 2.1b can be used to obtain the magnetic circuit equation;

$$F_1 = F_2 + \phi \, \Re$$

However, for an 'ideal' transformer, we assume that $\mu_r = \infty$ and thus $\Re = 0$.
Therefore;

$$F_1 = F_2$$

Or;

$$N_1 i_1 = N_2 i_2$$

The preceeding equation can be rewritten as;

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{N}$$

Where:

$$N \equiv \frac{N_1}{N_2}$$

The results of the preceding equations for an ideal transformer can be summarized as;

$$v_1 = Nv_2$$

$$i_1 = \frac{i_2}{N}$$

Thus current on one side of a transformer can be 'reflected' to the other side by dividing by the turns ratio. Conversely voltage on one side of a transformer can be 'reflected' to the other side by multiplying by the turns ratio.

## 2.1.2 Load Transformation

The electrical equivalent circuit for an ideal transformer is shown in Figure 2.2. Again the dot convention indicates that if current goes into the dotted end of one winding it will come out the dotted end of the other winding. By convention we refer to one winding as the primary and the other winding as the secondary. The primary winding is usually the one that has power flowing into it. The secondary winding is usually the one that has power flowing out of it.

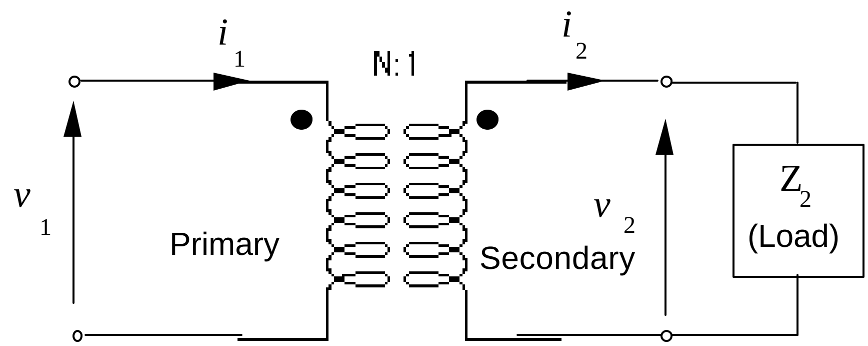



**Figure 2.2 Basic Equivalent Circuit For A Transformer**

In the circuit shown in **Figure 2.2** the secondary winding has a load of impedance $Z_2$ connected to it. Thus on the secondary side of the transformer;

$$v_2 = i_2 Z_2$$

Substitute for $v_2$ and $i_2$ to obtain;

$$\frac{v_1}{N} = i_1 N Z_2$$

This equation can be rewritten as;

$$\frac{v_1}{i_1} = N^2 Z_2$$

This equation represents an impedance $Z_1$ on the primary side of the transformer, where;

$$Z_1 = N^2 Z_2$$

Thus an impedance on one side of the transformer can be reflected to the other side by multiplying it by the square of the turns ratio.

## 2.2 Non-ideal Elements of a Transformer

### 2.2.1 Magnetizing Current

In a real transformer the equation for the magnetomotive force in the magnetic path cannot be simplified because $\mu_r \neq \infty$ and thus $\Re \neq 0$ and therefore;

$$F_1 = F_2 + \phi\, \Re_m$$

Where $\Re_m$ represents the reluctance of the path through the magnetic material.

Substitute for;

$$F_1 = N_1\, i_1$$

$$F_2 = N_2\, i_2$$

to obtain;

$$N_1\, i_1 = N_2\, i_2 + \phi\, \Re_m$$

It is convenient, for modelling purposes, to substitute for;

$\phi\, \Re_m = F_m$ the magnetomotive force "across" the iron

$= N_1\, i_m$

Where $i_m$ is referred to as the magnetizing current, the current required to maintain a magnetomotive force across the iron. In otherwords the magnetizing current is the current required to maintain magnetic flux in a real material that has less than infinite permeability.

Substitute for $\phi\, \Re_m$ into the equation for magnetomotive force in the magnetic path to obtain;

$$N_1\, i_1 = N_2\, i_2 + N_1\, i_m$$

Which can be simplified to;

$$i_1 = \frac{N_2}{N_1} i_2 + i_m$$

$$= \frac{i_2}{N} + i_m$$

The preceding equation can best be represented by incorporating an inductance, referred to as the magnetizing inductance $L_m$ , connected in parallel across the input voltage as shown in the equivalent circuit in **Figure 2.3**

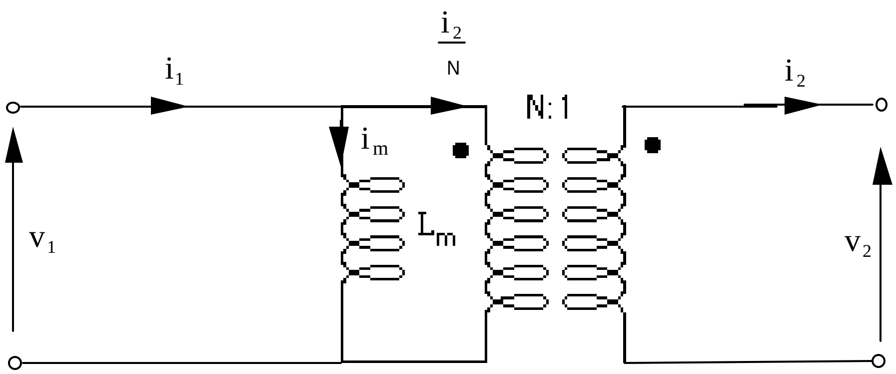



**Figure 2.3 Transformer Circuit With Magnetizing Inductance**

Note that in general $L_m$ can be shown on either the primary side or the secondary side and can be readily reflected from one side to the other by appropriately applying the turns ratio.

Furthermore, $L_m$ can be calculated from the basic expression for inductance;

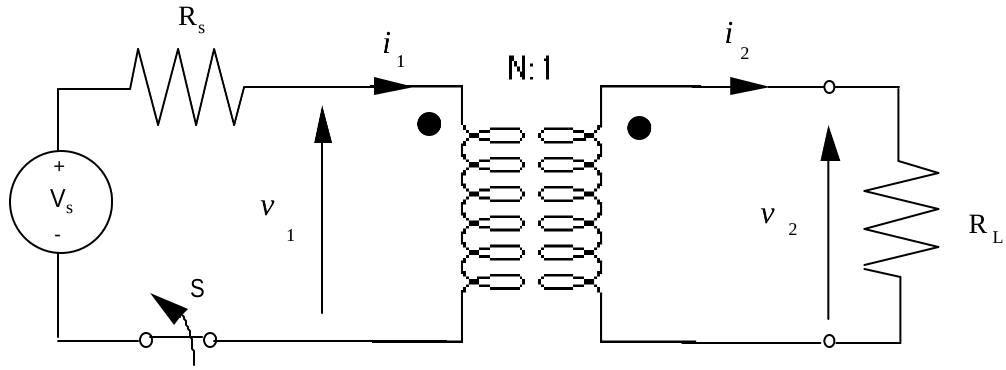
$$L_m = \frac{N_1^2}{\Re_m}$$

$$= \frac{N_1^2 \mu_0 \mu_r A}{\lambda}$$

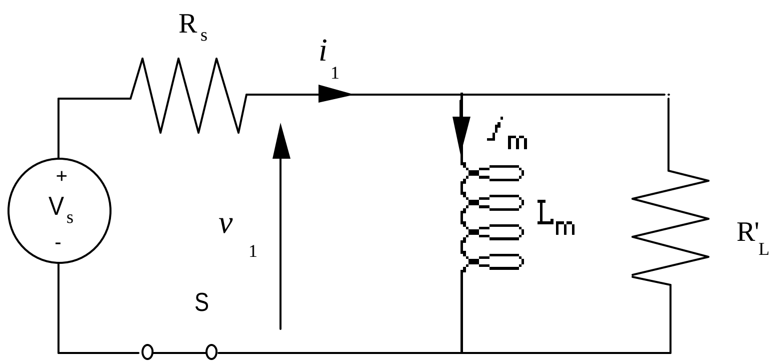
Example 2.1

A transformer has a magnetizing inductance of 100µH, referred to the primary. The turns ratio is 5:1. It is used in the circuit shown in **Figure 2.4a**, where switch S has been closed long enough to reach steady state and is opened at t=0. Determine the expression for the output voltage, $V_2$ for t > 0. Given that:

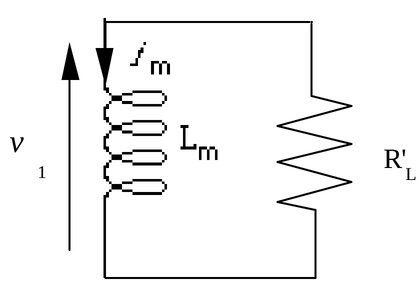
$R_L = 4\,\Omega$, $R_s = 10\,\Omega$, $V_s = 50$ V,




a) Circuit for example problem 2.1




b) Equivalent circuit, reflected to the primary, before switch S opens




c) Equivalent circuit, reflected to the primary, after switch S opens

**Figure 2.4 Transformer Circuit for Example Problem 2.1**

Solution to example 2.1

The equivalent circuit for $t=0^-$ is shown in **Figure 2.4b**. The steady state current $i_m(t)$ can be determined;

$$i_m(t) = \frac{V_s}{R_s} = \frac{50}{10} = 5\text{ A}$$

Also the input voltage to the transformer, $V_1$ can be determined;

$$V_1 = L_m \frac{\partial i_m}{\partial t} = 0$$

And, therefore;

$$V_2 = \frac{V_1}{N} = 0$$

At $t=0^+$ the switch opens, and the equivalent circuit becomes as shown in **Figure 2.4c**. The current $i_m(0^+)$ can not change instantaneously, therefore;

$$i_m(0^+) = i_m(0^-) = 5\text{ A}$$

The magnetizing current, $i_m(t)$, will now have to flow through the circuit consisting of the magnetizing inductance, $L_m$, and the load resistance, $R_L$, (reflected to the primary). Thus the expression for $i_m(t)$ becomes;

$$i_m(t) = 5\,\varepsilon^{-R'_L t/L_m}$$

Where $R'_L$ is the load resistance reflected to the primary side.

