CHAPTER 1
MAGNETIC CIRCUITS

1.1 Basic Magnetic Circuit Analysis

1.1.1 Magnetic Circuit Definitions

The simplest method of analyzing magnetic systems such as inductors, transformers, solenoids etc., is to derive a magnetic circuit model and then apply basic magnetic circuit analysis on the model. Basic magnetic circuit analysis is analogous to basic electric circuit analysis once some simple definitions are established. **Figure 1.1** shows a simple magnetic system consisting of a magnetic structure with a winding, around it. A current, I, flows through the coil. There are N turns in the winding.
The magnetic circuit model is shown in **Figure 1.2a** whereas the analogous electrical circuit model is shown in **Figure 1.2b**.

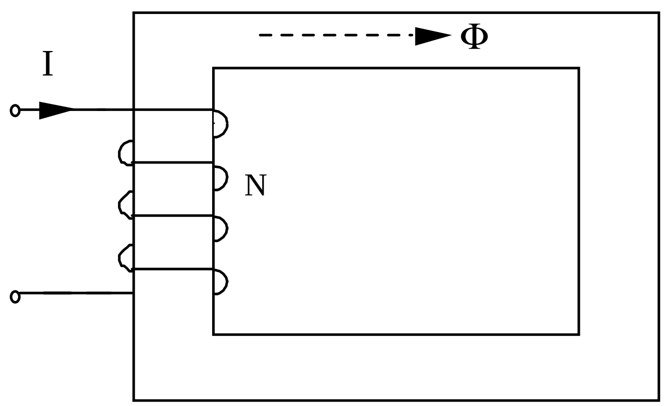
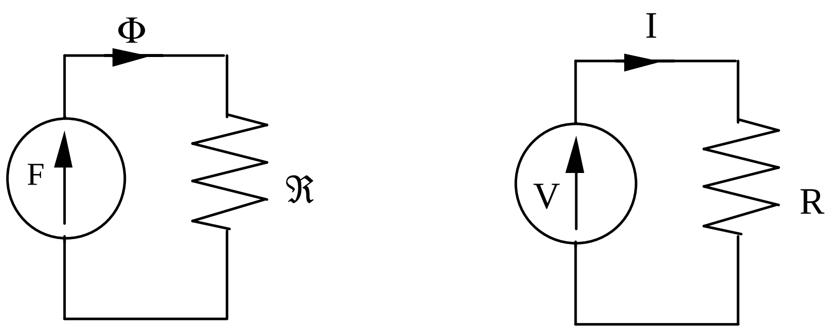


Figure 1.1 A Simple Magnetic System



a) Magnetic Equivalent Circuit

b) Analogous Electrical Circuit

Figure 1.2 Magnetic and Electrical Equivalent Circuit Models for the Magnetic System of Figure 1.1

In the analysis of the electrical circuit model the governing equation is:

$$V = IR$$

Where:

V is the electric force, in volts
I is the electric current, in amperes
R is the electrical resistance, in ohms

In the analysis of the magnetic circuit model the governing equation is:

$$F = \Phi \Re$$

Where:

F is the magnetomotive force, mmf, in ampere-turns
Φ is the magnetic flux, in webers
$\Re$ is the magnetic reluctance, in amper-turns/weber

Also, in most cases, the magnetomotive force is produced by the electric current in the coil, as given by the equation:

$$F = NI$$

Where:

N is the number of turns in a winding
I is the current in the winding

In electrical circuits the resistance, R, is usually known and V or I are to be determined. However in magnetic circuits the reluctance, $\Re$, has to be determined from fundamental physical properties of the materials involved.

The analysis of magnetic circuits is further complicated by the fact that the most relevant physical properties of magnetic materials are not functions of the bulk external variables, F or Φ, per se but of related internal variables, H and B, where H is the magnetic field intensity, (or magneto-motive force per unit length), and B is the magnetic flux density per unit of cross sectional area. The physical properties of the material will determine how much flux density will be produced by a given level of field intensity. In other words;

$$B = \mu H$$

Where μ is the permeability of the material and is usually expressed as:

$$\mu = \mu_o \mu_r$$

And:

μ_o is the permeability of free space, $4\pi \times 10^{-7}$
μ_r is the relative permeability of the specific material involved.
$\mu_r = 1$ for air
$1 << \mu_r < 10{,}000$ for magnetically useful materials

The fundamental relationships between B, H and Φ, F are as follows.

$$\Phi = \int B \partial A$$

Where B is the flux density in Webers/m^2 , (or Tesla), and A is the cross sectional area of the material. This equation can be reduced in simple cases where B does not vary with area:

$$\Phi = \int B \partial A = BA \quad \text{if B is constant with A}$$

Also:

$$F = \int H \partial \lambda$$

Where H is the magnetic field intensity in ampere-turns/meter and λ is the length of the magnetic path. This equation can be also reduced in simple cases where F does not vary with length:

$$F = \int H \partial \lambda = H\lambda \quad \text{if H is constant with } \lambda$$

Expressions for the reluctance, $\Re$, can now be determined from:

$$\Re = \frac{F}{\Phi}$$

In geometrically simple cases, we can substitute for:

$F = H\lambda$ and $\Phi = BA$ to obtain:

$$\Re = \frac{H\lambda}{BA}$$

Another substitution can be made for

$$B = \mu H = \mu_o \mu_r H$$

To obtain:

$$\Re = \frac{\lambda}{\mu_0 \mu_r A}$$

This equation is valid only for uniform materials with geometrically simple (usually rectangular) shapes.

Example 1.1

Determine the magnetic flux in a toroid of inner radius r_1 and outer radius r_2. The toroid has a rectangular cross section of width W. A winding of N turns is uniformly distributed over the entire toroid and carries a current of I amps.

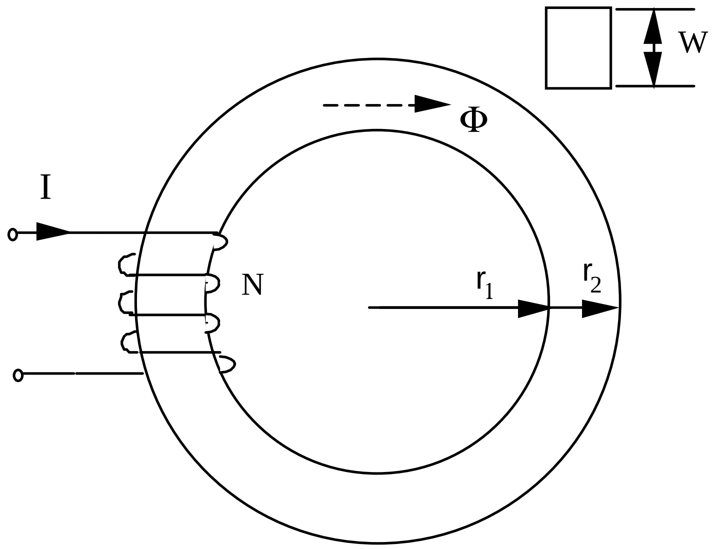


Figure 1.3 A Magnetic System Consisting of a Toroid

Solution to example 1.1

The toroid is shown in **Figure 1.3**. The magnetic flux can be determined from:

$$\Phi = \int B\partial A = \int_{r_1}^{r_2} BW\partial r = W\int_{r_1}^{r_2} B\partial r$$

Substitute for $B = \mu_o\mu_r H$ to obtain:

$$\Phi = W\int_{r_1}^{r_2} \mu_o\mu_r H\partial r = \mu_o\mu_r W\int_{r_1}^{r_2} H\partial r$$

