ELG41257: Robust Control Systems

Feedback control systems are widely used in manufacturing, mining, automobile and other hardware applications. In response to increased demands for increased efficiency and reliability, these control systems are being required to deliver more accurate and better overall performance in the face of difficult and changing operating conditions.

In order to design control systems to meet the needs of improved performance and robustness when controlling complicated processes, control engineers will require new design tools and better control theory. A standard technique of improving the performance of a control system is to add extra sensors and actuators. This necessarily leads to a multi-input multi-output (MIMO) control system. Accordingly, it is a requirement for any modern feedback control system design methodology that it be able to handle the case of multiple actuators and sensors.

Robust means durable, hardy, and resilient

Why Robust?

- When we design a control system, our ultimate goal is to control a particular system in a real environment.

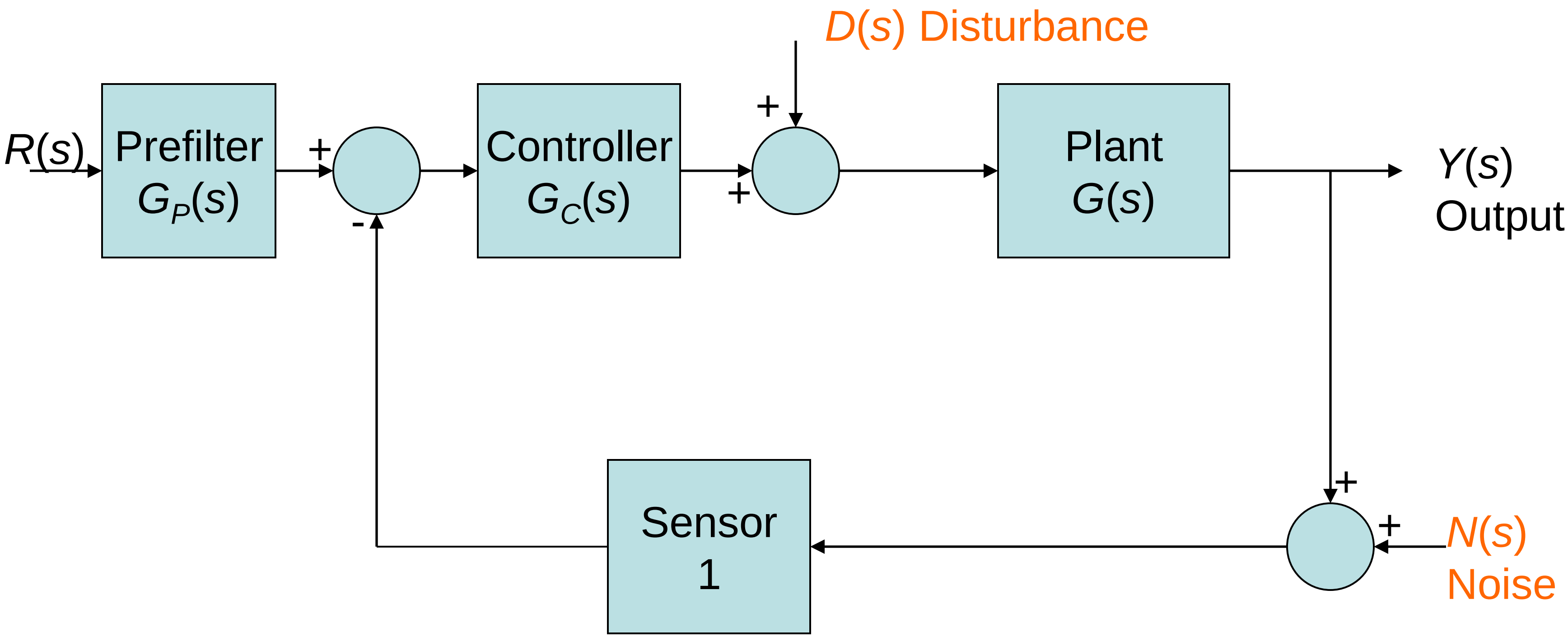
- When we design the control system we make numerous assumptions about the system and then we describe the system with some sort of mathematical model.

- Using a mathematical model permits us to make predictions about how the system will behave, and we can use any number of simulation tools and analytical techniques to make those predictions.

- Any model incorporates two important problems that are often encountered: a **disturbance signal** is added to the control input to the plant. That can account for wind gusts in airplanes, changes in ambient temperature in ovens, etc., and **noise** that is added to the sensor output.

A robust control system exhibits the desired performance despite the presence of significant plant (process) uncertainty

The goal of robust design is to retain assurance of system performance in spite of model inaccuracies and changes. A system is robust when it has acceptable changes in performance due to model changes or inaccuracies.



Why Feedback Control Systems?

- Decrease in the **sensitivity** of the system to variation in the parameters of the process *G*(*s*).

- Ease of control and adjustment of the **transient response** of the system.

- Improvement in the rejection of the **disturbance** and **noise** signals within the system.

- Improvement in the reduction of the **steady-state error** of the system

Sensitivity of Control Systems to Parameter Variations

- A process, represented by *G*(*s*), whatever its nature, is subject to a changing environment, aging, ignorance of the exact values of the process parameters, and the natural factors that affect a control process.
- The sensitivity of a control system to parameter variations is very important. A main advantage of a closed-loop feedback system is its ability to reduce the system's sensitivity.
- The system sensitivity is defined as the ratio of the percentage change in the **system** transfer function to the percentage change of the **process** transfer function.

$$T(s) = \frac{Y(s)}{R(s)}; \quad S(\text{sensitivity}) = \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)} = \frac{\partial T/T}{\partial G/G}$$

$$T(s) = \frac{G(s)}{1+GH(s)}; \; S_T^G = \frac{\partial T}{\partial G}.\frac{G}{T} = \frac{1}{(1+GH)^2}.\frac{G}{G/(1+GH)}$$

$$S_T^G = \frac{1}{1+G(s)H(s)} \left(\text{Less } S \text{ for larger } GH\right)$$

The sensitivity of the feedback system to changes in the feedback element *H*(*s*) is

$$S_H^T = \frac{\partial T}{\partial H} . \frac{H}{T} = \left(\frac{G}{1+GH} \right)^2 . \frac{-H}{G/\left(1+GH\right)} = \frac{-GH}{1+GH}$$

Often we need to determine S_α^T, where α is a **parameter** within the transfer function of a block G. Use the chain rule

$$S_\alpha^T = S_G^T \, S_\alpha^G = \frac{\partial T / T}{\partial \alpha / \alpha} \text{ (System Sensitivity)}$$

$$S_\alpha^{r_i} = \frac{\partial r_i}{\partial \alpha / \alpha} \text{ (Root sensitivity)}$$

Robust Control Systems and System Sensitivity

A control system is robust when: it has low sensitivities, (2) it is stable over the range of parameter variations, and (3) the performance continues to meet the specifications in the presence of a set of changes in the system parameters.

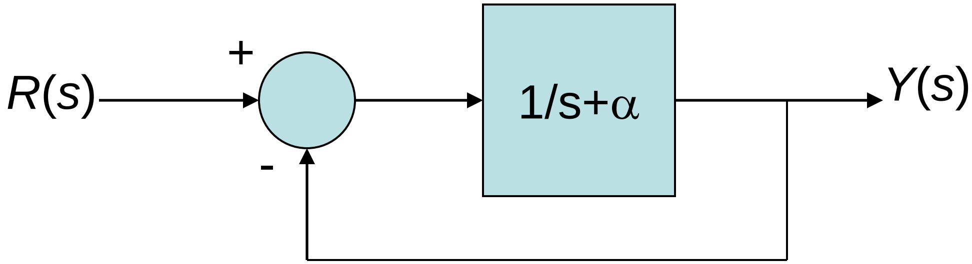
System sensitivity is: $S_\alpha^T = \dfrac{dT/T}{d\alpha/\alpha}$ (α is the parameter; T is the tranfer function)

Root Sensitivity is: $S_\alpha^{r_i} = \dfrac{dr_i}{d\alpha/\alpha}$ (zeros of $T(s)$ are independent of the parameter α)

$$S_\alpha^T = -\sum_{i=1}^{n} S_\alpha^{r_i} \cdot \frac{1}{(s+r_i)} = \frac{-\alpha}{s+\alpha+1}$$