And thus the expression for $V_2$ can be determined;

$$V_1 = L_m \frac{\partial i_m}{\partial t} = -5\,R'_L\,\varepsilon^{-R'_L t/L_m} = -5\,N^2 R_L\,\varepsilon^{-N^2 R_L t/L_m}$$

$$= 5 \times 5^2 \times 4 \times \varepsilon^{\left[-\frac{5^2 \times 4 \times t}{100 \times 10^{-6}}\right]} = -500\varepsilon^{-10^6 t}$$

$$V_2 = \frac{V_1}{N} = -\frac{500\varepsilon^{-10^6 t}}{5} = -100\varepsilon^{-10^6 t}$$

### 2.2.1 Core Losses

Any real magnetic component such as a transformer will have losses associated with the magnetic flux. These losses are called core losses because they occur in the flux carrying magnetic material which is usually the "core" of a transformer. They are also sometimes called the 'iron' losses because the cores of tranformers used to be made of iron or similar material. The core losses consist of two parts; hysteresis losses and eddy current losses.

Hysteresis losses, $P_h$, are due to the non-ideal nature of the B-H curve in all practical magnetic materials, where;

$$P_h = f\,\text{Vol.}\int H\partial B \propto fIV \propto fV^2 \propto V^2$$

Where Vol. represents the volume of the magnetic material, and $f$ represents the frequency of the magnetic flux, which, in linear systems, is the same as the frequency of the applied voltage, $V$.

Eddy current losses, $P_e$, are the losses in the magnetic material due to electric currents induced in the magnetic material by the changes in magnetic flux. The expression for $P_e$ is rather complex but suffice it say that;

$$P_e \propto f^2 \hat{B}^2 \propto V^2$$

Thus, the total core losses, $P_c$, can be expressed as;

$$P_c = P_e + P_h \propto V^2$$

Which can be represented as;

$$P_c = \frac{V^2}{R_c}$$

Where $R_c$ is a resistor representing the core losses. To satisfy the above equation this resistor would have to be connected in parallel across the input voltage as shown in the equivalent circuit in **Figure 2.5**

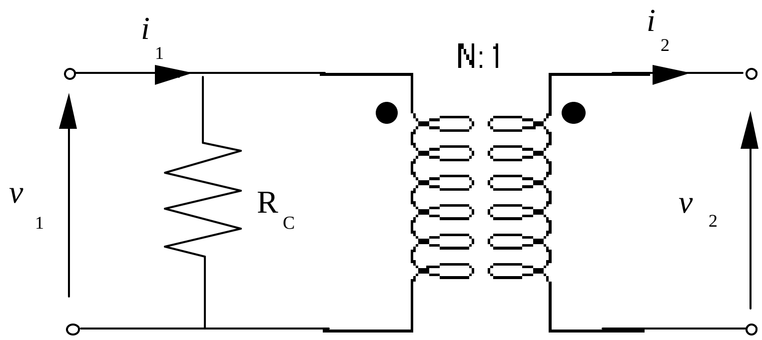



**Figure 2.5 Transformer Equivalent Circuit With Core Loss Resistance, $R_c$**

Note that in general $R_c$ can be shown on either the primary side or the secondary side and can be readily reflected from one side to the other by appropriately applying the turns ratio.

Example Problem 2.2

A transformer has an open circuit on the secondary and 115 Vac, 60 Hz on the primary. The total power drawn by the transformer is 4 watts. Determine $R_c$ .

Solution;

The equivalent circuit for this problem is the same as in **Figure 2.5** and the only power dissipating element is $R_c$ . Thus the power drawn by the transformer is;

$$P = \frac{V^2}{R_c}$$

Solve for;

$$R_c = \frac{V^2}{P} = \frac{115^2}{4} = 3{,}306\ \Omega$$

### 2.2.3 Leakage Inductance

The leakage inductance in a transformer is used to represent the effects of magnetic flux which is produced by one winding but does not couple with the other winding. This flux is referred to as leakage flux and is shown diagramatically in **Figure 2.6**.

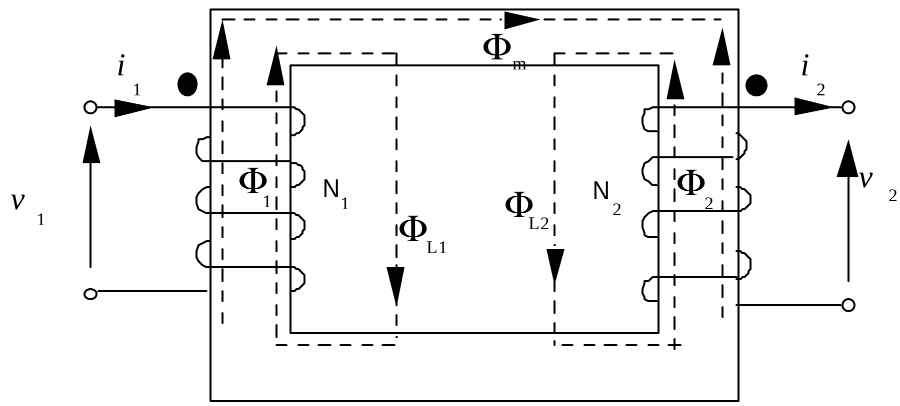



**Figure 2.6 Leakage Flux In A Transformer**

The expressions for voltages induced in each winding become;

$$v_1 = N_1\frac{\partial\Phi_1}{\partial t} = N_1\frac{\partial(\Phi_m + \Phi_{L1})}{\partial t} = N_1\frac{\partial\Phi_m}{\partial t} + N_1\frac{\partial\Phi_{L1}}{\partial t}$$

$$v_2 = -N_2\frac{\partial\Phi_2}{\partial t} = -N_2\frac{\partial(\Phi_{L2} - \Phi_m)}{\partial t} = N_2\frac{\partial\Phi_m}{\partial t} - N_2\frac{\partial\Phi_{L2}}{\partial t}$$

Where $\phi_m$ is the flux that is common to both windings and $\phi_{L1}$ is the flux produced by the primary winding that is not coupled to the secondary winding and similarly $\phi_{L2}$ is the flux produced by the secondary winding that is not coupled to the primary winding. In a linear system $\phi_{L1}$ and $\phi_{L2}$ are proportional to $i_1$ and $i_2$ respectively and thus the preceeding equations can be rewritten in the form;

$$v_1 = N_1\frac{\partial\Phi_m}{\partial t} + L_1\frac{\partial i_1}{\partial t}$$

$$v_2 = N_2\frac{\partial\Phi_m}{\partial t} - L_2\frac{\partial i_2}{\partial t}$$

The equation for $v_2$ can be solved for;

$$\frac{\partial \Phi_m}{\partial t} = \frac{v_2}{N_2} + \frac{L_2}{N_2}\frac{\partial i_2}{\partial t}$$

Substitute for $\frac{\partial \Phi_m}{\partial t}$ into the equation for $v_1$ to obtain;

$$v_1 = \frac{N_1}{N_2} v_2 + L_1 \frac{\partial i_1}{\partial t} + \frac{N_1}{N_2} L_2 \frac{\partial i_2}{\partial t}$$

The equivalent circuit for the above equation is shown in **Figure 2.7**. This circuit represents the effects of leakage inductance but ignores magnetizing inductance.

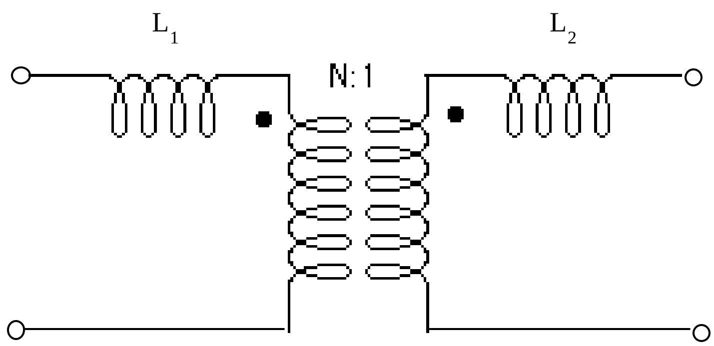



**Figure 2.7 Leakage Inductance In A Transformer**

From the analysis of section 2.2.1 we can superimpose the magnetizing inductance as shown in **Figure 2.8**.