Now we can derive H from:

$$F = NI = \int H\partial\lambda$$

Where λ is the path length, or in this case, the circumference for a given radius r. Since the windings are uniformly distributed over the entire toroid, then H will not vary over any given path length, even though the path length does vary with radius. Thus H will vary with radius, r, but for any given r, H will be constant over

the entire circumference, λ .
Thus:

$$F = NI = \oint H \partial\lambda = H\lambda \quad \text{because } H \text{ is constant with } \lambda$$

Substitute for:

$$\lambda = 2\pi r$$

To obtain:

$$F = NI = H2\pi r$$

Rearrange to obtain:

$$H = \frac{NI}{2\pi r}$$

Substitute for H into the last equation for Φ to obtain:

$$\Phi = \mu_o \mu_r W \int_{r_1}^{r_2} H \partial r = \mu_o \mu_r W \int_{r_1}^{r_2} \frac{NI}{2\pi r} \partial r = \frac{\mu_o \mu_r WNI}{2\pi} \int_{r_1}^{r_2} \frac{\partial r}{r}$$

Solve the integral to obtain:

$$\Phi = \frac{\mu_o \mu_r WNI}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

In examples such as this there is often no need to determine $\Re$, nevertheless:

$$\Re = \frac{F}{\Phi} = \frac{NI}{\Phi} = \frac{2\pi}{\mu_o \mu_r WN \ln\left(\frac{r_2}{r_1}\right)}$$

1.1.2 Series/Parallel Circuits

Most practical magnetic circuits do not consist of simple, uniform magnetic paths. However, most of them can be broken down into a number of simple paths each of which can be approximately uniform. For example the magnetic circuit shown in **Figure 1.4** can be represented by the equivalent circuit shown in **Figure 1.5** which consists of a number of magnetic elements, $\Re_a$, $\Re_b$, $\Re_c$, $\Re_d$, $\Re_x$, $\Re_y$, $\Re_{g1}$, $\Re_{g1}$.

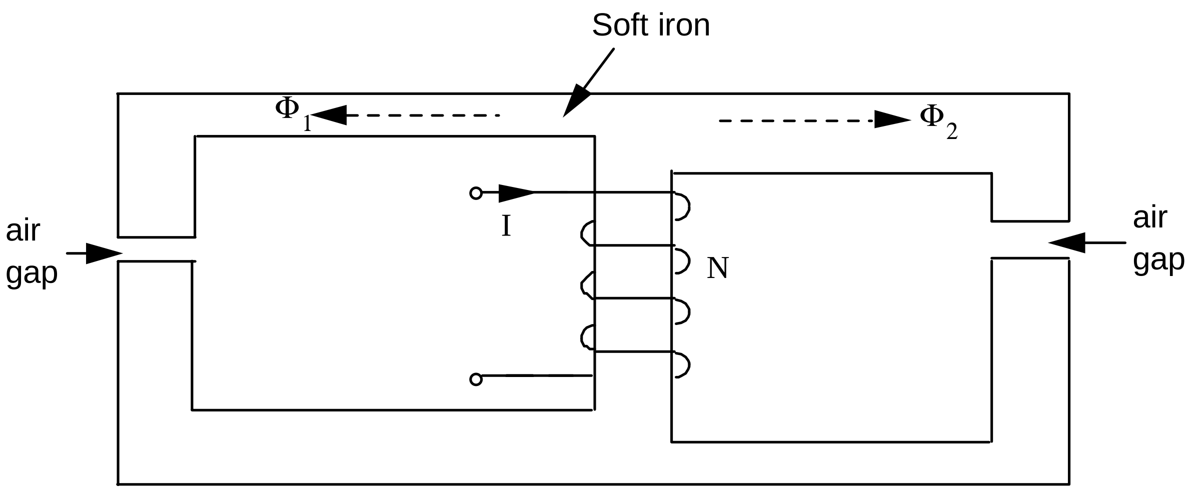


Figure 1.4 Magnetic System Consisting of Several Elements

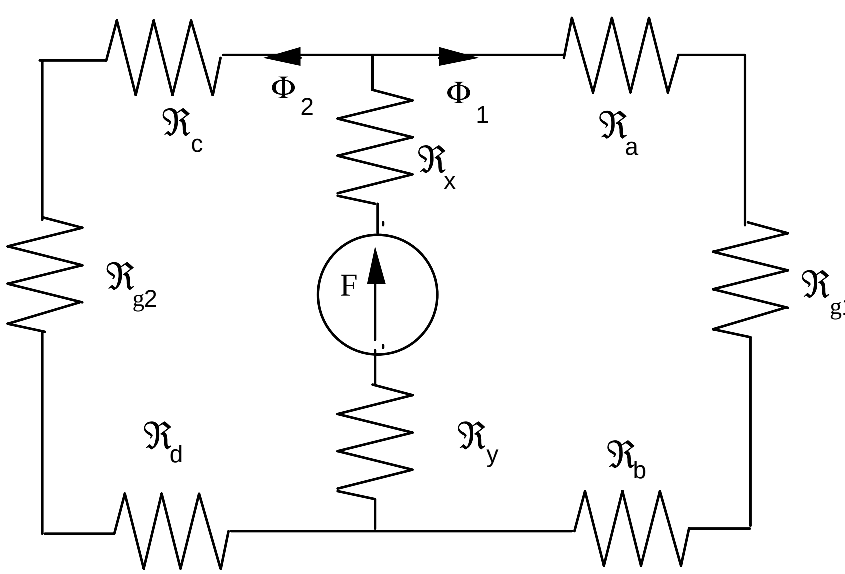


Figure 1.5 Magnetic Equivalent Circuit for the System of Figure 1.4

Each of these elements has a simple, rectangular geometry and will have an approximately uniform magnetic flux density and field intensity. Where $\Re_a$, $\Re_b$, $\Re_c$, $\Re_d$, $\Re_x$, $\Re_y$ represent the reluctance of the various iron members, and $\Re_{g1}$, $\Re_{g2}$ represent the reluctance of the air gaps.
Therefore if;

$$\mu_r >> 5{,}000$$

Then,

$$\Re_a, \Re_b, \Re_c, \Re_d, \Re_x, \Re_y << \Re_{g1}, \Re_{g2}$$

Thus the equivalent circuit can be adequately represented by the circuit shown in **Figure 1.6a** in which only the air gap reluctances, $\Re_{g1}$, $\Re_{g2}$, are included. This equivalent circuit can be further simplified, by combining reluctances to produce the circuit shown in **Figure 1.6c**, which consists of a single reluctance, $\Re$, where:

$$\Re = \frac{\Re_{g1}\Re_{g2}}{\Re_{g1} + \Re_{g2}}$$

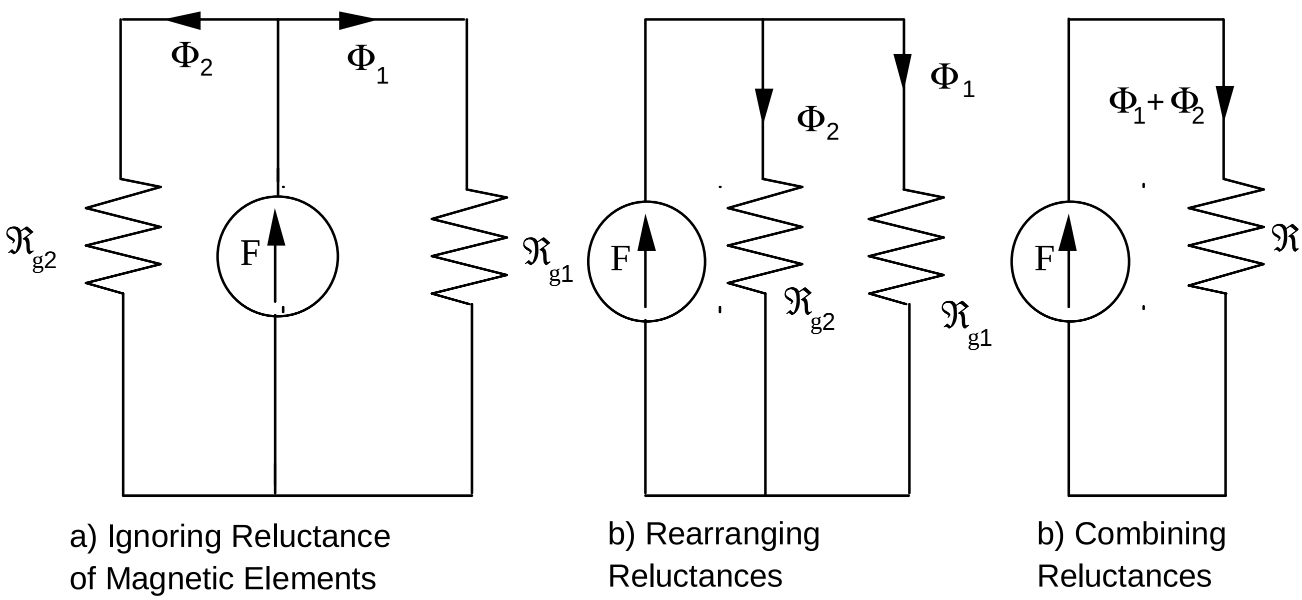


Figure 1.6 Simplifying the Magnetic Circuit of Figure 1.5

1.2 Electrical Characteristics of Magnetic Circuits

1.2.1 Inductance

In most electrical circuits an important parameter of magnetic elements is their inductance, L, which is defined as:

$$L = \frac{\lambda}{I}$$

Where λ is the flux linkage, or magnetic charge, in the magnetic system and is analogous to the electrical charge, Q in a capacitor.

The magnetic charge, λ, is defined as:

$$\lambda = N\Phi$$

Substitute for λ to obtain:

$$L = \frac{N\Phi}{I}$$

Substitute for Φ, to obtain:

$$L = \frac{N^2}{\Re}$$

$$= \frac{N^2\mu_o\mu_r A}{\lambda}$$ for uniform and geometrically simple structures

It is often convenient to derive expressions for voltage, current, power, and energy as functions of inductance.
For example,

$$V = \frac{\partial\lambda}{\partial t} = \frac{\partial(LI)}{\partial t} = L\frac{\partial I}{\partial t} + I\frac{\partial L}{\partial t}$$

$$= L\frac{\partial I}{\partial t}$$ for L constant

Conversely,

$$I = \frac{1}{L}\int V\partial t$$

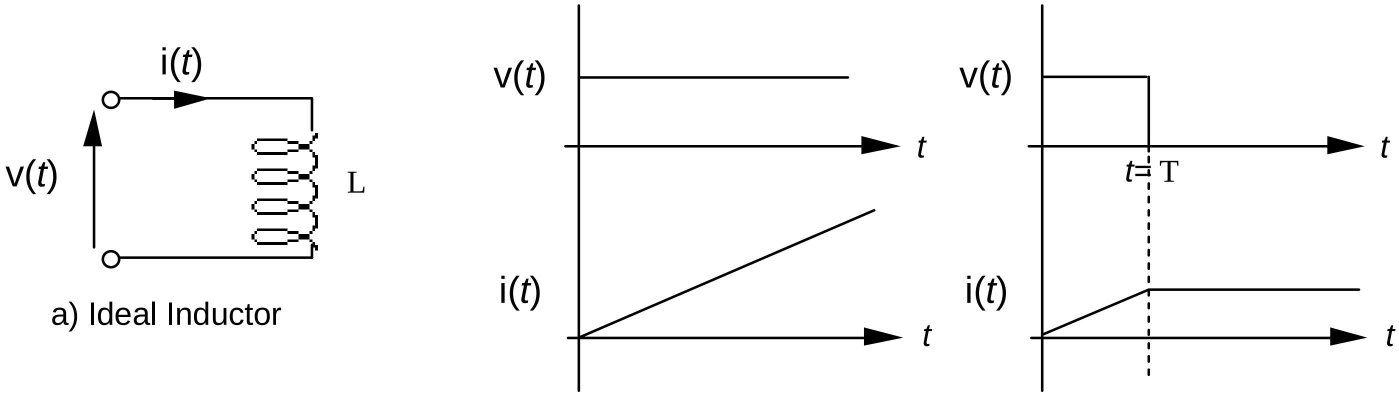
Also:

$$W = \int P\partial t = \int (VI)\partial t = \int (L\frac{\partial I}{\partial t})\partial t = \int LI\partial I$$

$W = \frac{1}{2} LI^2$ for L constant

1.2.2 Electrical Performance of Ideal Inductors

Given an ideal inductor, of inductance L, with a current i(*t*) and a voltage v(*t*) as shown in the circuit of **Figure 1.7a**.



a) Ideal Inductor

b) Voltage & Current for v(*t*) constant

c) Voltage & Current for v(*t*) “one shot” pulse

Figure 1.7 Voltage and Current in an Ideal Inductor

From the preceding analysis we can derive expressions for the voltage and current:

$$v(t) = L\frac{\partial i(t)}{\partial t}$$

$$i(t) = \frac{1}{L}\int v(t)\partial t$$

a) Assume the applied voltage is a constant DC level, V_{dc}. Therefore:

$$v(t) = V_{dc}$$

and thus,

$$i(t) = \frac{1}{L}\int v(t)\partial t = \frac{1}{L}\int v(t)\partial t = \frac{V_{dc}t}{L}$$

This expression indicates that the inductor current, i(*t*), will increase linearly with time as shown in **Figure 1.7b**. The current will continue to increase without bound. In real life the current will increase until something breaks, i.e. a fuse blows, or some external element limits the current.

b) Assume the applied voltage is a "single shot" pulse, as shown in **Figure 1.7c**.

In this case the expression for v(*t*) is:

$v(t) = V_p$ for $0 < t < T$

and

$v(t) = 0$ for $T < t < \infty$

From the preceding analysis we can determine that:

$$i(t) = \frac{V_p t}{L} \quad \text{for } 0 < t < T \text{ when } v(t) = V_p$$

Also at $t = T$:

$$i(T) = \frac{V_p T}{L}$$

And at $t = T+$:

$$v(t) = 0 = L\frac{\partial i(t)}{\partial t}$$

Thus:

$$\frac{\partial i(t)}{\partial t} = 0$$

Which means that the inductor current will not change from its preceding value even though the applied voltage has gone to zero. In other words:

$$i(t) = \frac{V_p T}{L} \quad \text{for } T < t < \infty \quad \text{even though } v(t) = 0$$

This indicates that the current through an ideal inductor will not change if the applied voltage is zero.

A noteworthy application of this principle is in super conducting magnets, in which the coils are cooled below the critical temperature such that the wire resistance disappears and the inductor becomes "ideal". Once a current is established in such an inductor it will continue without diminishing, and maintain a high magnetic field, even with no additional voltage or energy input, (provided the windings are maintained below their critical temperature).

An equally noteworthy, though adverse, effect of this principle is that when an inductor is switched in a switching circuit, eg. a power supply, the inductor current may not be zero even if its applied voltage has been removed.

c) Assume the applied voltage is a sinewave:

$$v(t) = V\sin(\omega t)$$

Therefore,

$$i(t) = \frac{1}{L}\int v(t)\partial t = \frac{1}{L}\int V\sin(\omega t)\partial t = \frac{-V}{\omega L}\cos(\omega t)\Big|_{t=0}^{t=t}$$

$$i(t) = \frac{V}{\omega L}\left[1 - \cos(\omega t)\right]$$

Thus the current through an ideal inductor can have a DC component even though the applied voltage is a pure sinewave. In an ideal inductor this DC current will continue indefinitely, but in a real inductor there will always be a winding resistance, R, that will cause the DC component of the current to decay with a time constant of R/L.