The root is $r_1 = +(\alpha+1)$

$$-S_\alpha^T = -\alpha; -S_\alpha^T = -S_\alpha^{r_i} \frac{1}{(s+\alpha+1)}$$

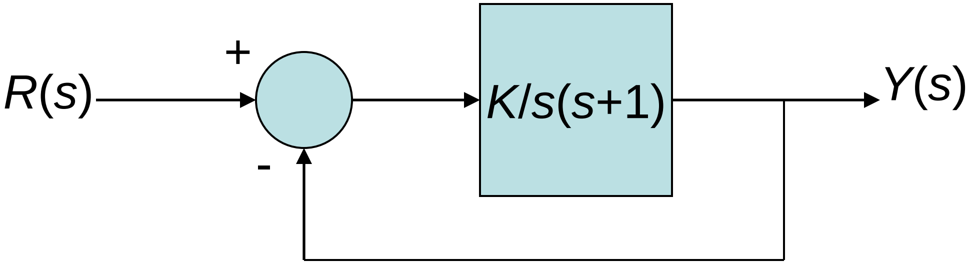


Let us examine the sensitivity of the following second-order system

$$T(s) = \frac{K}{s^2 + s + k}$$

We know from Eq. (4.12) that

$$S_K^T = \frac{1}{1 + GH(s)} = \frac{s(s+1)}{s^2 + s + K}$$



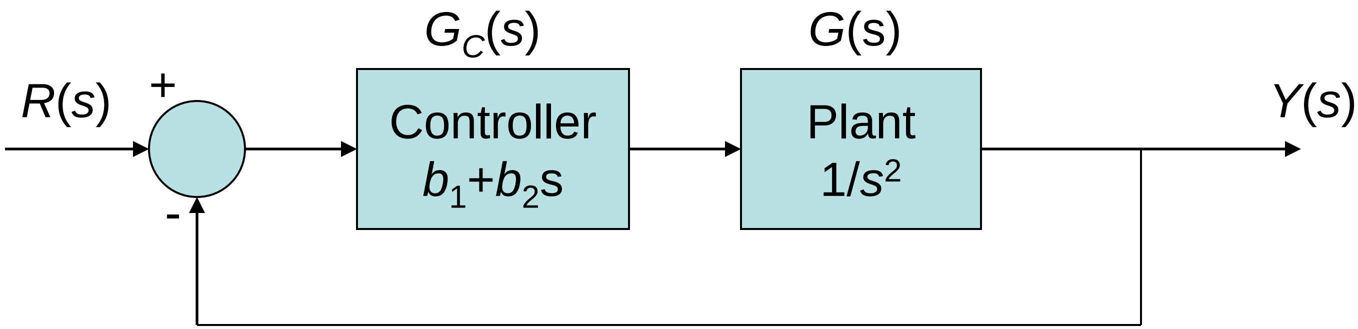
Sensitivity of a Controlled System

$G_C(s)$ is a proportional - derivative (PD) controller

$$S_G^T = \frac{1}{1+GG_C(s)} = \frac{s^2}{s^2+b_2 s+b_1}$$

$$T(s) = \frac{b_2 s+b_1}{s^2+b_2 s+b_1}$$

Consider the normal condition $\xi = 1$ and $\omega_n = \sqrt{b_1}$. Then $b_2 = 2\omega_n$ to get $\xi = 1$.



Plot $20\log|S|$ and $20\log|T|$ on a Bode diagram

Disturbance Signals in a Feedback Control System

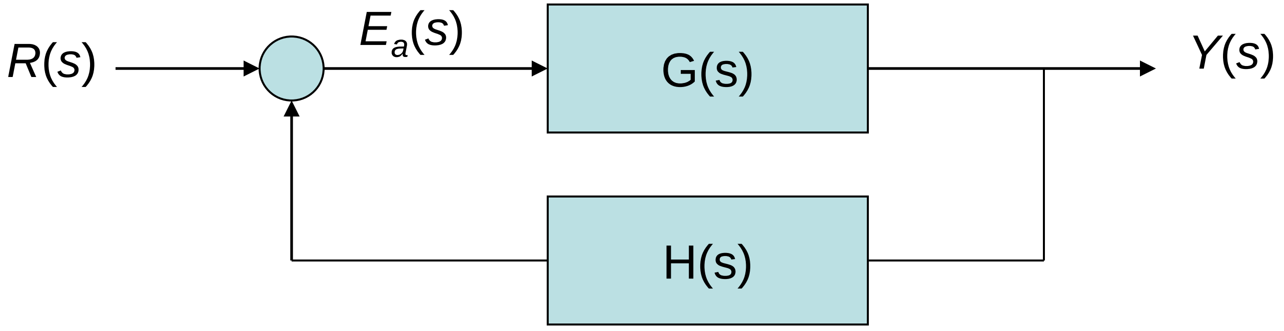
- Another important effect of feedback in a control system is the control and partial elimination of the effect of disturbance signal.

- A disturbance signal is an unwanted input signal that affects the system output signal. Electronic amplifiers have inherent noise generated within the integrated circuits or transistors; radar systems are subjected to wind gusts; and many systems generate all kinds of unwanted signals due to nonlinear elements.

- Feedback systems have the beneficial aspects that the effect of distortion, noise, and unwanted disturbances can be effectively reduced.

The Steady-State Error of a Unity Feedback Control System (5.7)

- One of the advantages of the feedback system is the reduction of the steady-state error of the system.
- The steady-state error of the closed loop system is usually several orders of magnitude smaller than the error of the open-loop system.
- The system actuating signal, which is a measure of the system error, is denoted as $E_a(s)$.



$$E(s) = R(s) - Y(s) = R(s) - \frac{G(s)}{1 + GH(s)} = \frac{1}{1 + G(s)} R(s) \text{ When } H(s) = 1$$

Compensator

- A feedback control system that provides an optimum performance without any necessary adjustments is rare. Usually it is important to compromise among the many conflicting and demanding specifications and to adjust the system parameters to provide suitable and acceptable performance when it is not possible to obtain all the desired specifications.

- The alteration or adjustments of a control system in order to provide a suitable performance is called **compensation**.

- A **compensator** is an additional component or circuit that is inserted into control system to compensate for a deficient performance.

- The transfer function of a compensator is designated as $G_C(s)$ and the compensator may be placed in a suitable location within the structure of the system.

Root Locus Method

- The root locus is a powerful tool for designing and analyzing feedback control systems.
- It is possible to use root locus methods for design when two or three parameters vary. This provides us with the opportunity to design feedback systems with two or three adjustable parameters. For example the PID controller has three adjustable parameters.
- The root locus is the path of the roots of the characteristic equation traced out in the s-plane as a system parameter is changed.
- Read Table 7.2 to understand steps of the root locus procedure.
- The design by the root locus method is based on reshaping the root locus of the system by adding poles and zeros to the system open loop transfer function and forcing the root loci to pass through desired closed-loop poles in the s-plane.

The root Locus Procedure

Step 1: The characteristic equation $1+GH(s)=1+\dfrac{K\left(\frac{1}{2}s+1\right)}{s\left(\frac{1}{4}s+1\right)}=0$

Step 2: The transfer function $GH(s)$ is written in terms of poles and zeros: $1+\dfrac{2K(s+2)}{s(s+4)}=0$

The multiplicative gain parameter is $2K$. To determine the locus of roots for the gain $0 \le K \le \infty$ (Step3) we locate the poles and zeros on the real axis.

Step 4: The angle criterion is satisfied on the real axis between the points 0 and -2, because the angle p1 at the origin is 180°, and the angle from the zero and pole p_2 at $s=-4$ is zero degrees.

The locus begins at the poles and ends at the zeros.

Step 5: Find the number of separate loci (equal to the number of poles).

Step 6: The root loci must be symmetrical with respect to the horizontal real axis.

Step 7: The loci proceed to the zeros at infinity along asymptotes centered at σ_A and with angle ϕ_A.