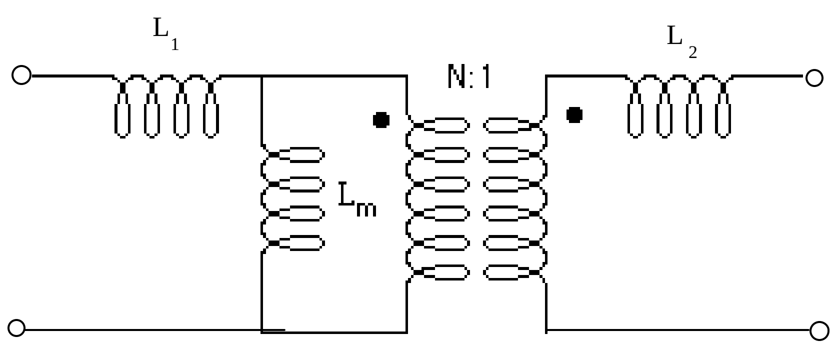



**Figure 2.8 Leakage and Magnetizing Inductances in a Tranformer**

For most transformers we can assume that, for any value of current;

$$\phi_{L1} , \phi_{L2} << \phi_m$$

And thus, also;

$$L_1 , L_2 << L_m$$

Therefore a good approximation can still be maintained by bringing the secondary leakage inductance, $L_2$ , to the primary side and combining it with the primary leakage inductance, $L_1$ , to form one combined leakage inductance, $L_L$ . The resultant equivalent circuit is shown in **Figure 2.9**.

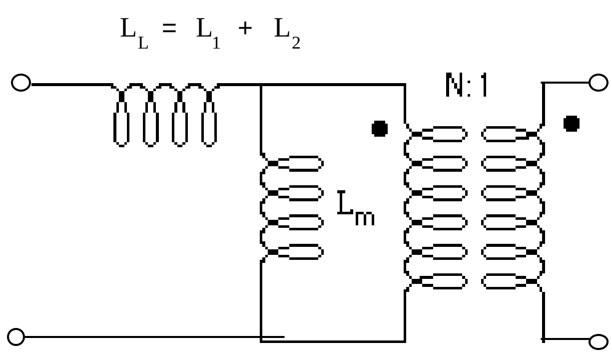



**Figure 2.9 Simplified Representation of Leakage Inductance**

Example problem 2.3

You are given a 60 Hz transformer with a turns ration of 5 ( high voltage is on the input side) and a leakage inductance of 140 mH, referred to the input. Assume that the core losses and magnetizing current are negligible. Determine the input voltage required to produce an output voltage of 120 V at 50 A.

Solution;

The equivalent circuit for this transformer is shown in **Figure 2.10**. The equation for the input voltage $v_{in}$ , can be determined;

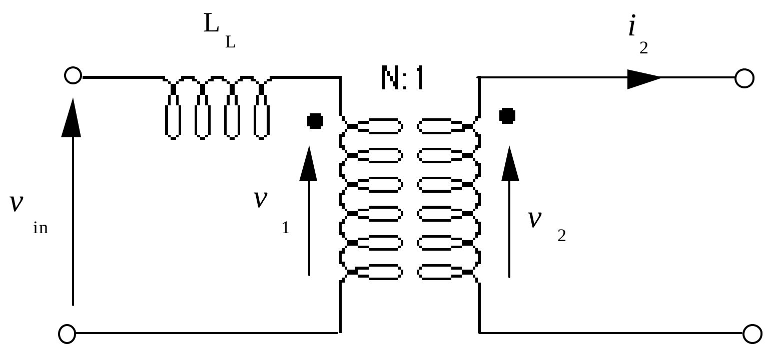



**Figure 2.10 Equivalent Circuit for Transformer of Example Problem 2.4**

$$v_{in} = Nv_2 + j\omega L_L \frac{i_2}{N}$$

Substitute for $v_2$ , $i_2$ , and $\omega$ to obtain;

$$v_{in} = 5\times120 + j2\pi\times60\times140\times10^{-3}\times\frac{50}{5}$$
$$= 600 + j528 = 799\angle^{41.3°}\ \mathrm{V}$$

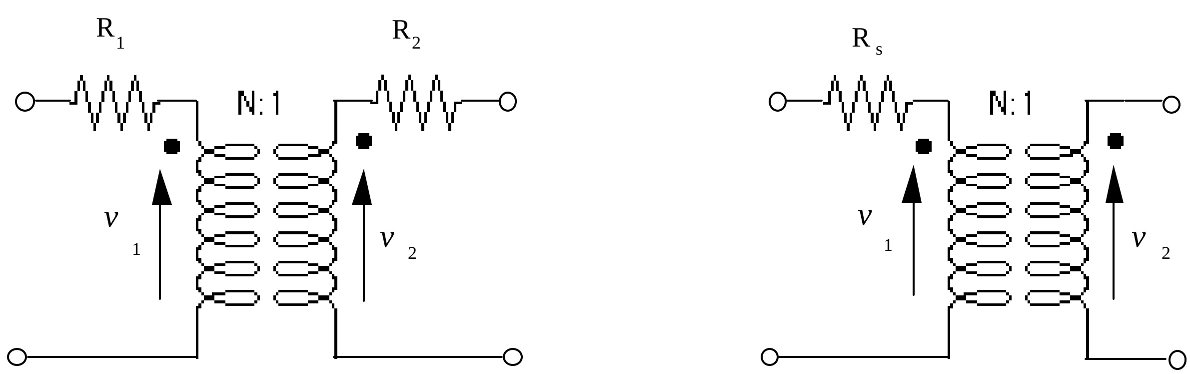
2.2.4 Wire Resistance

In all real transformers the windings consist of loops of wire which all have a finite resistance, except superconducting materials. The resistance of the wires is best represented in an equivalent circuit by series resistances, $R_1$ and $R_2$ as shown in **Figure 2.11a**. Again we can assume that $R_1$ and $R_2$ can be combined into a single equivalent series resistance $R_s$ as shown in **Figure 2.11b**, where:

$$R_s = R_1 + \left[\frac{N_1}{N_2}\right]^2 R_2 \qquad \text{referred to the primary side,}$$

or,

$$R_s' = R_2 + \left[\frac{N_2}{N_1}\right]^2 R_1 \qquad \text{referred to the secondary side.}$$




a) Equivalent circuit with wire resistances

b) Simplified equivalent circuit

**Figure 2.11 Transformer Equivalent Circuit For Wire Resistance**

The values of $R_1$ , $R_2$ and thus $R_s$ can be calculated from the length, diameter and composition of the wires but it is usually more convenient to determine $R_s$ experimentally.

Example problem 2.4:

A 500V/240V transformer has 115Vac applied to the primary side while the secondary side is short circuited. The secondary current is 12A and the primary power drawn in 150W. Assume that the core losses, and magnetizing current are negligible. Determine the series resistance and leakage reactance, referred to the secondary side.

Solution:

The equivalent circuit for the short circuit condition is shown in **Figure 2.12**.

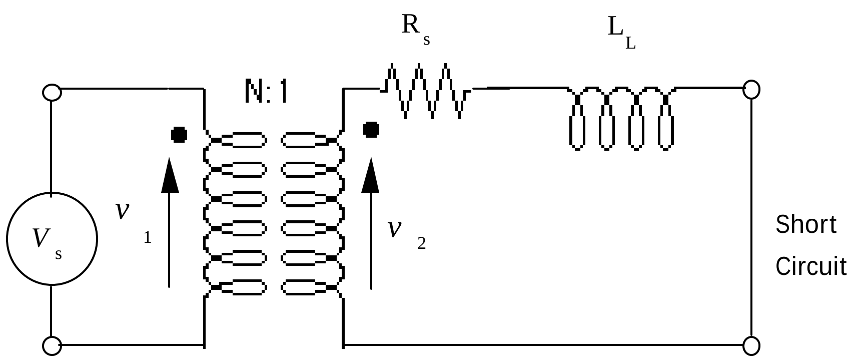



**Figure 2.12 Equivalent Circuit for Example Problem 2.5**

$$N = 500/240 = 2.08$$

Since the only power dissipating element in the equivalent circuit is $R_s$ , therefore;

$$P = i_2^2 R_s$$

Thus, solve for $R_s$ ;

$$R_s = \frac{P}{i_2^2} = \frac{150}{12^2} = 1.04\,\Omega$$

Also:

$$X_L = R_s \tan(\theta)$$

Where:

$$\theta = \cos^{-1}\left[\frac{P_{in}}{V_{in}I_{in}}\right] = \cos^{-1}\left[\frac{PN}{V_s I_2}\right] = \cos^{-1}\left[\frac{150 \times 2.08}{115 \times 12}\right] = 76.9°$$

Substitute for $\theta$ into the equation for $X_L$ to obtain:

$$X_L = R_s \tan(\theta) = 1.04 \times \tan(76.9°) = 4.48\,\Omega$$

## 2.3 Transformer Tests

### 2.3.1 Background

The transformer equivalent circuit derived in the preceding sections is shown in **Figure 2.13** and is valid for linear circuits at low frequencies $\omega$ < 1 kHz, which includes most commercial power applications. In most cases the parameters of the equivalent circuit are most easily determined by experimental measurements.