1.2.3 Electrical Interaction Between Two Ideal Inductors

Assume that two inductors are connected as shown in the circuit of **Figure 1.8** and the initial conditions, at $t = 0^-$ are:

$i_1(0^-) = 0$
$i_2(0^-) = I$
Switch S is closed

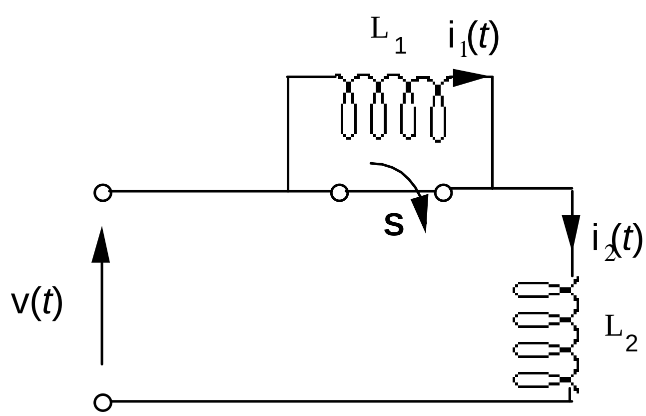


Figure 1.8 Two Ideal Inductors Connected by a Switch

At t=0 switch S is opened, which has the effect of forcing the current through each inductor to be the same, i.e.:

$$i_1(0^+) = i_2(0^+) = i(0^+)$$

The new value of inductor current is determined by the conservation of flux linkage, (analogous to the conservation of charge in capacitors):

$$\sum\lambda(0^-) = \sum\lambda(0^+)$$

or:

$$i_1(0^-)\,L_1 + i_2(0^-)\,L_2 = i_1(0^+)\,L_1 + i_2(0^+)\,L_2$$
$$0 + IL_2 = i(0^+)L_1 + i(0^+)L_2$$

Solve for :

$$i(0^+) = \frac{IL_2}{L_1 + L_2}$$

Thus i_1 goes from 0 to $IL_2/(L_1+L_2)$ instantaneously and i_2 goes from I to $IL_2/(L_1+L_2)$ instantaneously, which seems contrary to the previous conclusion that the current through an ideal inductor cannot change instantaneously. However, in this case the overriding law is that the total flux linkages in the inductor pair cannot change, just as the total charge in a capacitor pair cannot change.

1.2.4 Magnetic vs Capacitive Analogies

	Magnetic	Capacitive
Physical Parameter	L	C
Voltage	$v = L\frac{\partial i}{\partial t}$	$v = \frac{1}{C}\int i\,\partial t$
Current	$i = \frac{1}{L}\int v\,\partial t$	$i = C\frac{\partial v}{\partial t}$
Energy	$W = \frac{1}{2}LI^2$	$W = \frac{1}{2}CV^2$
Charge	$\lambda = LI$	$Q = CV$
Charge Derivative	$v = \frac{\partial \lambda}{\partial t}$	$i = \frac{\partial Q}{\partial t}$
Conservation Equation	$\sum\lambda(0^-) = \sum\lambda(0^+)$	$\sum Q(0^-) = \sum Q(0^+)$

1.3 Non-Ideal Magnetic Systems

1.3.1 Electrical Performance of Real Inductors

As a first approximation, the main difference between a real inductor and an ideal inductor can be represented by incorporating a resistance, R, in series with an ideal inductance, L, as shown in **Figure 1.9**. The value of R is usually the resistance of the wires in the inductor.

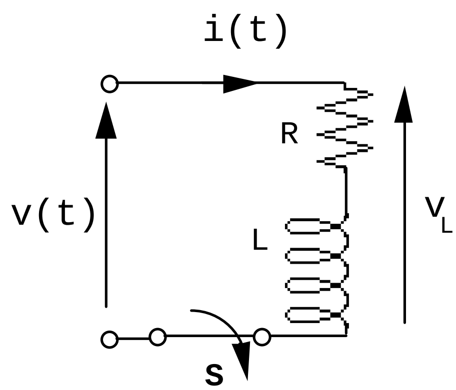


Figure 1.9 Circuit Model of a "Real" Inductor (First Approximation)

The governing equation now becomes:

$$v(t) = Ri(t) + L\frac{\partial i}{\partial t}$$

Assume that:

$$v(t) = V_{dc} \quad \text{constant DC level}$$

the switch S has been closed for a sufficiently long time such that steady state conditions have been reached and thus:

$$i(t) = V/R$$

Assume now that the switch is opened at t=0, forcing the inductor current to go to zero in a short time interval of Δt. Thus:

$$v_L(0^+) = L\frac{\partial i}{\partial t} \approx L\frac{\Delta I}{\Delta t}$$

Thus if the time interval, Δt, is sufficiently small, then the voltage across the inductor, v_L, could be very large. This voltage is usually sufficient to break down the air gap across a mechanical switch and cause arcing. It is also sufficient to break down the semiconductor in solid state switches and destroy them unless precautions are taken.

Effectively, the energy stored in the inductor has to be dissipated in the switch or in radiated energy, or both. In real applications this results in excessive stresses on the switching components, usually transistors, and/or excessive radiated noise.

1.3.2 Non-Linear Magnetics

Most practical magnetic applications use ferrous or ferrite materials that have non-linear magnetic characteristics. The non-linearity mainly applies to the B vs H curve as shown in **Figure 1.10** which demonstrates saturation and **Figure 1.11** which demonstrates hysteresis as well as saturation.