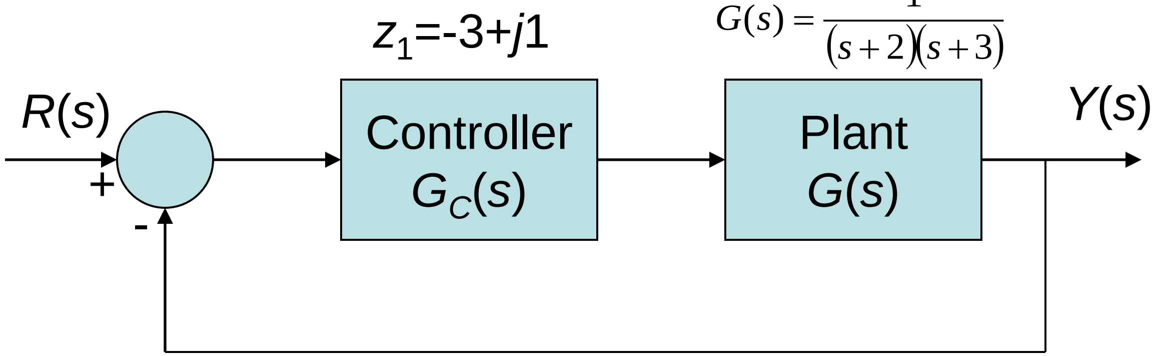
Step 8: Determine the point at which the locus crosses the imaginary axis.

Step 9: Determine the breakway point on the real axis.

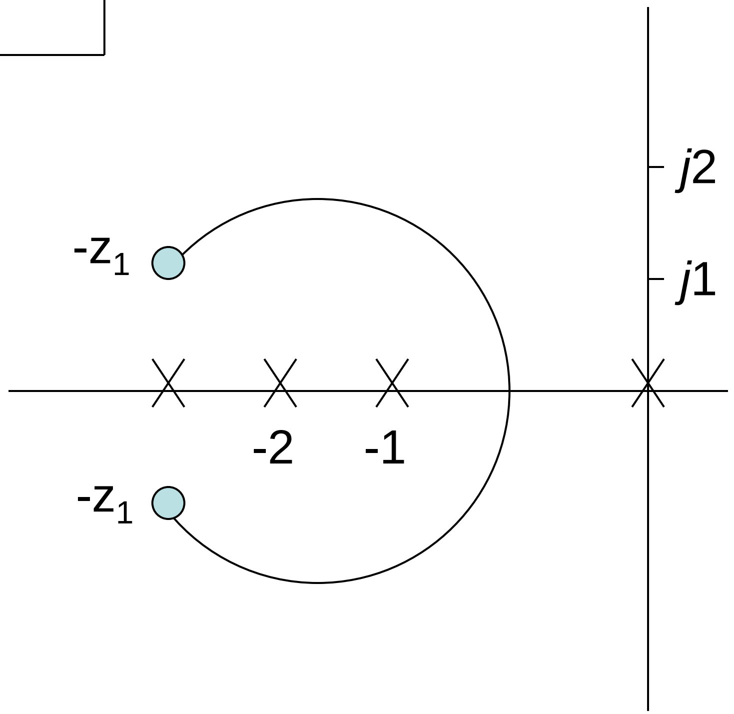
Step 10: Determine the angle of departure of the locus from a pole and the angle of arrival at a zero.

Example

$$G(s) = \frac{1}{(s+2)(s+3)}$$



$$T(S) = \frac{G(s)G_C(s)}{1+G(s)G_C(s)} = \frac{K3(s+z_1)(s+\hat{z}_1)}{(s+r_2)(s+r_1)(s+\hat{r}_1)}$$



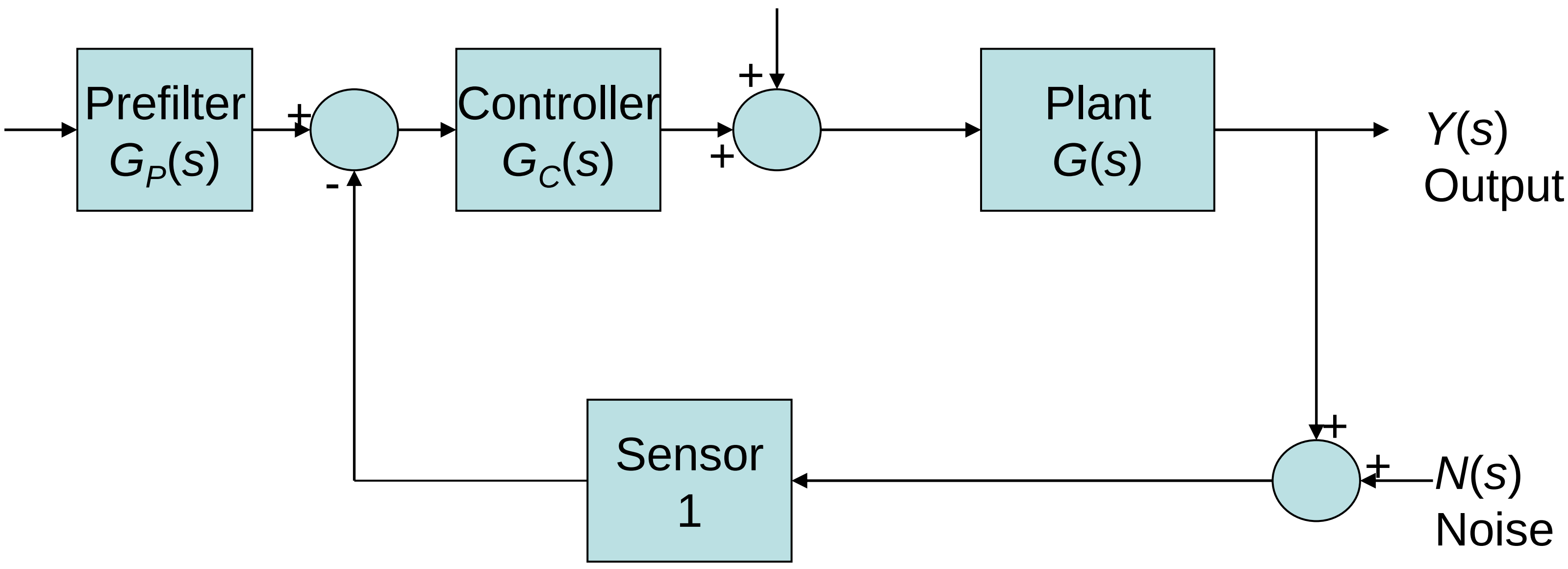
Analysis of Robustness

System goals: Small tracking error $[e(t) = r(t) - y(t)]$ for an input $r(t)$ and keep the output $y(t)$ small for a disturbance $d(t)$.

Sensor noise $n(t)$ must be small to $r(t)$ so $|r| \rangle\rangle |n|$

$S(s) = [1 + G_C(s)G(s)]^{-1}$. The closed-loop transfer function

$T(s) = \dfrac{G_C(s)G(s)}{1 + G_C(s)G(s)}$; When $G_P(s) = 1$, then $S(s) + T(s) = 1$; Better $S(s)$ small.



The Design of Robust Control Systems

- The design of robust control systems is based on two tasks: determining the structure of the controller and adjusting the controller's parameters to give an optimal system performance. This design process is done with complete knowledge of the plant. The structure of the controller is chosen such that the system's response can meet certain performance criteria.

- One possible objective in the design of a control system is that the controlled system's output should exactly reproduce its input. That is the system's transfer function should be **unity**. It means the system should be presentable on a Bode gain versus frequency diagram with a 0-dB gain of infinite bandwidth and zero phase shift. Practically, this is not possible!

- Setting the design of robust system requires us to find a proper compensator, $G_C(s)$ such that the closed-loop sensitivity is less than some tolerance value.