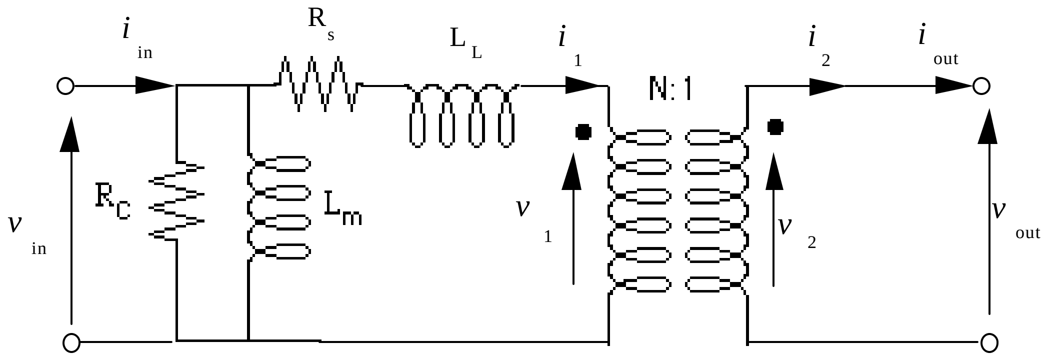



**Figure 2.13 Basic Transformer Equivalent Circuit**

The equivalent circuit consists of low values of series impedances, $R_s$ , $X_L$ and high values of shunt impedances, $R_c$ , $X_m$ and thus lends itself to short circuit and open circuit test to determine the values of these parameters.

2.3.2 Open Circuit Test

In the open circuit test one winding is open circuited and a voltage is applied to the other winding . The voltage is usually, but not necessarily, rated voltage, and the winding is referred to as the powered winding.

Thus, in the equivalent circuit in **Figure 2.13,**

$$i_2 = i_{out} = 0$$

Also;

$$R_s , j\omega L_L << R_c , j\omega L_m$$

Thus, for open circuit tests the equivalent circuit simplifies as shown in **Figure 2.14**.

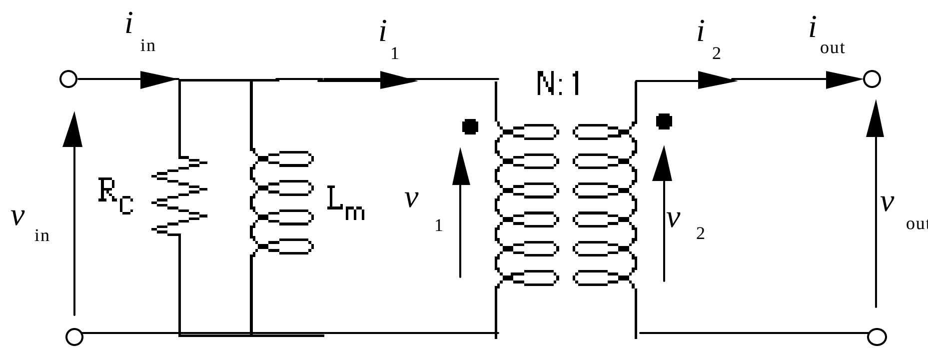



**Figure 2.14 Transformer Equivalent Circuit for Open Circuit Test**

The following parameters can be determined by measuring the input voltage, $V_{in}$ , input current, $I_{in}$ , and input power, $P_{in}$ , in the powered winding and the voltage, $V_{out}$ , in the open circuited winding:

$$N = \frac{V_1}{V_2} = \frac{V_{in}}{V_{out}}$$

$$R_c = \frac{V_{in}^2}{P_{in}} = \frac{V_1^2}{P_{in}} = \frac{N^2 V_2^2}{P_{in}}$$

$$X_m = \omega L_m = \frac{R_c}{\tan(\theta)}$$

Where;

$$\theta = \cos^{-1}\left[\frac{P_{in}}{V_{in} I_{in}}\right]$$

## 2.3.3 Short Circuit Test

In the short circuit test a voltage is applied to one winding and the other winding is short circuited. The voltage is usually much lower than rated voltage to prevent excessive currents in the windings. The voltage is usually (but not always), adjusted such that rated current is applied to the windings. Thus, in the equivalent circuit in **Figure 2.13,**

$$v_2 = v_{out} = 0$$

Also;

$$R_s \, , j\omega L_L \ll R_c \, , j\omega L_m$$

Thus, for short circuit tests the equivalent circuit simplifies as shown in **Figure 2.15**.

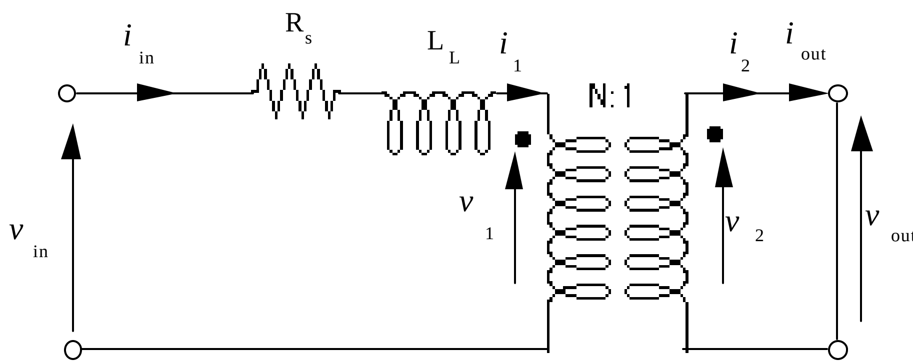



**Figure 2.15 Transformer Equivalent Circuit for Short Circuit Test**

The following parameters can be determined by measuring the input voltage, $V_{in}$, input current, $I_{in}$, and input power, $P_{in}$, in the powered winding and the current, $I_{out}$, in the short circuited winding:

$$N = \frac{I_{out}}{I_{in}}$$

$$R_s = \frac{P_{in}}{I_{in}^2} = \frac{N^2 P_{in}}{I_{out}^2}$$

$$X_L = \omega L_L = R_s \tan(\theta)$$

Where;

$$\theta = \cos^{-1}\left[\frac{P_{in}}{V_{in} I_{in}}\right]$$

`Example problem 2.6:

A 100kVA, 5000V/500V, 60Hz, transformer came with the following test data (measured on the L.V. side);

O.C. TEST: 500V, 2.8A, 1250W
S.C. TEST: 400V, 20A, 4000W

a) Determine the equivalent circuit parameters referred to the primary side.
b) Determine the equivalent circuit parameters referred to the secondary side.

Solution:

The equivalent circuit is shown in **Figure 2.13**. From the ratings data we can determine:

$$N = \frac{V_1}{V_2} = \frac{5000}{500} = 10$$

a) Equivalent circuit parameters on the primary side:
From the open circuit test data we can determine:

$$R_c = \frac{V_1^2}{P_1} = \frac{N^2 V_2^2}{P_2} = \frac{10^2 \times 500^2}{1250} = 20{,}000\ \Omega \text{ on the primary side}$$

$$\theta = \cos^{-1}\left[\frac{P_1}{V_1 I_1}\right] = \cos^{-1}\left[\frac{P_2}{V_2 I_2}\right] = \cos^{-1}\left[\frac{1250}{500 \times 2.8}\right] = 26.8°$$

$$X_m = \frac{R_c}{\tan(\theta)} = \frac{20000}{\tan(26.8°)} = 39{,}650\ \Omega \text{ on the primary side}$$

From the short circuit test data we can determine:

$$R_s = \frac{P_1}{I_1^2} = \frac{N^2 P_2}{I_2^2} = \frac{10^2 \times 4000}{20^2} = 1000\ \Omega \text{ on the primary side}$$

$$\theta = \cos^{-1}\left[\frac{P_1}{V_1 I_1}\right] = \cos^{-1}\left[\frac{P_2}{V_2 I_2}\right] = \cos^{-1}\left[\frac{4000}{400 \times 20}\right] = 60°$$

$$X_L = R_s \tan(\theta) = 1000\tan(60°) = 1732\,\Omega \text{ on the primary side}$$

b) Equivalent circuit parameters on the secondary side:

$$R'_c = \frac{R_c}{N^2} = \frac{20000}{10^2} = 200\,\Omega \text{ on the secondary side}$$

$$X'_m = \frac{X_m}{N^2} = \frac{39650}{10^2} = 397\,\Omega \text{ on the secondary side}$$

$$R'_s = \frac{R_s}{N^2} = \frac{1000}{10^2} = 10\,\Omega \text{ on the secondary side}$$

$$X'_L = \frac{X_L}{N^2} = \frac{1732}{10^2} = 17\,\Omega \text{ on the secondary side}$$

## 2.4 Performance at Power Frequencies

### 2.4.1 Complex Notation

For most commercial power applications the operating frequency is a constant 50 or 60 Hz, (for aircraft 400 or 1000 Hz are more common). Thus the equivalent circuit model of **Figure 2.13** is valid. The input voltage is very close to sinusoidal and it is therefore convenient to express the model in the frequency domain as shown in **Figure 2.16**.