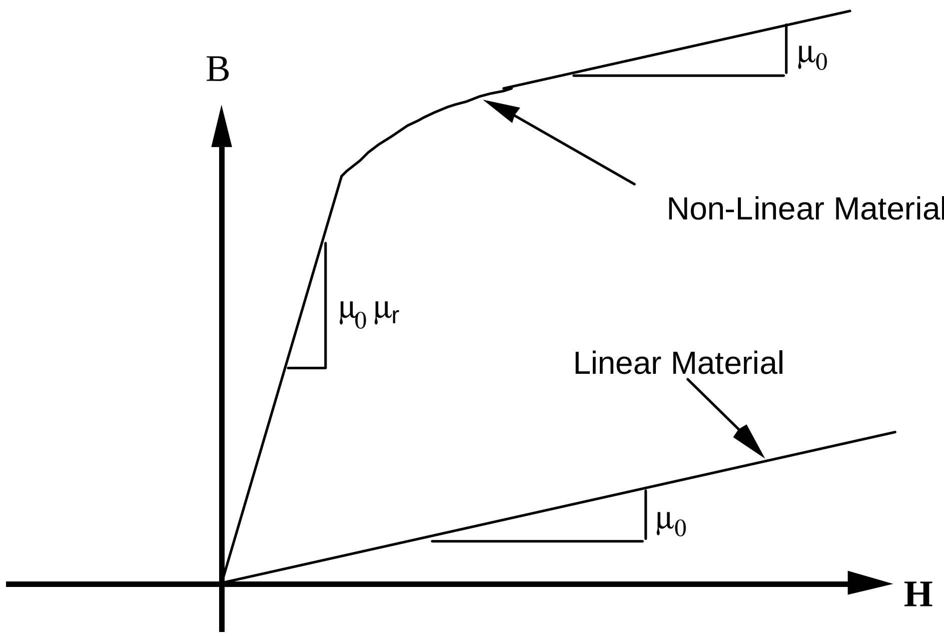
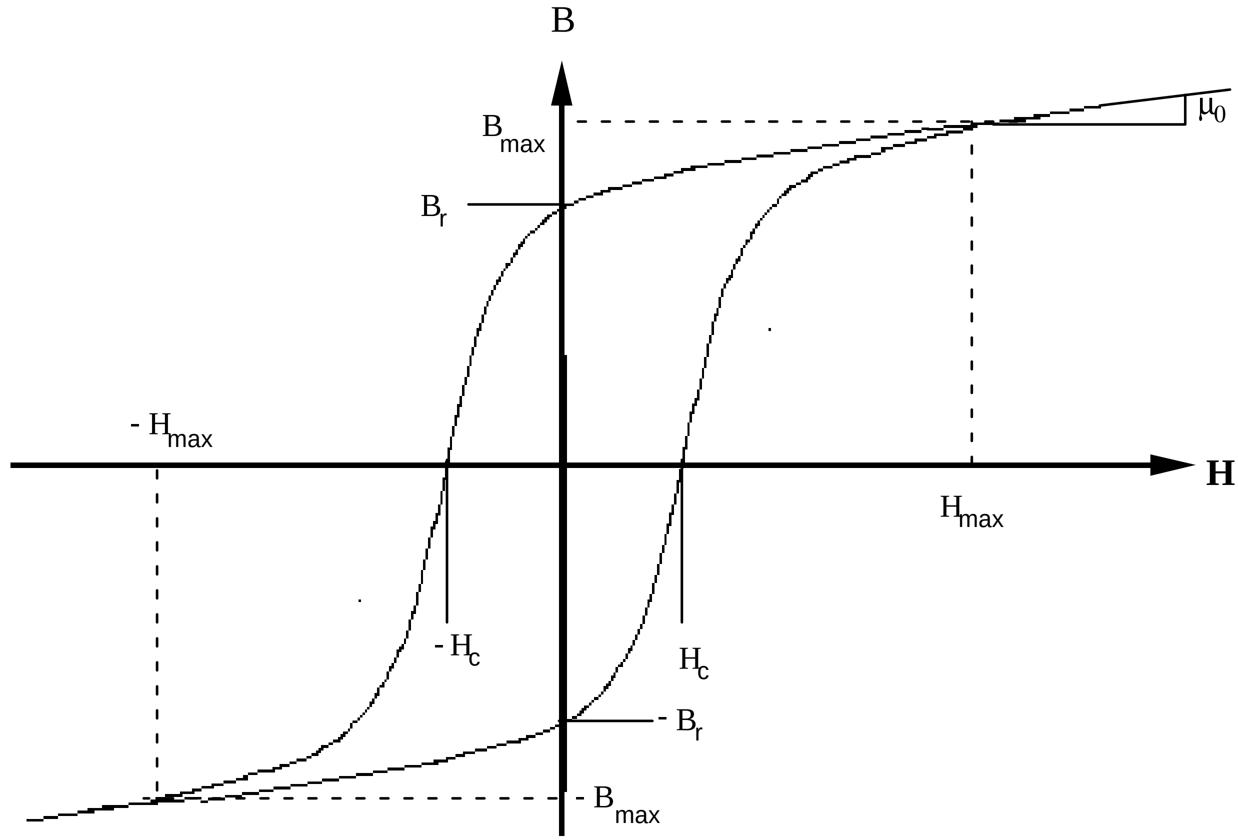
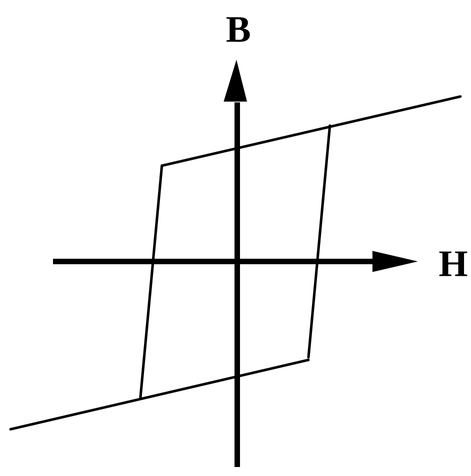


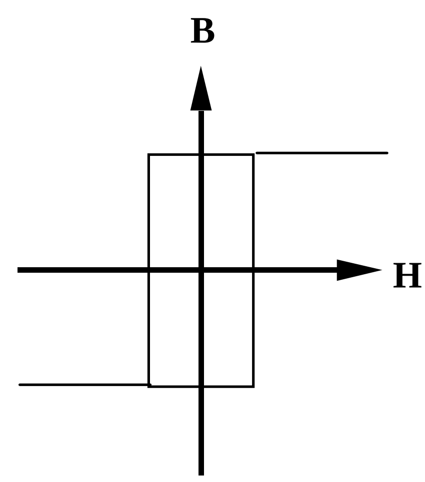
Figure 1.10 B vs H Curve Demonstrating Saturation



a) Generalized Hysteresis Curve



b) Piecewise Linear Approximation



c) Simplified Approximation

Figure1.11 B vs H Curves Demonstrating Saturation and Hysteresis

In general, a non-linear magnetic characteristic means that the relative permeability of the material, μ_r , is not constant but varies with H. Saturation means that the effective value of μ_r decreases abruptly to unity, i.e. the magnetic material becomes no more effective than free air. Hysteresis means that the magnetic flux density, B, can assume more than one value for a given value of field intensity, H. The actual value of B will be dependent on whether H is increasing or decreasing and on the preceding magnetic history of the material, i.e. hysteresis implies a magnetic memory effect.

Some key points on the B vs. H curve of **Figure 1.11** are:

B_{max} is called the saturation flux density and is defined as the flux density when μ_r goes to 1.

H_{max} is called the maximum field intensity and is defined as the magnetic field intensity when μ_r goes to 1.

B_r is called the residual flux density and is defined as the flux density when the magnetic field intensity, H, is zero.

H_c is called the coercive force and is defined as the magnetic field intensity required to bring the flux density, B, to zero.

As will be shown in section 1.4.1 that the energy dissipated in a magnetic material is given by the expression:

$$W_\Phi = m^3 \int_{B_1}^{B_2} H \partial B$$

and, if there is hysteresis, then :

$$W_\Phi = m^3 \int_{B_1}^{B_2} H \partial B > 0 \quad \text{even if } B_1 = B_2$$

Thus a certain amount of energy is dissipated in a magnetic material each time it is "cycled through" the B H curve. In AC systems this occurs once during each cycle. This energy is referred to as hysteresis losses and becomes very significant at high frequencies.

It is important to note that all magnetic materials demonstrate saturation and hysteresis to varying degrees. The more ideal applications operate in the region where B is much less than B_{max} and the more ideal materials demonstrate less hysteresis, eg, B_r and H_c are close to zero. In choosing the right magnetic material for a given application various tradeoffs are required in selecting the magnetic characteristics of the material to suit the application. Some desirable characteristics for specific applications are:

Application	Desirable characteristic
permanent magnets	high B_r
measuring instruments	very low B_r
motors/generators	high B_{max} , low H_{max}
magnetic memories	high B_r , repeatable H_c
high frequency operation	low $\int_{B_1}^{B_2} H \partial B$
DC operation	high H_{max}

Example 1.1

Given the magnetic system shown in **Figure 1.12**, assume the soft iron element has the B vs H characteristic shown in **Figure 1.11** and that the cross-sectional area of the iron is A_f , and the effective cross-sectional area of the air gap is A_g , the mean length in the soft iron is λ and the air gap width is g.
Note that μ_r is comparable to μ_0 and λ is comparable to g.