PID Controllers

PID stands for Proportional, Integral, Derivative. One form of controller widely used in industrial process is called a three term, or PID controller. This controller has a transfer function:

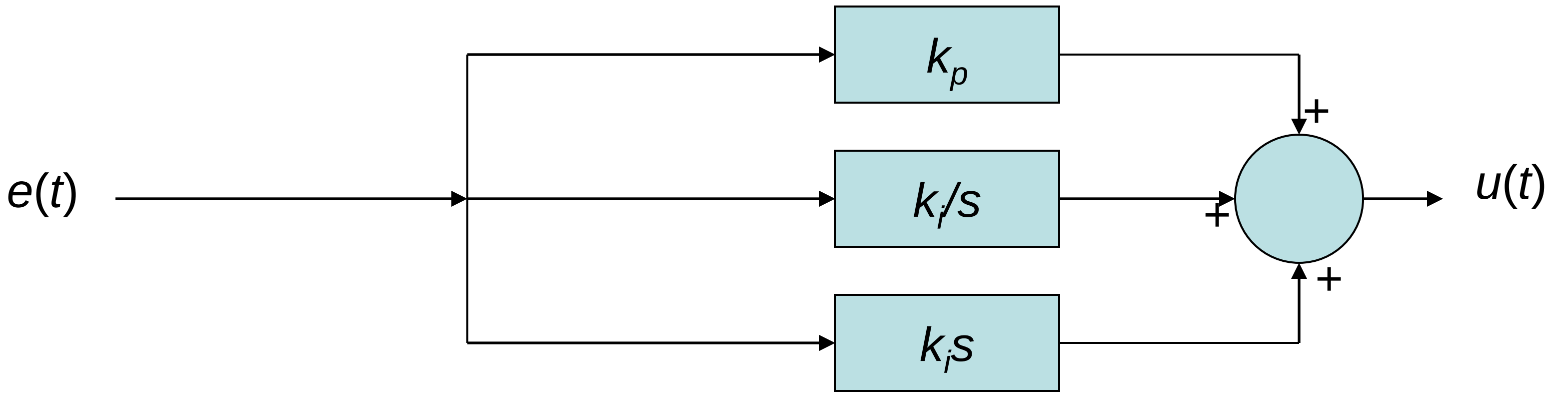
A proportional controller (K_p) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady state error. An integral control (K_I) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control (K_D) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

$$G_C(t) = K_p + \frac{K_I}{s} + K_D s$$

The controller provides a proportional term, an integration term, and a derivative term

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt}$$

Proportional-Integral-Derivative (PID) Controller



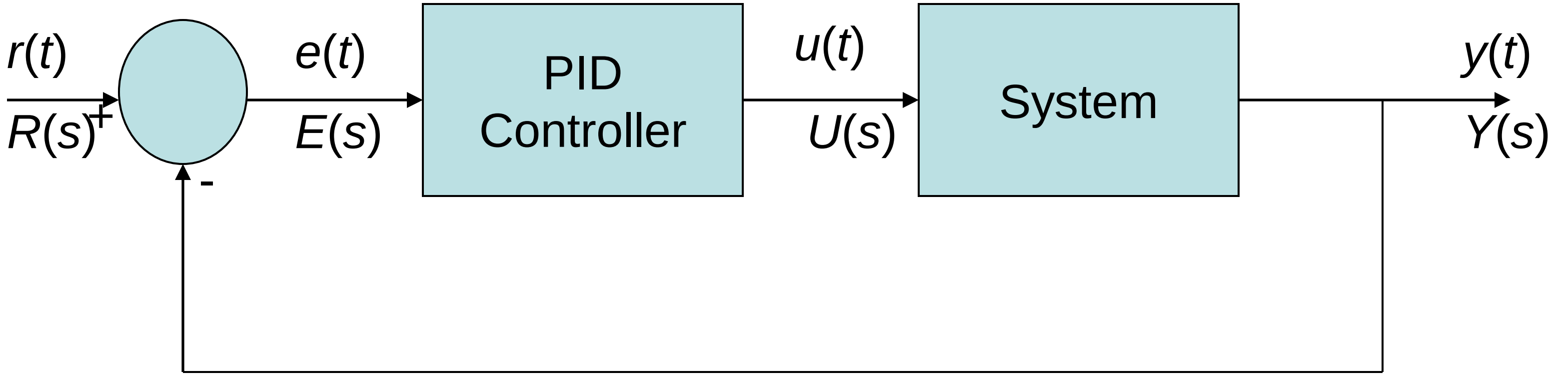
$$u(t) = K_p e(t) + K_I \frac{e(t)}{s} + K_D s e(t)$$

$e(t) = r(t) - y(t)$ is the error between the reference signal and the system output; K_P, K_I, and K_D are the proportional, integral, and derivative feedback gains, respectively.

$$U(s) = (K_p + \frac{K_I}{s} + K_D s)E(s)$$

$$G_{PID}(s) = \frac{U(s)}{E(s)} = \frac{K_D s^2 + K_P s + K_I}{s}$$

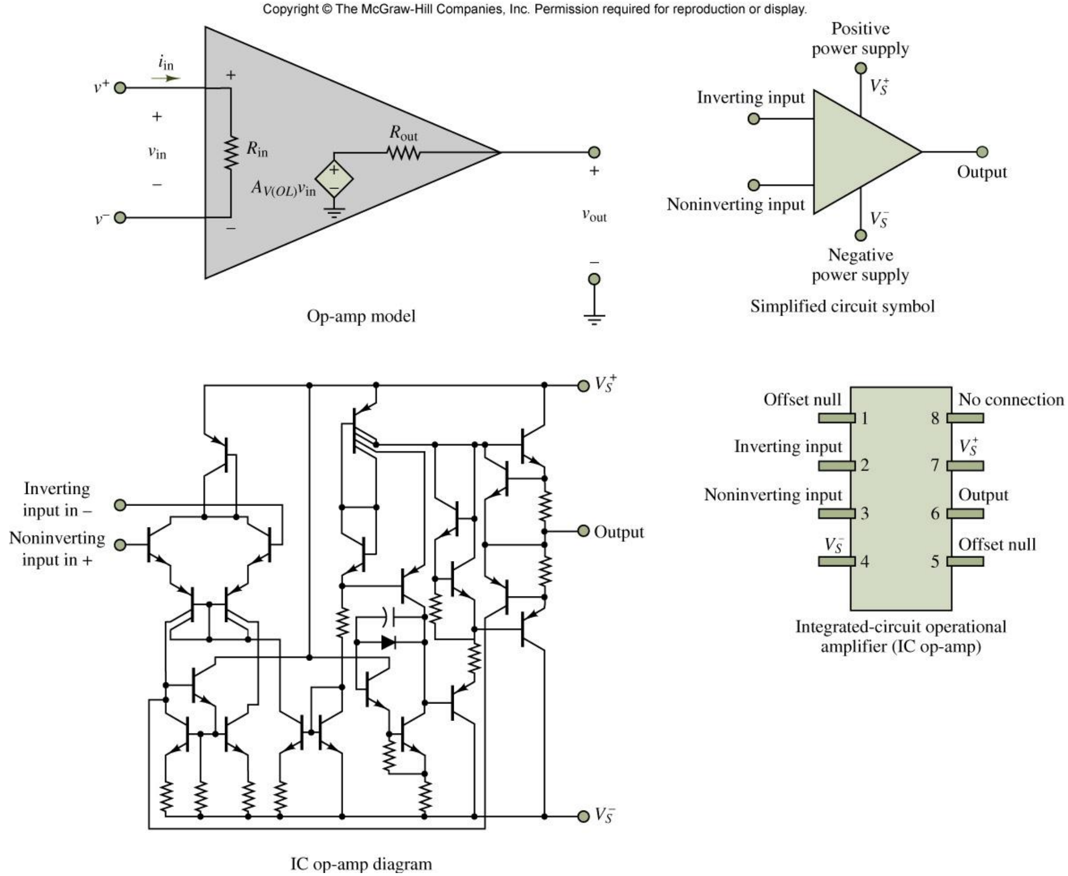
Time- and *s*-domain block diagram of closed loop system