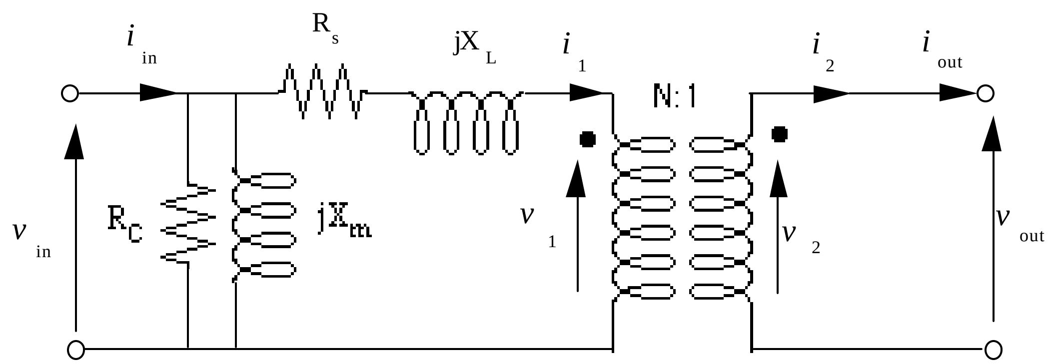



**Figure 2.16 Transformer Equivalent Circuit in Complex Notation**

Solutions for $i_1$, $i_2$ and $v_2$ have to be derived in the complex plane by conventional complex circuit analysis.

## 2.4.2 Per Unit Notation

In many utilities and power systems the absolute values of the equivalent circuit parameters are not as important as their relative value. This is particularly the case in power distribution systems in which there are several distribution voltage levels coupled by transformers. In such cases the actual values of voltages and currents will vary at each distribution level and circuit impedances will vary as the square of the voltage level whereas the *relative* values of circuit parameters will not vary much at all. Also relative values of circuit parameters will be the same on the primary and secondary of any transformer. For power transformers the relative value is referred to as the 'per unit' value.

The per unit values are derived by dividing the absolute value of each parameter by the corresponding value of the transformer rating:

- all voltages are divided by the rated voltage, $V_0$
- all currents are divided by the rated current, $I_0$
- all impedances are divided by the rated impedance, $Z_0$ . Where;

$$Z_0 = \frac{V_0}{I_0}$$

- all VA's (and watts), are divided by the rated VA, $V_0\, I_0$

Example problem 2.7:

Determine the equivalent circuit parameters, in per unit, on both the primary and secondary side for the example problem 2.7.

Solution:

The rated values on the primary side are:

$$V_0 = 5000 \text{ V}$$

$$I_0 = \frac{P_o}{V_0} = \frac{100000}{5000} = 20 \text{ A}$$

Thus;

$$Z_0 = \frac{V_0}{I_0} = \frac{5000}{20} = 250$$

Thus on the primary side the circuit parameters in per unit values are:

$$R_c^0 = \frac{R_c}{Z_0} = \frac{20000}{250} = 80 \text{ p.u. on the primary side}$$

$$X_m^0 = \frac{X_m}{Z_0} = \frac{39650}{250} = 159 \text{ p.u. on the primary side}$$

$$R_s^0 = \frac{R_s}{Z_0} = \frac{1000}{250} = 4 \text{ p.u. on the primary side}$$

$$X_L^0 = \frac{X_L}{Z_0} = \frac{1732}{250} = 6.9 \text{ p.u. on the primary side}$$

The rated values on the secondary side are:

$$V'_0 = 500 \text{ V}$$

$$I'_0 = \frac{P_o}{V'_0} = \frac{100000}{500} = 200 \text{ A}$$

Thus;

$$Z'_0 = \frac{V'_0}{I'_0} = \frac{500}{200} = 2.5$$

Thus on the secondary side the circuit parameters in per unit values are:

$$R_c^{0'} = \frac{R'_c}{Z'_0} = \frac{200}{2.5} = 80 \text{ p.u. on the secondary side}$$

$$X_m^{0'} = \frac{X'_m}{Z'_0} = \frac{396.50}{2.5} = 159 \text{ p.u. on the secondary side}$$

$$R_s^{0'} = \frac{R'_s}{Z'_0} = \frac{10}{2.5} = 4 \text{ p.u. on the secondary side}$$

$$X_L^{0'} = \frac{X'_L}{Z'_0} = \frac{17.32}{2.5} = 6.9 \text{ p.u. on the secondary side}$$

### 2.4.3 Voltage Regulation

Voltage regulation, VR, is defined as the change in output voltage from no load to full load, usually expressed as a fraction of rated voltage, or;

$$VR = \frac{V_{oc} - V_L}{V_L}$$

$$\approx \frac{I_o Z_s}{V_L} = \frac{Z_s}{Z_o}$$

$$= Z_s^0 \text{ p.u.}$$

Where $Z_s^0$ is in per unit and:

$$Z_s^0 = R_s^0 + jX_L^0$$

The voltage regulation can also be expressed as a percentage of full load voltage:

$$VR\% = 100\frac{V_{oc} - V_L}{V_L}$$

$$\approx 100\frac{I_o Z_s}{V_L} = 100\frac{I_o Z_s}{I_o Z_o} = 100\frac{Z_s}{Z_o}$$

$$\approx 100 Z_s^0 \text{ p.u.}$$

## 2.4.5 Variable Frequency Operation

A transformer will continue to operate as a transformer only as long as the core doesn't saturate. This can be expressed in equation form:

$$\hat{B} \le B_{max}$$

And since:

$$\hat{B} = \frac{\hat{\Phi}}{A}$$

Therefore:

$$\hat{\Phi} \le \Phi_{max}$$

The expression for transformer flux is:

$$\hat{\Phi} = \frac{1}{N}\int_0^p V(t)\partial t$$

Assume that;

$$V(t) = \hat{V}\,\text{sine}(\omega t)$$

Therefore:

$$\hat{\Phi} = \frac{2V}{\omega N} \le \Phi_{max}$$

In other words, for any particular transformer, the ratio $V/\omega$ must not exceed a limit that is determined by the physical structure of that transformer; its turns ratio, effective cross-sectional area and peak flux density rating of its magnetic material.

Thus one can increase the voltage applied to a transformer if the frequency is increased. Conversely, for a given voltage, the transformer size can be decreased if the frequency is increased. This is one of the main reasons for using higher switching frequencies in switched mode power supplies; the transformer becomes smaller.

On the other hand if the frequency is decreased, the voltage will also have to be decreased to keep the core from saturating.

## 2.5 Common Tranformers

### 2.5.1 Center Tapped Transformer

A center tapped transformer consists of a single primary winding and a secondary winding which has its centerpoint or 'center tap' brought out. The secondary winding can thus be thought of as two secondary windings connected in series as shown in **Figure 2.17**.

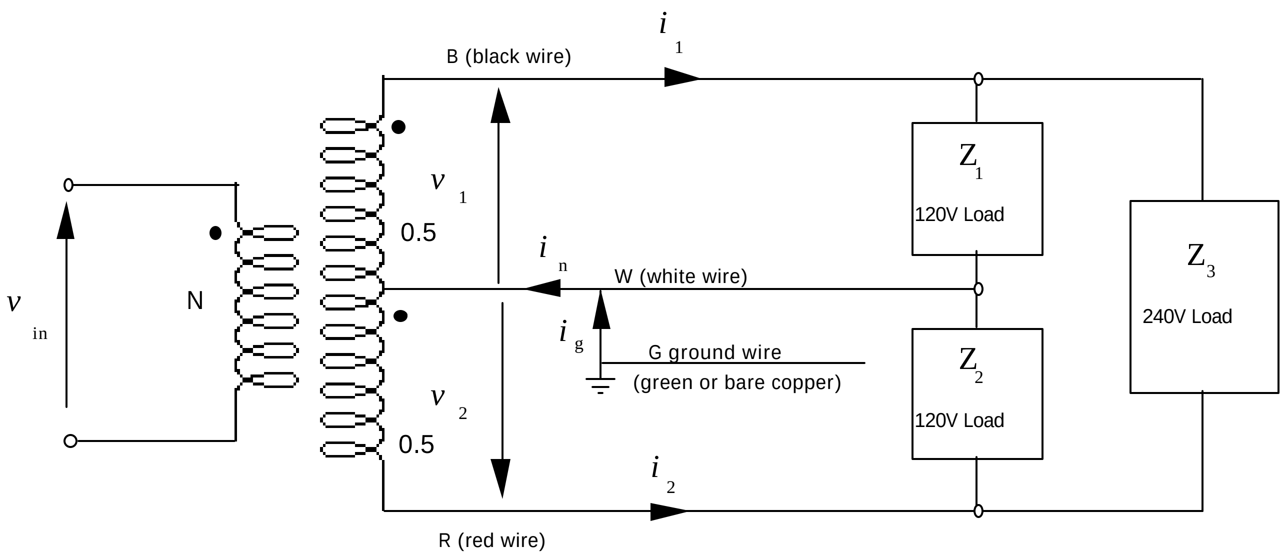



**Figure 2.17 Centre-Tapped Transformer as used in North American 120V/240V Residential Power Distribution**

The windings are connected such that:

$$V_1 = -V_2 = \frac{V_{in}}{2N}$$

In other words, $V_1$ and $V_2$ are equal in amplitude but180° out of phase with each other. Also;

$$V_3 = -V_2 + V_1 = 2V_1 = \frac{V_{in}}{N}$$

Thus $V_3$ is in the same phase as $V_1$ and double the amplitude of $V_1$ or $V_2$ .