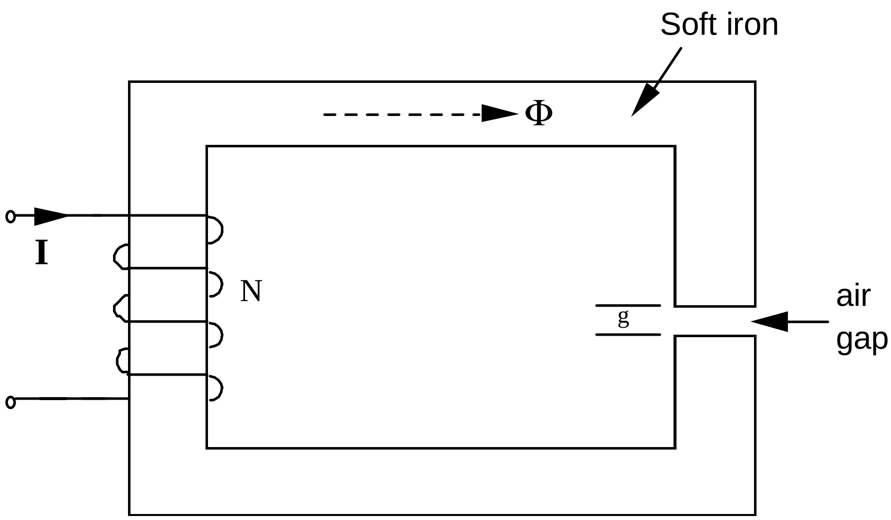


Figure 1.12 A Magnetic System with an Air Gap and Soft Iron having the B H Characteristics shown in Figure 1.11

Solution:
The equivalent magnetic circuit for this system is shown in **Figure 1.13** where $\Re_f$ represents the reluctance of the iron element and $\Re_g$ represents the reluctance of the air gap.

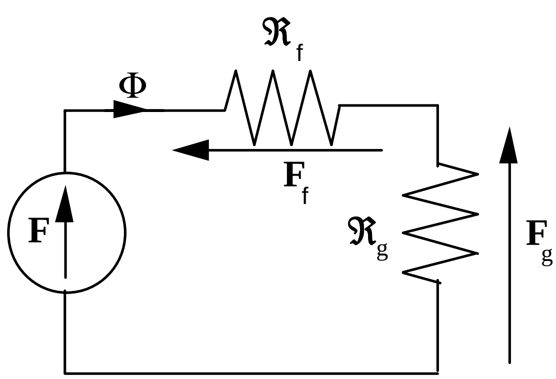


Figure 1.13 Equivalent Magnetic Circuit of System Shown in Figure 1.12

The actual value of $\Re_f$ cannot be analytically determined because it is dependent on the μ_r of the iron which is a non-linear function of the field intensity applied to the iron. This non-linearity is represented by the non-linear B vs. H curve for the iron. Nevertheless, since the B vs. H curve for the iron is available it is possible to determine magnetic parameters for this system graphically. The actual values of B and H for the iron will have to satisfy two curves: one curve being the B vs. H curve for the iron as shown in **Figure 1.11a**, the second curve being the B vs. H relationship for the magnetic system as a whole.

This second curve can be determined from the equations derived from the magnetic equivalent circuit shown in **Figure 1.13** as follows:

The basic equations for the air gap are:

$$F_g = \Phi \Re_g$$

$$\Re_g = \frac{g}{\mu_g A_g}$$

$$\Phi = B_g A_g$$

And for the iron:

$$F_f = H_f \lambda$$

$$\Phi = B_f A_f$$

Furthermore, the same flux, Φ, is in both the iron and the air gap, or:

$$\Phi = B_g A_g = B_f A_f$$

The basic equation for the system is:

$$F = N I = F_f + F_g$$

Substitute for F_f ,F_g , and Φ. into the above equation to obtain:

$$NI = H_f \lambda + B_f A_f \Re_g$$

Solve for:

$$B_f = \frac{(NI - H\lambda)}{A_f \Re_g}$$

This is the second B vs. H curve for the iron. It's actually a straight line representing the relationships imposed by the air gap. The characteristics of this line are:

Vertical intersect: $B = \dfrac{H\lambda}{A_f \Re_g}$

Horizontal intersect: $H = \dfrac{NI}{\lambda}$

Slope is given by:

$$\frac{\partial B}{\partial H} = \frac{\partial \left[\dfrac{NI - H_f \lambda}{A_f \Re_g} \right]}{\partial H} = \frac{-\lambda}{A_f \Re_g} = -\mu_0$$

Figure 1.14 shows the superposition of the two B vs. H curves.

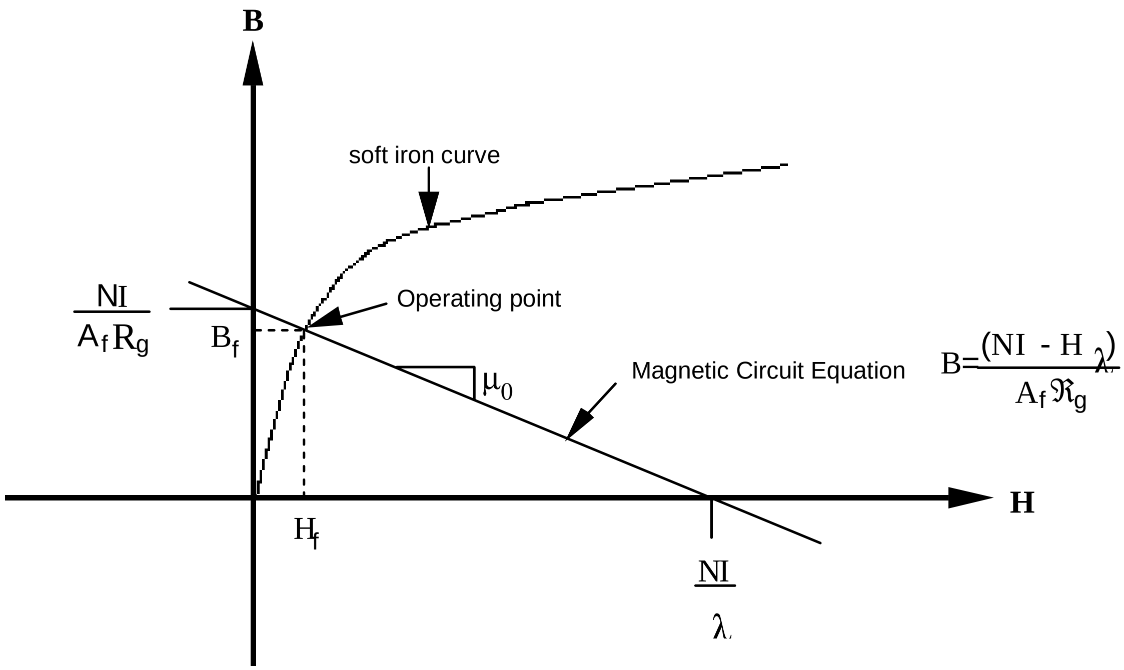


Figure 1.14 B vs H curves for the soft iron and the magnetic system shown in Figure 1.12

1.4 Energy and Force in Magnetic Systems

1.4.1 Energy Stored in a Magnetic Field

To determine the electrical energy stored in a magnetic element we have to determine the voltage applied to the element. We can utilize Lenz's Law which states that:

$$V_\Phi = \frac{\partial \lambda}{\partial t} = \frac{\partial N\Phi}{\partial t} = N\frac{\partial \Phi}{\partial t}$$

The energy, W_Φ, that was required to generate the magnetic flux is given by the equation:

$$W_\Phi = \int V_\Phi I \partial t$$

Substitute for V_Φ to obtain:

$$W_\Phi = \int N\frac{\partial \Phi}{\partial t} I \partial t = \int N I \partial \Phi$$