$$G(s)=\frac{Y(s)}{R(s)}=\frac{G_{sys}(s)G_{PID}(s)}{1+G_{sys}(s)G_{PID}(s)}$$

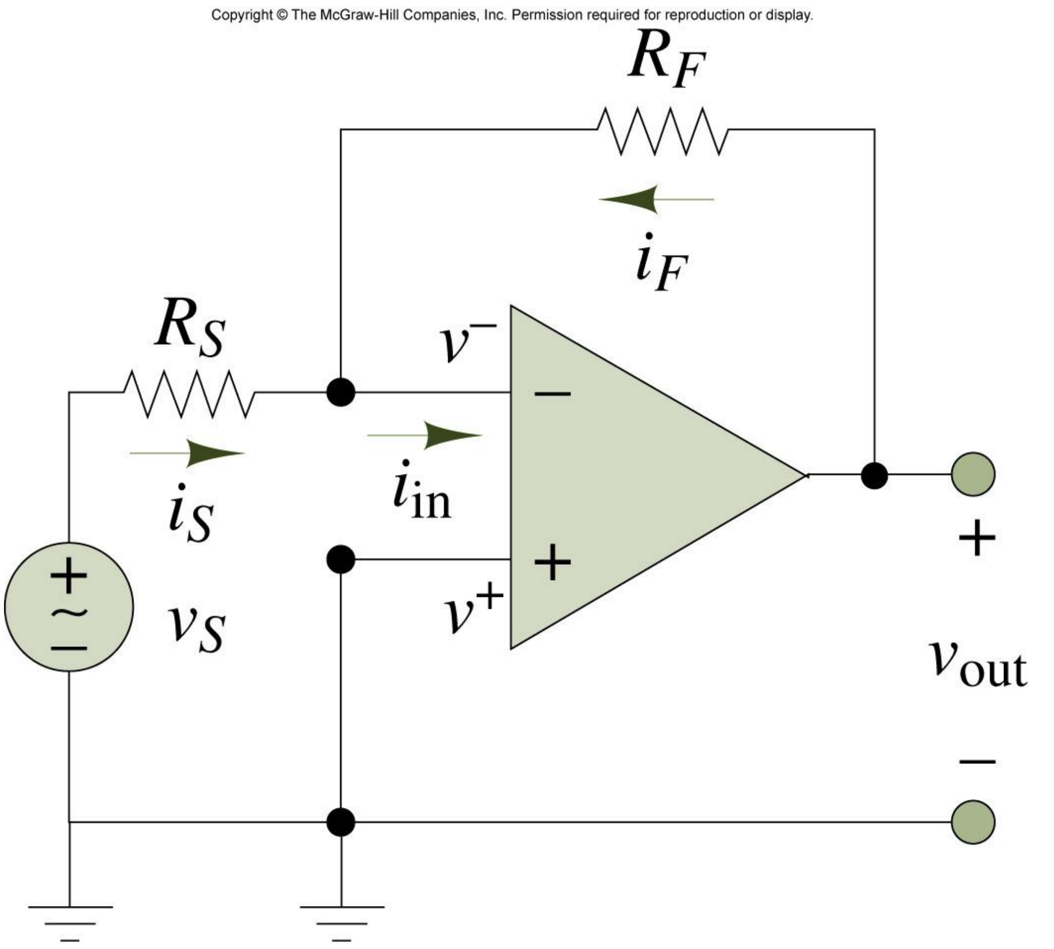
PID and Operational Amplifiers

A large number of transfer functions may be implemented using operational amplifiers and passive elements in the input and feedback paths. Operational amplifiers are widely used in control systems to implement PID-type control algorithms needed.



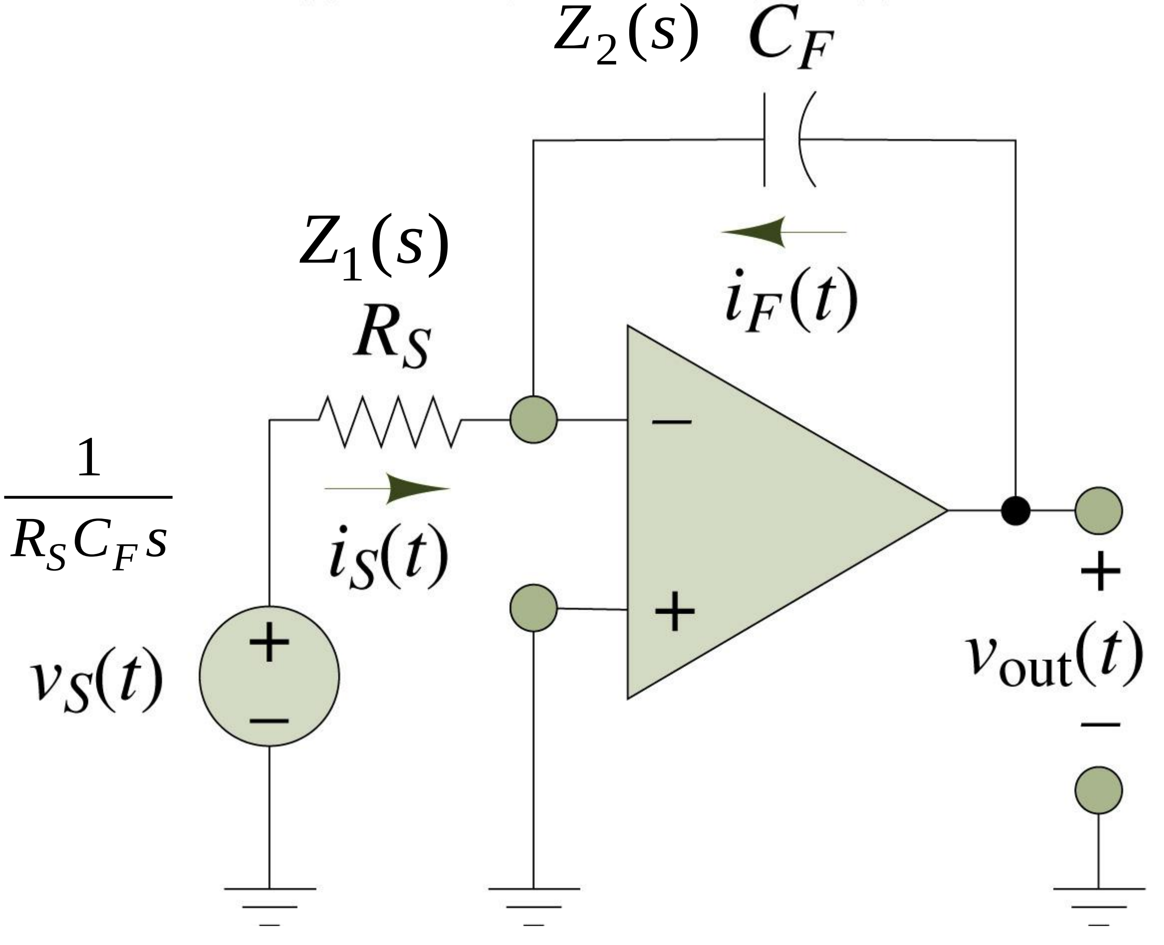
Inverting amplifier

$$\frac{V_o(t)}{V_S(t)} = -\frac{R_2}{R_1}$$



Op-amp Integrator

$$G(s) = \frac{V_{out}(s)}{V_s(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{1}{R_S C_F s}$$