Center tapped transformers are used in North America to provide 120V/240V for household use. The red (R) and black (B) wires are called "hot" or "live" leads and are at 120V with respect to ground. The white (W) wire is called the neutral and is at zero volts with respect to ground but is not the same wire as ground. The "ground" (G) wire is green or bare copper and connects all accessible metallic parts to ground.

All 120V appliances such as stereos, toasters, household lights, PC's etc. are connected between white and either red or black. High power 240V appliances such as stoves, dryers, etc. are connected between red and black.

Note that in a perfectly balanced system the load on $V_1$ would be equal to the load on $V_2$ and thus:

$$I_1 = I_2$$

And thus;

$$I_n = 0$$

2.5.2 Three Phase Transformers

Three phase transformers consist basically of three windings on the primary and three windings on the secondary. However, a three phase transformer does not have to consist of three single phase transformers. A three phase transformer can take advantage of the 120° phase shift between each phase to reduce the overall size of magnetic material as compared to three single phase transformers. Also there is an inherent savings in copper wiring in a three phase system as compared to a single phase system of the same power handling capacity.

The simplified circuit diagram for a three phase transformer is shown in **Figure 2.18**.

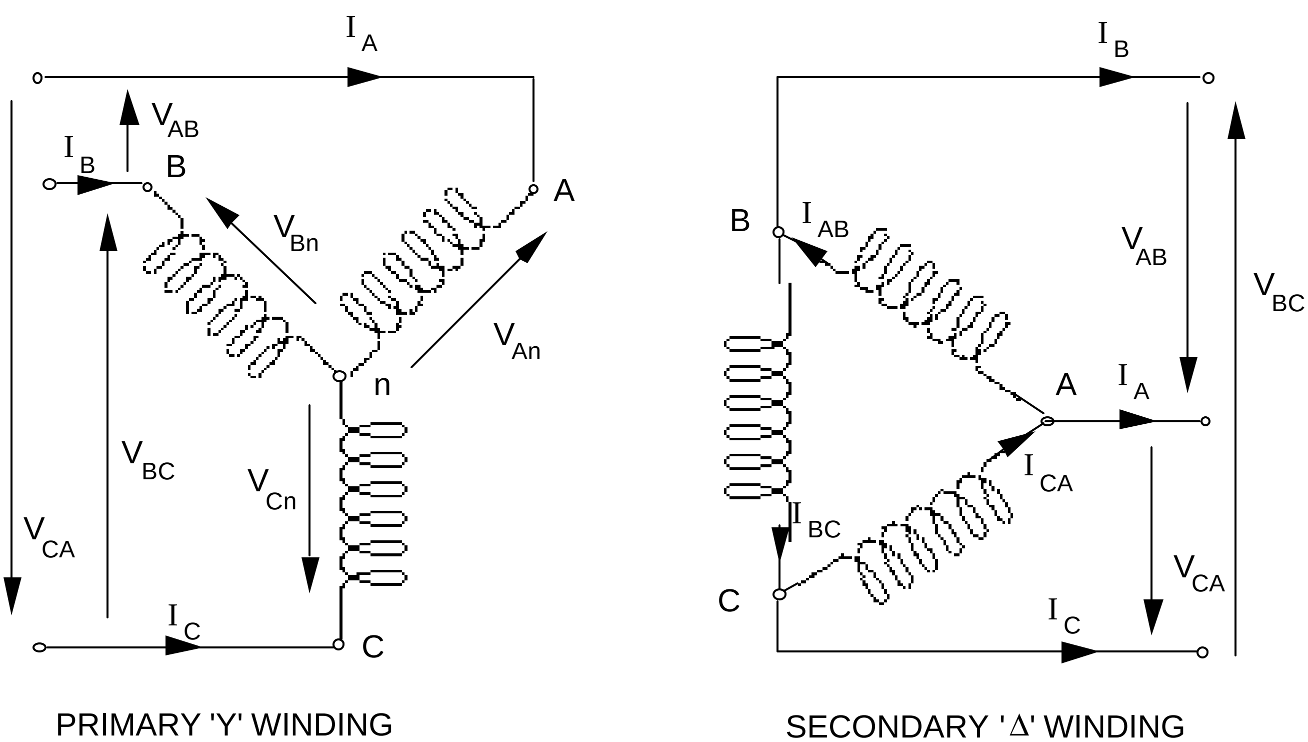



**Figure 2.18 Three Phase Transformer**

In this case the primary is connected in 'Y'" and the secondary is connected in 'Δ'. This configuration is called a 'Y to Δ' configuration. In general either winding can be connected in either 'Y' or 'Δ'. Thus transformers can be wired up in 'Y to Y', or 'Δ to Δ', or 'Δ to Y' configurations.

In a balanced three phase system the following relationships are valid:

$$V_{CA} = V_{BC}\angle^{120°} = V_{AB}\angle^{240°}$$

The voltages; $V_{AB}$ , $V_{BC}$ , and $V_{CA}$ , are referred to as 'phase to phase' voltages, or 'line to line' voltages or sometimes just 'line voltages'.

Also:

$$I_C = I_B \angle^{120^\circ} = I_A \angle^{240^\circ}$$

For a 'Y' configuration:

$$V_{AB} = \sqrt{3}\, V_{An}$$

$$I_A = I_{An}$$

And:

$$P = \sqrt{3}\, V_{AB}\, I_A \cos(\theta)$$

Where $\theta$ is the angle between $I_{An}$ and $V_{An}$ , and thus $\theta$ is also the load power factor angle.

The voltages; $V_{An}$ , $V_{Bn}$ , and $V_{Cn}$, are referred to as 'phase to neutral' voltages, or 'line to neutral' voltages or sometimes just 'phase voltages'.

For a '$\Delta$' configuration:

$$I_A = \sqrt{3}\, I_{AB}$$

And:

$$P = \sqrt{3}\, V_{AB}\, I_A \cos(\theta)$$

Where $\theta$ is the angle between $I_{AB}$ and $V_{AB}$ , and thus $\theta$ is also the load power factor angle.

In North America the common three phase commercial voltage is a 'Y' connected 120V/208V. This means the line to neutral voltage is 120V and the line to line voltage is 208V. In most of the rest of the world the corresponding voltages are 220V/380V. In the United Kingdom, Australia, Malaysia and other relics of the British Empire the voltages are 240V/415V.

It is important to note that for a three phase system, the amount of copper wire required to transmit a given power level of $P$ at a fixed voltage of $V_{AB}$ is;

$$\mathrm{Cu} \propto 3I_A = 3\frac{P}{\sqrt{3}V_{AB}} = \frac{\sqrt{3}P}{V_{AB}} = \frac{1.73P}{V_{AB}}$$

Whereas for a two wire single phase system, the amount of copper would be;

$$\mathrm{Cu} \propto 2I_A = 2\frac{P}{V_{AB}} = \frac{2P}{V_{AB}}$$

Thus, for the same amount of transmitted power, a three phase system requires 1.73/2, or 0.866 as much copper as a single phase system. That represents a 13.4% savings in copper costs, which is significant for a large power distribution system.

Also virtually all high power machinery, including all AC power generators are three phase, as discussed in chapter 4. Thus all commercial and industrial AC power in the world is three phase.

### 2.5.3 Instrument Transformers

#### a) Voltage Transformers

A voltage tranformer is a simple transformer used to 'step down' a high AC voltage to produce a low AC voltage for measurement and sensing purposes.  This may be required for safety reasons if the original voltage is very high and/or because a transformer may be more economical than a high voltage voltmeter.

The requirements for a voltage transformer are a high degree of linearity and a high magnetizing impedance.  These translate into a very low flux density and a high number of turns.

#### b) Current Transformers

A current transformer is used to transform a high AC current into a low AC current for measurement and sensing purposes.  This is usually required because a current transformer is more economical than a high current ammeter.

The requirements for a current transformer are a high degree of linearity and a low impedance.  These translate into a very low flux density and a low number of turns on the primary, (usually one turn), vs. a high number of turns on the secondary, (typically a hundred or more).  Since the primary often requires only one turn it is economical to build a current transformer with a hole in the core and put the primary current carrying conductor through the hole as shown in **Figure 2.19**.