Substitute for NI to obtain:

$$W_\Phi = \int \Re \Phi \partial \Phi$$

For linear systems in which $\Re$ is independent of Φ the integral simplifies to:

$$W_\Phi = \frac{\Re \Phi^2}{2}$$

For nonlinear systems the integral can still be simplified by first substituting F for $\Re\Phi$ to obtain:

$$W_\Phi = \int \Re \Phi \partial \Phi = \int F \partial \Phi$$

Furthermore, for geometrically simple cases, we can substitute for:

$$F = H\lambda \text{ and } \Phi = BA$$

to obtain:

$$W_\Phi = \int H\lambda\,\partial(BA) = \int H\lambda A\,\partial(B) = \int \lambda A H\,\partial B$$

Note, however, that the expression, λA, is the product of the length and cross-sectional area of the magnetic path. This is the volume of the magnetic path, eg.

$$W_\Phi = \int \text{volume} \times H\partial B = m^3 \int H\partial B$$

or:

$$\frac{W_\Phi}{m^3} = \int H\partial B$$

Similarly, to change the magnetic flux density in a material from B_1 to B_2 will require an energy input of:

$$\frac{W_\Phi}{m^3} = \int_{B_1}^{B_2} H\partial B$$

It was shown in section 1.3.2 that in most magnetic materials, due to hysteresis:

$$\frac{W_\Phi}{m^3} = \int_{B_1}^{B_2} H\partial B > 0 \qquad \text{even if } B_1 = B_2$$

1.3.2 Energy Stored in an Air Gapped System

A simple magnetic system with two air gaps is shown in **Figure 1.5**. It consist of two iron elements: a horseshoe and a bar. The bar is separated from the horseshoe by a gap of length g. The effective cross-sectional area of the air gap is A. There is a coil of N turns carrying a current of I amps.

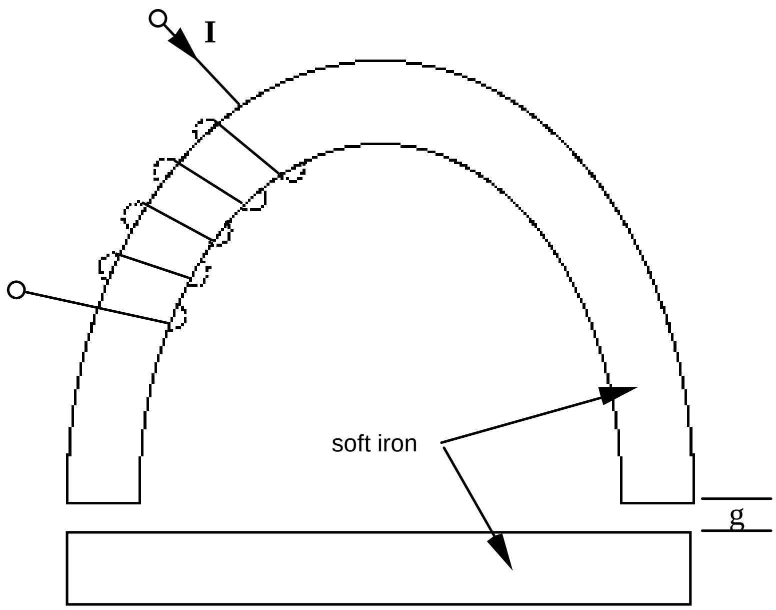


Figure 1.15 Simple Horeshoe Magnet

The inductance, L_g, of each air gap is:

$$L_g = \frac{N^2 \mu_0 \mu_r A}{g}$$

The energy stored in each air gap can be determined from:

$$W = \frac{1}{2} L I^2$$

Thus:

$$W_g = \frac{1}{2} L_g I^2 = \frac{N^2 I^2 \mu_0 \mu_r A}{2g}$$

The preceding expression for energy in each air gap can be used to determine the force, F_g, acting on the bar to try to close the air gap. The expression for force as a function of energy is given by:

$$F = \frac{\partial W}{\partial g}$$

Substitute for W_g to obtain:

$$F_g = \frac{\partial W_g}{\partial g} = -\frac{N^2 I^2 \mu_0 \mu_r A}{2g^2}$$ in each gap

Thus the total force acting on the bar is:

$$F_g = -\frac{N^2 I^2 \mu_0 \mu_r A}{g^2}$$ (sum of both air gaps)

The above expression is negative indicating that the force is acting to make g smaller.

1.4.3 Energy Dissipation Between Two Ideal Inductors.

In the example shown in **Figure 1.8** and discussed in section 1.2.3 it was shown that the law of conservation of flux linkage, λ, determines the resultant current when two inductors are connected together. That is to say that the total flux linkage in the system does not change, however, the total energy, W, stored in the inductors does change:

$$W(0^-) = \tfrac{1}{2} L_1\, i_1^2(0^-) + \tfrac{1}{2} L_2\, i_2^2(0^-)$$
$$= \tfrac{1}{2} L_2\, I^2$$

However,

$$W(0^+) = \tfrac{1}{2} L_1\, i_1^2(0^+) + \tfrac{1}{2} L_2\, i_2^2(0^+)$$
$$= \tfrac{1}{2} L_1 \left[\frac{IL_2}{L_1+L_2}\right]^2 + \tfrac{1}{2} L_2 \left[\frac{IL_2}{L_1+L_2}\right]^2$$
$$= \tfrac{1}{2} L_2\, I^2 \left[\frac{L_2}{L_1+L_2}\right]$$
$$= W(0^-) \left[\frac{L_2}{L_1+L_2}\right]$$

Thus:

$$W(0^+) < W(0^-)$$

The energy lost in the system was dissipated in the contact resistance of the switch and as radiated energy if there is a high $\partial i/\partial t$.

1.5 Transforming Magnetic Circuits to Electrical Circuits

Magnetic equivalent circuits can be transformed into electrical equivalent circuits by a form of topological transformation in which magnetic circuit loops are "mapped" into electrical circuit loops.

Assume we have a generalized magnetic system as shown in **Figure 1.16**, consisting of several magnetic paths and two coils.

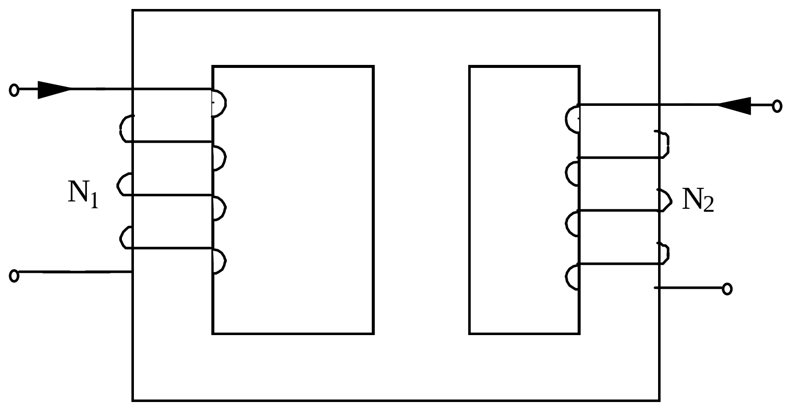


Figure 1.16 Generalized Magnetic System to be mapped into an Electrical Circuit.

The steps are as follows:

Step1: Draw the equivalent magnetic circuit. This is done in **Figure 1.17**.