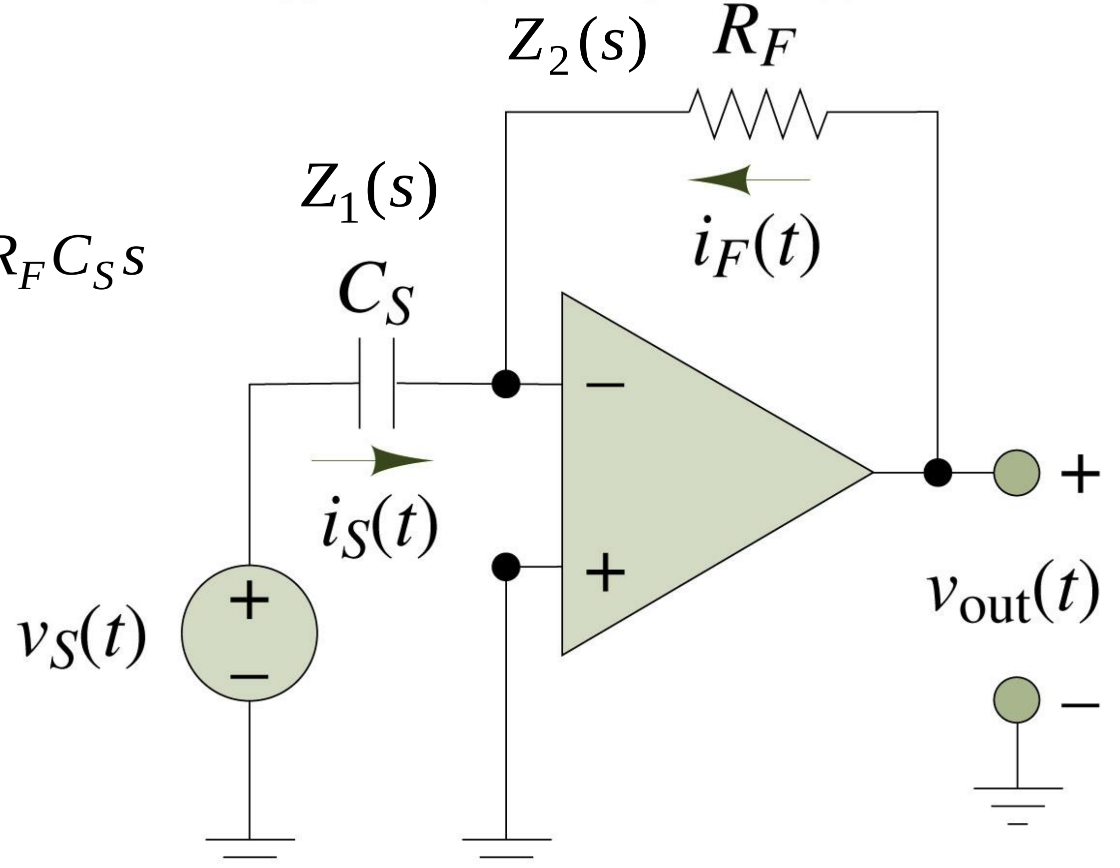


Op-amp Differentiator

The operational differentiator performs the differentiation of the input signal. The current through the input capacitor is $CS\ d_v s(t)/dt$. That is the output voltage is proportional to the derivative of the input voltage with respect to time, and

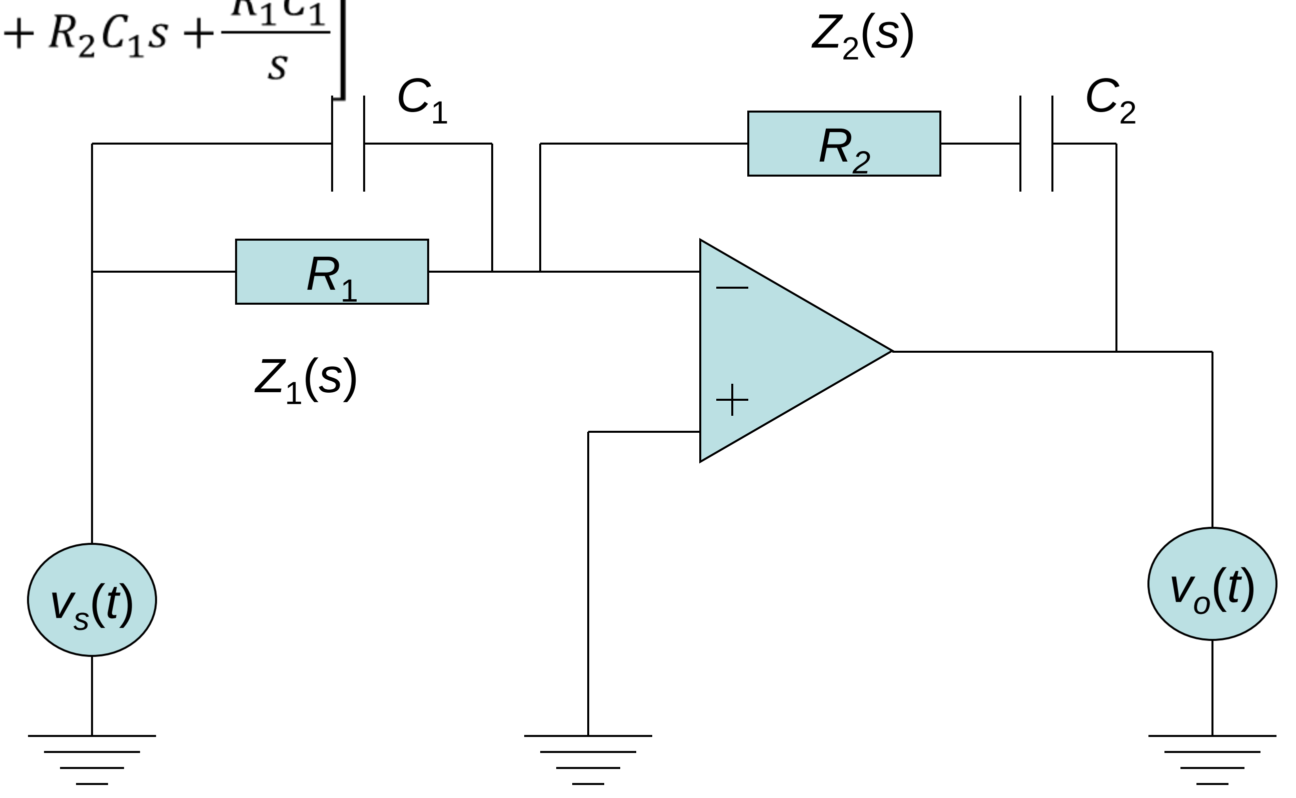
$$V_o(t) = -R_F C_S\, dv_s(t)/dt$$

$$G(s) = \frac{V_o(s)}{V_S(s)} = -\frac{Z_2(s)}{Z_1(s)} = -R_F C_S s$$



Linear PID Controller

$$\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_1}}{s}\right]$$



$$G(s)=\frac{V_o(s)}{V_S(s)}=-\frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1C_2s}=\frac{R_2C_1s^2+\frac{R_1C_1+R_2C_2}{R_1C_2}s+\frac{1}{R_1C_2}}{s}$$

$$G_{PID}(s)=\frac{K_Ds^2+K_Ps+K_I}{s};KP=-\frac{R_1C_1+R_2C_2}{R_1C_2};K_I=-\frac{1}{R_1C_2};K_D=-R_2C_1$$

Tips for Designing a PID Controller

When you are designing a PID controller for a given system, follow the following steps in order to obtain a desired response.

- Obtain an open-loop response and determine what needs to be improved
- Add a **proportional** control to improve the **rise time**
- Add a **derivative** control to improve the **overshoot**
- Add an **integral** control to eliminate the **steady-state error**
- Adjust each of Kp, K_I, and K_D until you obtain a desired overall response.

- It is not necessary to implement all three controllers (proportional, derivative, and integral) into a single system, if not needed. For example, if a PI controller gives a good enough response, then you do not need to implement derivative controller to the system.

The popularity of PID controllers may be attributed partly to their robust performance in a wide range of operation conditions and partly to their functional simplicity, which allows engineers to operate them in a simple manner.

$$G_C(t) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3 s^2 + K_1 s + K_2}{s}$$

$$= \frac{K_3\left(s^2 + as + b\right)}{s} = \frac{K_3\left(s + z_1\right)\left(s + z_2\right)}{s}$$

Where $a = K_1 / K_3$; and $b = K_2 / K_3$. Accordingly the PID introduces a transfer function with one pole at the origin and two zeros that can be located anywhere in the left-hand s-plane

Root Locus

Root locus begins at the poles and ends at the zeros.