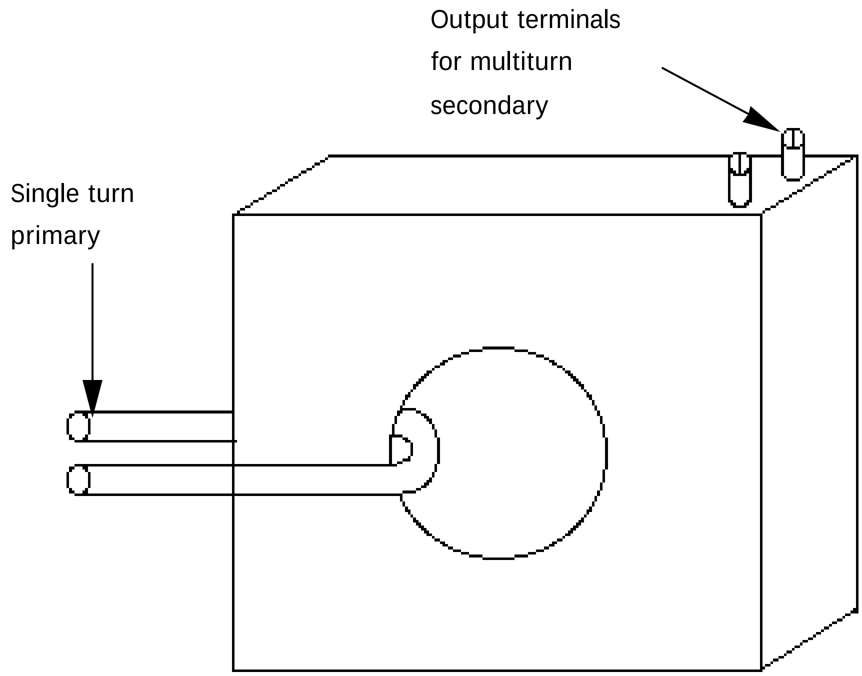



**Figure 2.19 Current Transformer**

It is important to note that a current transformer is designed to carry a high primary current at a low primary voltage. This can only be achieved if a low impedance is connected across the secondary. If in the extreme case the secondary is open circuited then a relatively high impedance will be reflected to the primary and thus a relatively high voltage will appear across the primary. This will result in a very high voltage across the secondary due to the high secondary to primary turns ratio. This voltage could be high enough to cause arcing and/or a safety hazard. Furthermore because the core of a current transformer is not designed for a high voltage it will probably saturate and overheat.

## 2.5.4 Impedance Matching Transformers

Most signal amplifiers (such as stereo amplifiers and cable TV amplifiers) are often designed to produce maximum signal power and/or minimum distortion when driving a particular impedance. The 'load' for such amplifiers (a speaker for a stereo or a TV receiver for cable TV) will not always have this optimum impedance. In some cases it is therefore necessary to use a transformer of the proper turns ratio to 'match' the load impedance to the amplifier requirements. For example coax TV transformers typically "match" a 75 $\Omega$ TV receiver to a 300 $\Omega$ cable TV system. That means that the transformer will change 75 $\Omega$ on the secondary to 300 $\Omega$ on the primary.

## 2.5.5 Autotransformers

An autotransformer can be considered to be a transformer in which the primary and secondary windings are connected together in series as shown in **Figure 2.20**.

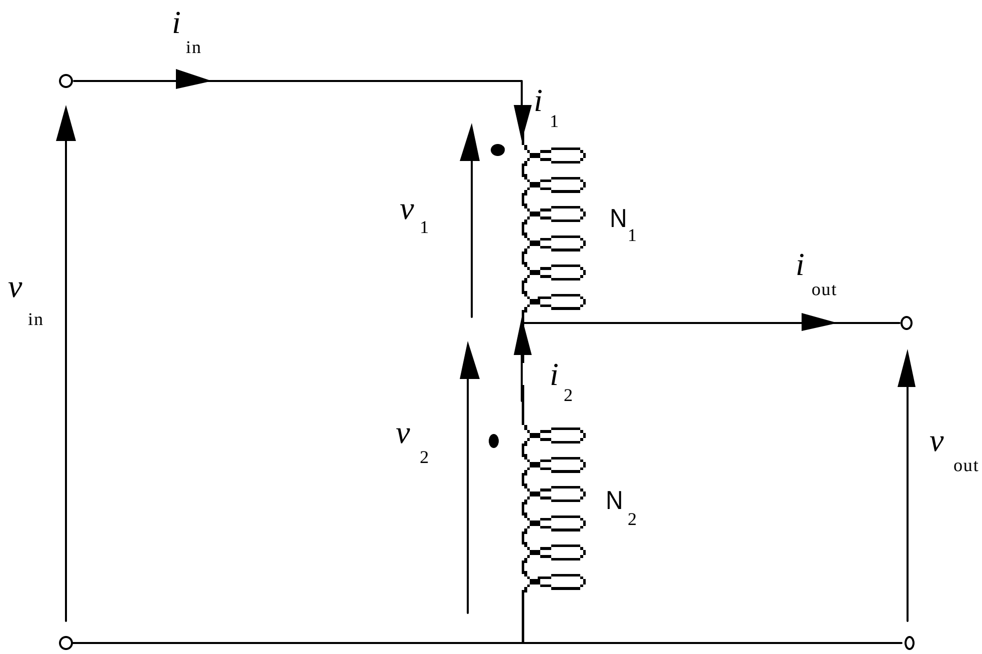



**Figure 2.20 Simplified Circuit Diagram for an Autotransformer**

The transformer equation now becomes:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

And:

$$V_{in} = V_1 + V_2$$

$$V_{out} = V_2$$

The above equations can be solved for $V_{in}$ ,$V_{out}$ ;

$$V_{in} = \frac{N_1+N_2}{N_2} V_{out} = N' V_{out}$$

Where N' is defined as:

$$N' \equiv \frac{N_1+N_2}{N_2}$$

In other words, an autotransformer is similar to a standard transformer with a turns ratio of $\frac{N_1+N_2}{N_2}$.

By a similar analysis it can be shown that:

$$I_{in} = \frac{N_2}{N_1+N_2} I_{out} = \frac{I_{out}}{N'}$$

In general, autotransformers are smaller and cheaper than a standard transformer of the same power handling capacity, however they do not provide the electrical isolation between input and output that a standard transformer provides.

A rough indicator of the size and cost of a transformer is its Volt-Ampere rating, usually referred to as its VA, where:

$$VA = V_1 I_1 + V_2 I_2$$

For a standard transformer the above equations can be simplified by substituting for;

$$V_2 = \frac{V_1}{N}$$

$$I_2 = NI_1$$

$$V_1 = V_{in}$$

$$I_1 = I_{in}$$

To obtain:

$$VA = V_1 I_1 + NI_1 \frac{V_1}{N} = 2V_1 I_1$$

$$= 2V_{in} I_{in}$$

Thus for a standard transformer the VA rating is proportional to the input power rating.

For an autotransformer the substitutions are:

$$V_1 = V_{in} - V_{out}$$

$$I_1 = I_{in}$$

$$V_2 = V_{out}$$

$$I_2 = I_{out} - I_{in}$$

$$V_{in} = \frac{N_1+N_2}{N_2} V_{out}$$

$$I_{in} = \frac{N_2}{N_1+N_2} I_{out}$$

And the basic equation for VA becomes:

$$VA = (V_{in} - V_{out})I_{in} + V_{out}(I_{out} - I_{in}) = V_{in} I_{in} + V_{out} I_{out} - 2V_{out} I_{in}$$

$$= 2V_{in} I_{in} - 2V_{out} I_{in}$$

$$= 2I_{in}(V_{in} - V_{out})$$

Thus for an autotransformer the VA rating is proportional to the *difference* between the input and output voltage.

## 2.6 Problems

### 2.6.1 Turns Ratios

1. You have an audio amplifier that can be modeled by an AC voltage source of $V_s$ Vrms in series with a resistance of $R_s$ . You have a speaker with a resistance of $R_L$ . Determine the turns ratio of an ideal transformer that will enable maximum power to be delivered to the speaker. ($\sqrt{R_s/R_L}$ )

2. You have bought an audio amplifier capable of delivering 100 W into a properly matched load of 4000 Ω. Your speaker has an impedance of 10 Ω. Assuming audio transformers have negligible losses, what turns ratio would your transformer have to have to properly match your speaker to your amplifier. (20)

3. The output coax cable from Rogers Cable requires a 75Ω termination for optimum picture quality. Your TV set input has an impedance of 300Ω. Assuming video transformers have negligible losses, what turns ratio would your transformer have to have to properly match your TV to the coax cable. (0.5)

### 2.6.2 Open Circuit/Short Circuit Tests

4. The results of open circuit and short circuit tests, at rated frequency, on a 500 KVA, 40,000/2400 V, 60 Hz transformer are:

Open circuit test, voltage applied to low voltage side, measurements taken on the low voltage side;

2400 V, 5.1 A, 2334 W

Short circuit test, low voltage side shorted, measurements taken on the high voltage side;

1200 V, 12.1A, 4750 W.

a) Determine the equivalent circuit parameters referred to the high voltage side. (N= 16.67, $R_s$ = 32.4Ω, $X_s$ = 93.6Ω, $R_c$ = 686KΩ, $X_m$ = 133KΩ)

b) Determine the equivalent circuit parameters referred to the low voltage side. (N= 16.67, $R_s$ = 0.117Ω, $X_s$ = 0.337Ω, $R_c$ = 2,469Ω, $X_m$ = 407Ω)

c) Determine the equivalent circuit parameters in per unit quantities. ($Z_0$ = 3,200Ω, $R_{so}$ = 0.010, $X_{so}$ = 0.029, $R_{co}$ = 214, $X_{mo}$ = 35.0)