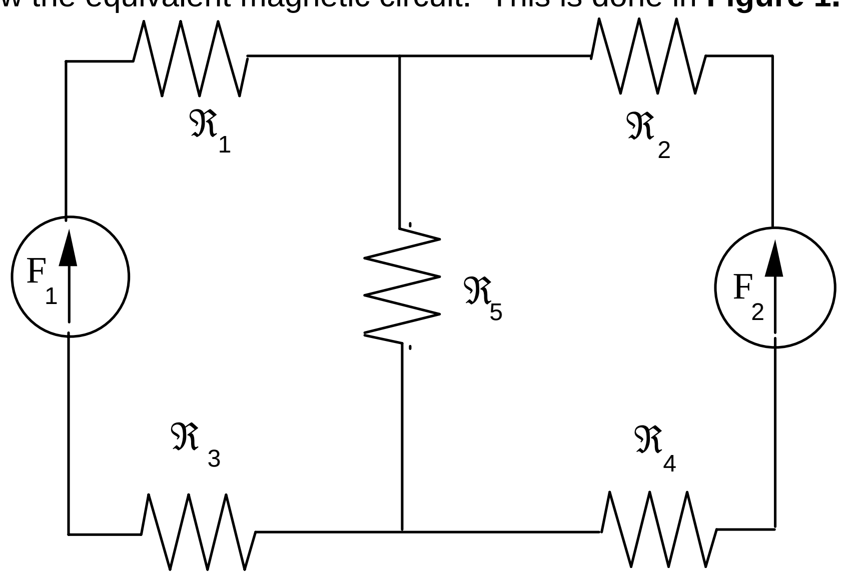


Figure 1.17 Magnetic Equivalent Circuit for the system shown in Figure 1.16

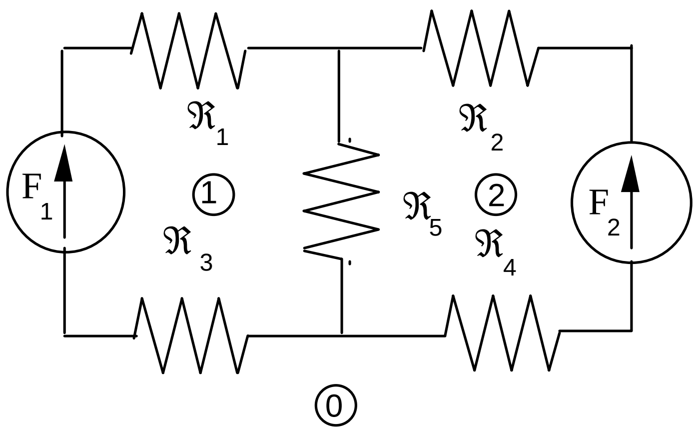
Step 2: Assign a number to each circuit loop in the magnetic equivalent circuit, as shown in **Figure 1.18a**, with the outside loop assigned the number 0.

Step 3: Assign the corresponding numbers to each node in the electrical equivalent circuit, as shown in **Figure 1.18b**.

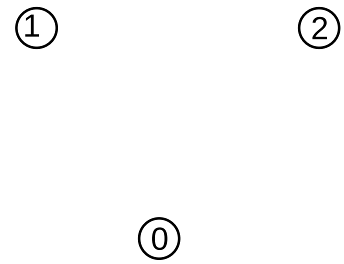
Step 4: Identify each magnetic circuit element between any two magnetic loops as shown in **Figure 1.18c**.

Step 5: Insert a corresponding electrical circuit element between the corresponding electrical nodes, as shown in **Figure 1.18d**, but changing all reluctances into "internal" inductors, L', and all coils into ideal transformers. The value of each internal inductor is determined by:

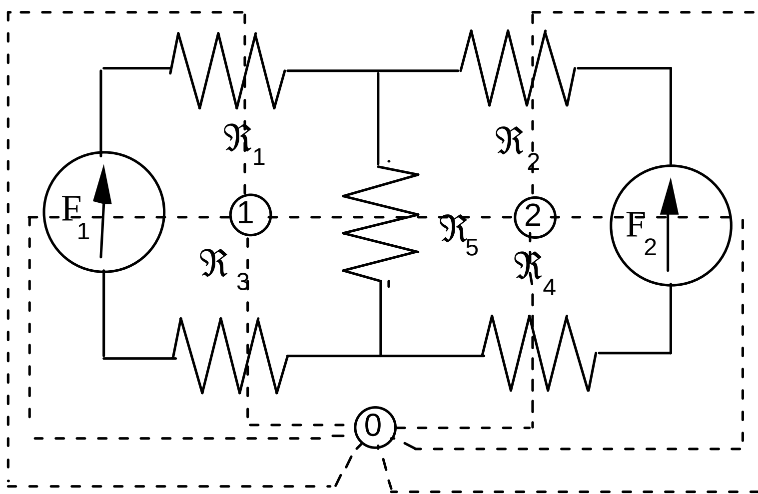
$$L' = \frac{1}{\Re}$$



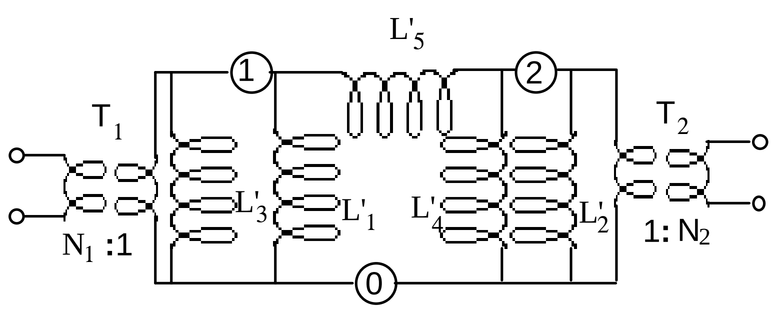
a) Magnetic Equivalent Circuit
with Loops Numbered



b) Electrical Circuit with
Corresponding Nodes Numbered



c) Reluctances and Coils between Loops
Identified



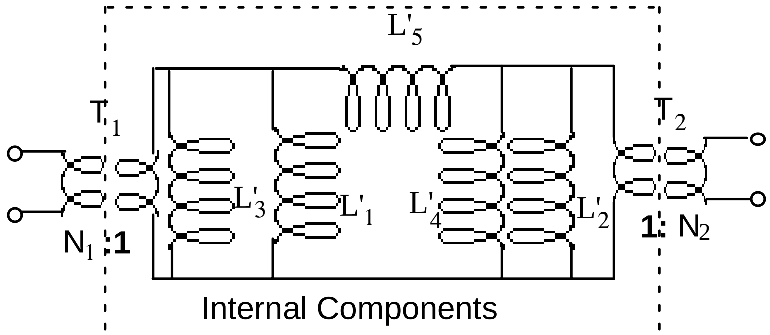
d) Corresponding Inductors
and Tranformers between Nodes

Figure 1.18 Transforming a Magnetic Circuit into an Electrical Circuit

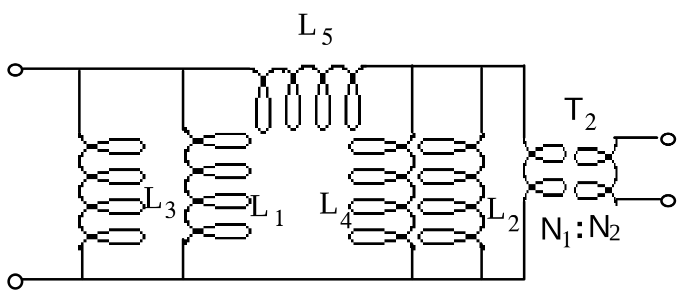
The "internal" inductors of **Figure 1.18c** can be "externalized" by "pulling" them through one or the other transformers and changing the value by:

$$L = N^2 L' = \frac{N^2}{\Re}$$

The resultant electrical equivalent circuit is shown in **Figure 1.19**.



a) Electrical Circuit with "Internal" Magnetic Elements



b) "Internal" Magnetic elements brought out through transformer T_1

Figure 1.19 Electrical Equivalent Circuit for the Magnetic System of Figure 1.16