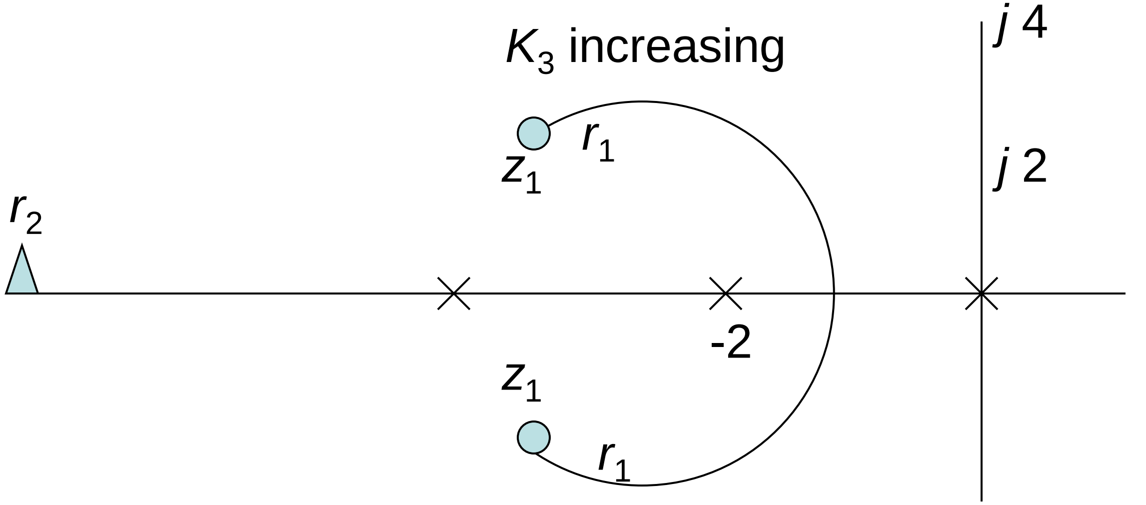
$$G(s) = \frac{1}{(s+2)(s+5)}$$

Assume we use a PID controller with complex zeros, we can plot the root locus. As K3 of the controller increases, the complex roots approaches the zero. The closed loop transfer function is

$$T(s) = \frac{G(s)G_C(s)G_P(s)}{1+G(s)G_C(s)} = \frac{K_3(s+z_1)(s+\hat{z}_1)}{(s+r_2)(s+r_1)(s+r_1)}G_P(s) = \frac{K_3 G_P(s)}{(s+r_2)}$$

Because the zeros and the complex roots are approximately equal. Setting $G_P(s)=1$, we have

$$T(s) = \frac{K_3}{s+r_2} \approx \frac{K_3}{s+K_3}$$; If K_3 is large, the system will have a fast response and zero steady state error.



Design of Robust PID-Controlled Systems

The selection of the three coefficients of PID controllers is basically a search problem in a three-dimensional space. Points in the search space correspond to different selections of a PID controller's three parameters. By choosing different points of the parameter space, we can produce different step responses for a step input.

The first design method uses the (integral of time multiplied by absolute error (ITAE) performance index in Section 5.9 and the optimum coefficients of Table 5.6 for a step input or Table 5.7 for a ramp input. Hence we select the three PID coefficients to minimize the ITAE performance index, which **produces an excellent transient response to a step (see Figure 5.30c)**. The design procedure consists of the following three steps.

$$\mathrm{ITAE} = \int_0^T t\left|e(t)\right|dt$$

The Three Design Steps of Robust PID-Controlled System

- **Step 1:** Select the ω_n of the closed-loop system by specifying the settling time.

- **Step 2:** Determine the three coefficients using the appropriate optimum equation (Table 5.6) and the ω_n of step 1 to obtain $G_C(s)$.

- **Step 3:** Determine a prefilter $G_P(s)$ so that the closed-loop system transfer function, $T(s)$, does not have any zero, as required by Eq. (5.47)

$$T(s) = \frac{Y(s)}{R(s)} = \frac{b_o}{s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_o}$$

Input Signals; Overshoot; Rise Time; Settling Time

- Step: $r(t) = A \quad R(s) = A/s$
- *Ramp:* $r(t) = At \quad R(s) = A/s^2$
- The performance of a system is measured usually in terms of step response. The swiftness of the response is measured by the rise time, T_r, and the peak time, T_p.
- The settling time, T_s, is defined as the time required for the system to settle within a certain percentage of the input amplitude.
- For a second-order system with a closed-loop damping constant, we seek to determine the time, T_s, for which the response remains within 2% of the final value. This occurs approximately when

$$e^{-\xi\omega_n T_s} \langle 0.02; \quad \xi\omega_n T_s \cong 4; \; T_s = 4\tau = \frac{4}{\xi\omega_n} (\omega_n : \text{undamped natural frequency}; \; \xi : \text{damping ratio})$$

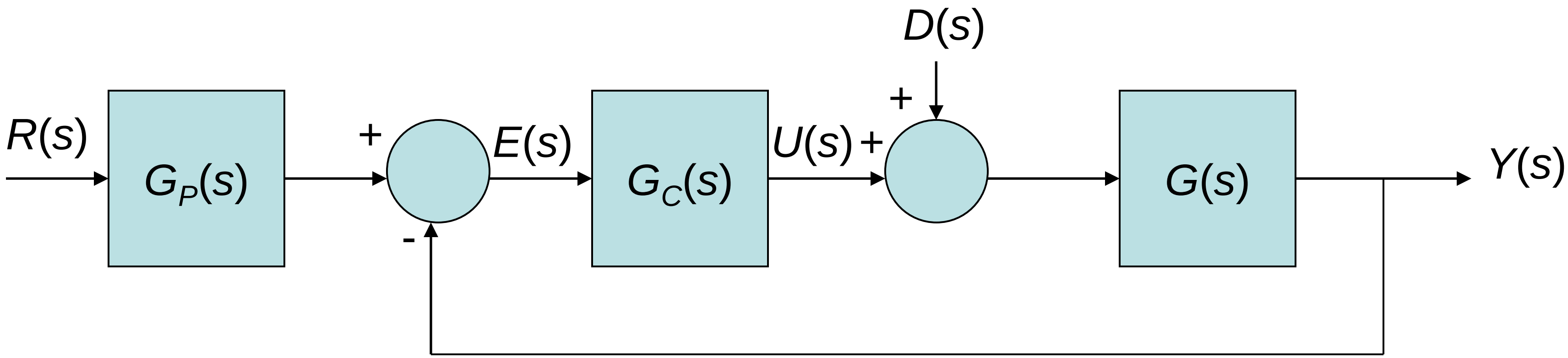
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ (Peak Time)}; M_p = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}} \text{ (Peak Response)}$$

$$\text{Percentage Overshoot (PO)} = 100e^{-\xi\pi/\sqrt{1-\xi^2}}$$

Effects of Poles and Zeros

- The response of a dominantly second order system is sped up by an additional zero and is slowed down by an additional pole.
- In the dominantly second-order system the added closed loop zero also has the important effect of increasing the amount of oscillation in the system while an added closed loop pole has the effect of decreasing the amount of oscillation.
- Added forward path zeros and added forward path poles have an opposite effect on the overshoot.
- A forward path pole which is too close to the origin may turn the closed loop system unstable.

Example: Robust Control of Temperature Using PID Controller employing ITAE performance for a step input and a settling time of less than 0.5 seconds.



$$G(s)=\frac{1}{(s+1)^2}=\frac{1}{s^2+2s+1}=\frac{1}{s^2+2\xi\omega_n s+\omega_n^2};\, e_{ss}=\frac{A}{1+K_p};\, K_p=\lim_{s\longrightarrow 0} G(s)=1;\, e_{ss}=\frac{1}{2}=50\%;\, \xi=\frac{2}{2}=1$$

If $G_C(s)=1$, the steady-state error is 50%, and the settling time (2% criterion) is 3.2 seconds for a step input.

We desire to obtain an optimim ITAE performance for a step input for a settling time of less than 0.5 seconds.