5. You are given an unmarked transformer with the following test data;

| INPUT: Volts | Amps | Watts | OUTPUT: Volts | Amps |
|---|---|---|---|---|
| 230 | 0.45 | 53 | 23 | O.C. |
| 120 | 1.80 | 135 | S.C. | 18 |

a)Determine the equivalent circuit of the transformer refered to the input side. (N= 10, $R_s$ = 41.7Ω, $X_s$ = 52.0Ω, $R_c$ = 998Ω, $X_m$ = 595Ω)

b)What would be the magnitude and phase of the output voltage if a 1 ohm resistor were connected across the output and 230V connected across the input? (14.63V @ -16.7°)

6. The results of open circuit and short circuit tests, at rated frequency, on a 500 KVA, 40,000/2400 V, 60 Hz transformer are:

Open circuit test, voltage applied to low voltage side;

2400 V, 9.1 A, 1925 W

Short circuit test, low voltage side shorted;

2300 V, 9.1A, 4075W.

a) Determine the equivalent circuit parameters referred to the high voltage side.(N= 16.67, $R_s$ = 49.2Ω, $X_s$ = 247.9Ω, $R_c$ = 831KΩ, $X_m$ = 73.5KΩ)

b) Determine the equivalent circuit parameters referred to the low voltage side.(N= 16.67, $R_s$ = 0.177Ω, $X_s$ = 0.892Ω, $R_c$ = 2992Ω, $X_m$ = 265Ω)

c) Determine the equivalent circuit parameters in per unit quantities. ($Z_0$ = 3200Ω, $R_{so}$ = 0.0153, $X_{so}$ = 0.0775, $R_{co}$ = 260, $X_{mo}$ = 23.0)

7. The results of open circuit and short circuit tests, at rated frequency, on a 500 KVA, 40,000/2400 V, 60 Hz transformer are:

Open circuit test, voltage applied to high voltage side;

40,000 V, 1.0 A, 1925 W

Short circuit test, high voltage side shorted;

200 V, 105 A, 4075 W.

a) Determine the equivalent circuit parameters referred to the H.V. side. (N= 16.67, $R_s$ = 102.7Ω, $X_s$ = 519Ω, $R_c$ = 831KΩ, $X_m$ = 40.04KΩ)

b) Determine the equivalent circuit parameters referred to the L.V. side.(N= 16.67, $R_s$ = 0.370Ω, $X_s$ = 1.86Ω, $R_c$ = 2992Ω, $X_m$ = 145Ω)

c) Determine the equivalent circuit parameters in per unit quantities. ($Z_0$ = 3200Ω, $R_{so}$ = .0321, $X_{so}$ = 0.162, $R_{co}$ = 260, $X_{mo}$ = 12.6)

8. You are given an unmarked transformer with the following test data;

| INPUT: Volts | Amps | Watts | OUTPUT: Volts | Amps |
|---|---|---|---|---|
| 230 | 0.45 | 53 | 23 | O.C. |
| 120 | 4.80 | 35 | S.C. | 48 |

a)Determine the equivalent circuit of the transformer referred to the input side. (N= 10, $R_s$ = 1.52Ω, $X_s$ = 25.0Ω, $R_c$ = 998Ω, $X_m$ = 595Ω)

b)What would be the output voltage if 10 ohms were connected across the output and 230 V connected across the input? (23V@ 10.1°)

9. You are given an unmarked transformer with the following test data;

| Input | | | Output | |
|---|---|---|---|---|
| Volts | Amps | Watts | Volts | Amps |
| 230 | 0.45 | 53 | 23 | O.C. |
| 120 | 4.80 | 35 | S.C. | 48 |

a)Determine the equivalent circuit of the transformer refered to the input side. (N= 10, $R_s$ = 1.519Ω, $X_s$ = 24.95Ω, $R_c$ = 998Ω, $X_m$ = 595Ω)

b)What would be the output voltage if 10 ohms were connected across the output and 230 V connected across the input? (23V@1.4°)

10. You are given a Power transformer labeled “10 KVA, 460V primary, 120V secondary”. The following test data is supplied;

Secondary Short Circuited, Primary: 11 Volts, rated current, 28 Watts.
Primary Open Circuited, Secondary: 2.1 Amps, rated voltage, 82 Watts.

a)Draw the transformer equivalent circuit.

b)Determine the circuit parameters referred to the high voltage (primary) side. (N= 3.83, $R_s$ = 0.059Ω, $X_s$ = 0.503Ω, $R_c$ = 2580Ω, $X_m$ = 888Ω)

11.You are given a transformer labeled “600V/480V, 60Hz, 12KVA”. The following test data is supplied;

| $V_{in}$ | $I_{in}$ | $P_{in}$ | $V_{out}$ | $I_{out}$ |
|---|---|---|---|---|
| 100V | 20A | 187.5W | 0 (s.c.) | 25A |
| 600V | 0.067A | 36W | 480V | 0 (o.c.) |

a)Draw the transformer equivalent circuit.

b)Determine the circuit parameters referred to the low voltage (secondary) side.(N= 1.25, $R_s$ = 0.3Ω, $X_s$ = 3.19Ω, $R_c$ = 6400Ω, $X_m$ = 12,879Ω)

c)Determine the capacitive impedance required on the output to produce unity power factor on the input. $X_c$ = -12879Ω

12. You are given an unmarked transformer with the following test data;

Primary shorted, secondary 15V, 6.2A, 24W

Secondary open, primary 216V, 0.1A, 8W, secondary 18V.

Determine the equivalent circuit of the transformer referred to the primary side. (N= 12, $R_s$ = 89.9Ω, $X_s$ = 336.6Ω, $R_c$ = 5832Ω, $X_m$ = 2325Ω)

13. You are given a transformer labeled “240V/120V, 60Hz, 6KVA”. The following test data is supplied;

| $V_{in}$ | $I_{in}$ | $P_{in}$ | $V_{out}$ | $I_{out}$ |
|---|---|---|---|---|
| 10V | 20A | 8.3W | 0 (s.c.) | 40A |
| 240V | 1A | 12.1W | 120V | 0 (o.c.) |

a)Draw the transformer equivalent circuit.

b)Determine the circuit parameters referred to the high voltage (primary) side. (N= 2, $R_s$ = 0.02075Ω, $X_s$ = 0.499Ω, $R_c$ = 4760Ω, $X_m$ = 240Ω)

c)Determine the voltage and current on the input required to produce rated voltage and rated current on the output, assuming a resistive load.

($V_1$ = 240.8 $\angle 2.98°$V, $I_1$ = 25.02 $\angle -2°$A)

### 2.6.3 Variable Frequency

14. The following no load data were obtained by exciting the 440V winding of a 60Hz transformer:

| Volts | Frequency | Core Losses |
|---|---|---|
| 440 | 60 Hz | 600 W |
| 220 | 30 Hz | 40 W |

Neglect the resistance of the windings.

a)Calculate the hysteresis losses and eddy current losses at each of the above tests. (40W,560W: 5W,35W)

b)Predict the total losses at 440 V and 400 Hz. (25,155W)

15. The following no load data were obtained by exciting the 440V winding of a 60Hz transformer:

| Volts | Frequency | Core Losses |
|---|---|---|
| 440 | 60 Hz | 144 W |
| 220 | 30 Hz | 13.5 W |

Assume hysteresis losses are proportional to frequency, and eddy current losses are proportional to the square of frequency.
Neglect the resistance of the windings.

a)Calculate the hysteresis losses and eddy current losses at each of the above tests. (72W,72W: 9W, 4.5W)

b)Predict the total losses at 440 V and 400 Hz. (3670W)

### 2.6.4 Transient Analysis

16. For the circuit shown in Figure 1 determine the angle, $\alpha$, at which the switch should be closed such that the DC component of the inductor current will be equal to one third of the peak current. Assume an ideal inductor and v(t) = Vsin(wt). ($\pi/3$ or $2\pi/3$)

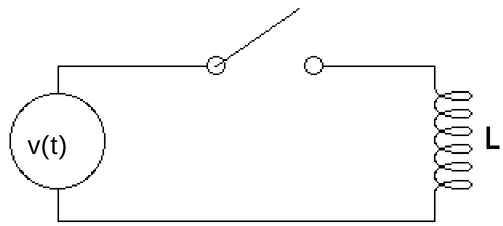


Figure 1

17. For the circuit shown in Figure 2 determine the time, $t_0$ , at which the switch should be closed such that the DC component of the inductor current will be equal to zero. Assume an ideal inductor and v(t) = Vsin(wt). ($\pi/2\omega$)

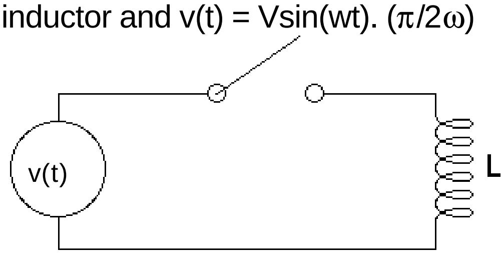


Figure 2