Using a PID controller : $G_C(s)=\dfrac{K_3s^2+K_1s+K_2}{s}$; The closed-loop transfer function with $G_P(s)=1$ is

$$T_1(s)=\frac{Y(s)}{R(s)}=\frac{G_CG(s)}{1+G_CG(s)}=\frac{K_3s^2+K_1s+K_2}{s^3+(2+K_3)s^2+(1+K_1)s+K_2}$$

The optimum coefficients of the characteristic equation for ITAE (Table 5.6) : $\left(s^3+1.7\omega_n s^2+2.15\omega_n^2 s+\omega_n^3\right)$

We need to select ω_n in order to meet the settling time requirement. $T_s = 4/\xi\omega_n$.
ξ is unknown but near 0.8, we set $\omega_n = 10$. Equate the denominator of the equation to the desired equation, we obtain the three coefficients as $K_1 = 214$, $K_3 = 15.5$, and $K_2 = 1000$.

$$T(s) = \frac{15.5s^2 + 214s + 1000}{s^3 + 17.5s^2 + 215s + 1000} = \frac{15.5(s + 6.9 + j4.1)(s + 6.9 - j4.1)}{s^3 + 17.5s^2 + 215s + 1000}$$

We select a prefilter $G_P(s)$ so that to achieve the desired ITAE response

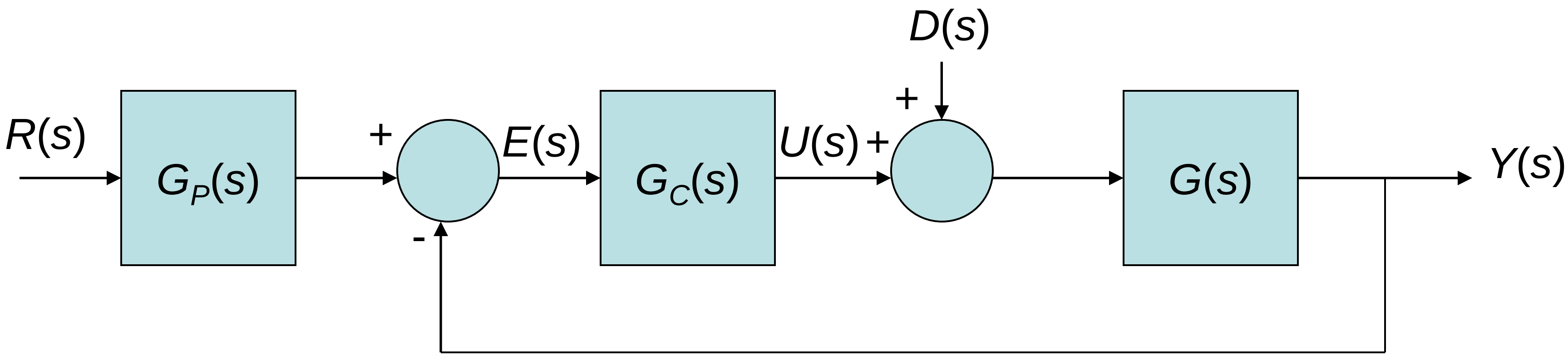
$$T(s) = \frac{G_c(s)GG_P(s)}{1 + GG_c(s)} = \frac{1000}{s^3 + 17.5s^2 + 215s + 1000}$$

Therefore we require $G_P(s) = \frac{64.5}{(s^2 + 13.8s + 64.5)}$ in order to eliminate the zeros in the previous equation and bring the overall numerator to 1000.

Results for Example

Controller	$\boldsymbol{G_C(s)=1}$	**PID &** $\boldsymbol{G_P(s)=1}$	**PID with** $\boldsymbol{G_P(s)}$ **Prefilter**
Percent overshoot	0	31.7%	1.9%
Settling time (s)	3.2	0.20	0.45
Steady-state error	50.1%	0.0%	0.0%
$y(t)/d(t)_{\text{maximum}}$	52%	0.4%	0.4%

Example: Using the ITAE performance method for step input, determine the required $G_C(t)$. Assume ω_n = 20 for Table 5.6. Determine the step response with and without a prefilter $G_P(s)$



$$G(s) = \frac{1}{s+1}; \text{Use a PI controller given by } G_C = K_1 + \frac{K_2}{s}$$

$$T(s) = \frac{G_C G(s)}{1 + G_C G(s)} = \text{Find it} = s^2 + (K_1 + 1)s + K_2$$

The ITAE characteristic equation is : $s^2 + 1.4\omega_n s + \omega_n^2$

When $\omega_n = 20$ then we have $K_1 = 27$ and $K_2 = 400$

Without a prefilter, the closed-loop system is $\frac{Y(s)}{R(s)} = \frac{27s + 400}{s^2 + 28s + 400}$

With a prefilter, the closed-loop gain is $\frac{Y(s)}{R(s)} = \frac{G_C(s) G Gp(s)}{1 + G_C G(s)} = \frac{400}{s^2 + 28s + 400}$

Where $G_P(s) = \frac{14.8}{s + 14.8}$; Draw the step response without and with the prefilter

A closed-loop unity feedback system has

$G(s) = \dfrac{9}{s(s+p)}$; $p = 3$; Find S_P^T and plot $T(j\omega)$ and $S(j\omega)$ on a Bode plot

The closed-loop transfer function is $T(s) = \dfrac{9}{s^2 + ps + 9}$

The sensitivity function is $S(s) = \dfrac{s^2 + ps}{s^2 + ps + 9}$

The sensitivity of T to changes in p is determined by

$$S_P^T = \frac{dT}{dp}\frac{p}{T} = -\frac{ps}{s^2 + ps + 9}$$

Then plot the relationship

A system has a plant $G(s)=\dfrac{15{,}900}{s\left(\dfrac{s}{100}+1\right)\left(\dfrac{s}{200}+1\right)}$ and a negative unity feedback with PD compensator $G_C(s)=K_1+K_2 s$. Design $G_C(s)$ so that the overshoot to a step is less than 20% and the settling time is less than 60 ms.

The open-loop transfer function is $GG_c=\dfrac{15900K_2\left(s+\dfrac{K_1}{K_2}\right)2\times10^4}{s(s+100)(s+200)}$

$$=\frac{K\left(s+\dfrac{K_1}{K_2}\right)}{s(s+100)(s+200)}$$ where $K=3.18\times10^8 K_2$. Select $K_1/K_2=100$

$$GG_c(s)=\frac{K}{s(s+200)}$$

The closed-loop transfer function is $T(s)=\dfrac{K}{s^2+200s+k}$

Let $\xi=0.5$ for P.O.$\langle$20%. $2\xi\omega_\mathrm{n}=200$; $\omega_\mathrm{n}=200$ and $\mathrm{K}=\omega_\mathrm{n}^2=40000$

The settling time is $T_S=\dfrac{4}{\xi\omega_n}=\dfrac{4}{100}=40$ ms; The controller $G_c(s)=0.00012(s+100)$

Example

$$G(s)=\frac{1}{s(s+10)} \text{(The space robot transfer function)}$$

(a) **Consider** $G_c(s)=K;\quad T(s)=\frac{GGc(s)}{1+GGc(s)}=\frac{K}{s^2+10s+K}$

For $PO \langle\, 4.5\%;\ \xi=0.702;\ K=50.73;\ Gc(s)=50.73$

(b) **Consider** PD controller : $G_c(s)=K_1+K_2 s$

$$T(s)=\frac{K_1+K_2 s}{s^2+(10+K_2)s+K_1}\text{; Use ITAE method : } K_1=100; K_2=4$$

$$G_c(s)=4s+100; G_p(s)=\frac{100}{4s+100}$$

$\omega_n = 10$

(c) **Consider** the PI controller : $G_c(s)=K_1+\frac{K_2}{s}=\frac{K_1 s+K_2}{s}$

$$T(s)=\frac{K_1 s+K_2}{s^3+10s^2+K_1 s+K_2}\text{; Use ITAE: } \omega_n=5.7; K_1=70.2; K_2=186.6$$

$$G_c(s)=70.2+186.6/s;\ G_p(s)=\frac{186.6}{70.2s+186.6}$$

$$\text{(d) } \textbf{Consider} \text{ PID controller}: G_c(s) = \frac{K_1 s^2 + K_2 s + K_3}{s}$$

$\omega_n = 10$

$$T(s) = \frac{K_1 s^2 + K_2 s + K_3}{s^3 + 10s^2 + K_1 s^2 + K_2 s + K_3}; \text{ Use ITAE with } \omega_n = 10$$

$$K_1 = 7.5; K_2 = 215; K_3 = 1000; G_c(s) = \frac{7.5s^2 + 215s + 1000}{s}$$

$$Gp(s) = \frac{1000}{7.5s^2 + 215s + 1000}$$

Find a summary of the performance for the four cases: K, PD, PI, and PID

Performance means overshoot, settling time, and peak time

$$PO = 100e^{-\xi\pi l\sqrt{1-\xi^2}} \text{ (Eq 5.15)}; \; t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ (Eq. 5.14)}$$

Physical Realization of PID Compensators

PI Controller

$$-\frac{R_2}{R_1}\frac{(s+\frac{1}{R_2C})}{s}$$

PD Controller

$$-R_2C\left(s+\frac{1}{R_1C}\right)$$

PID Controller

$$\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{\frac{1}{R_1C_1}}{s}\right]$$