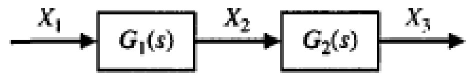
Table 2.6 Block Diagram Transformations

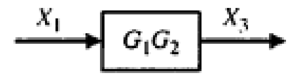
Transformation | **Original Diagram** | **Equivalent Diagram**

1. Combining blocks in cascade

Original Diagram:



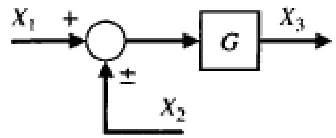
Equivalent Diagram:



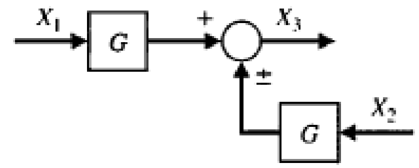
or

2. Moving a summing point behind a block

Original Diagram:

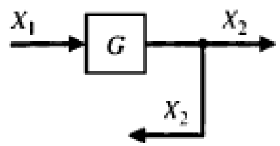


Equivalent Diagram:

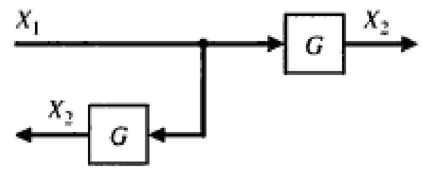


3. Moving a pickoff point ahead of a block

Original Diagram:

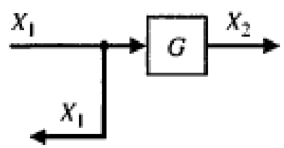


Equivalent Diagram:

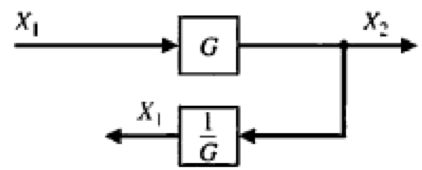


4. Moving a pickoff point behind a block

Original Diagram:

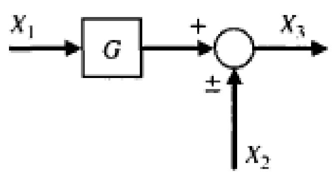


Equivalent Diagram:

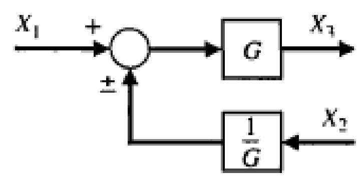


5. Moving a summing point ahead of a block

Original Diagram:

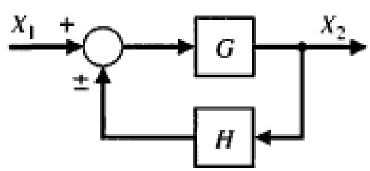


Equivalent Diagram:

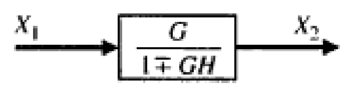


6. Eliminating a feedback loop

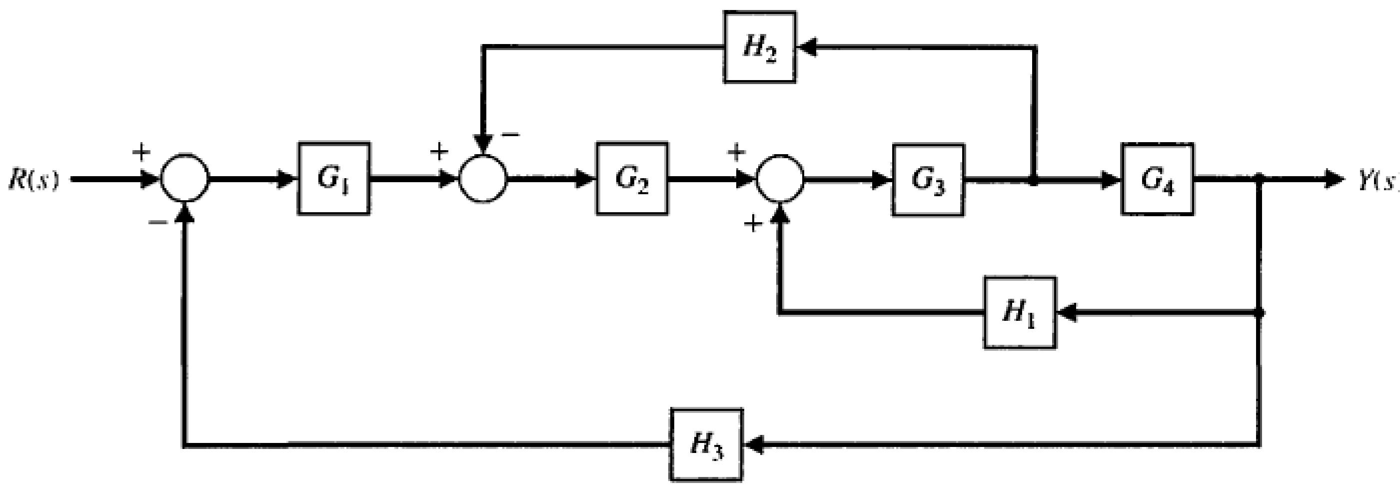
Original Diagram:

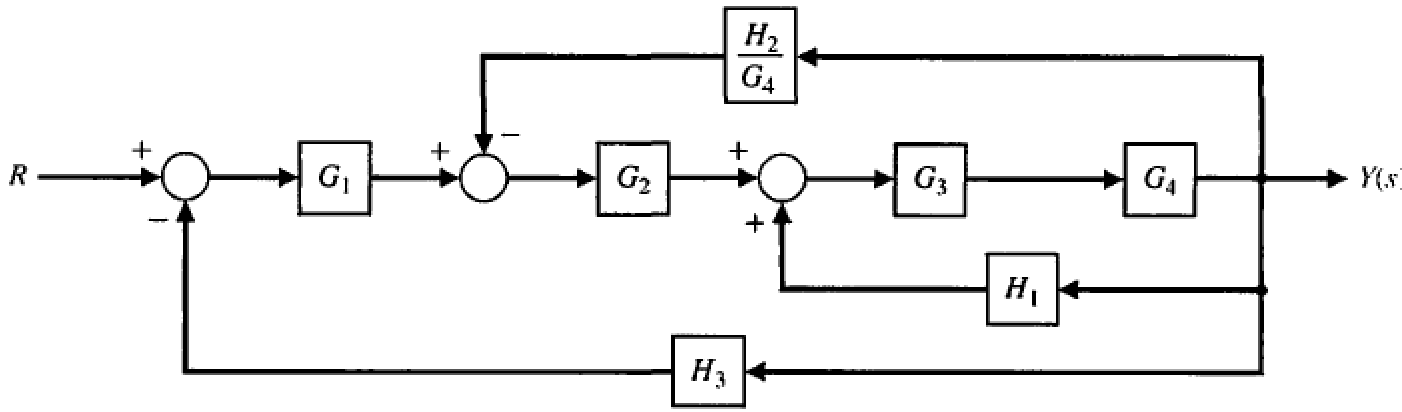


Equivalent Diagram:

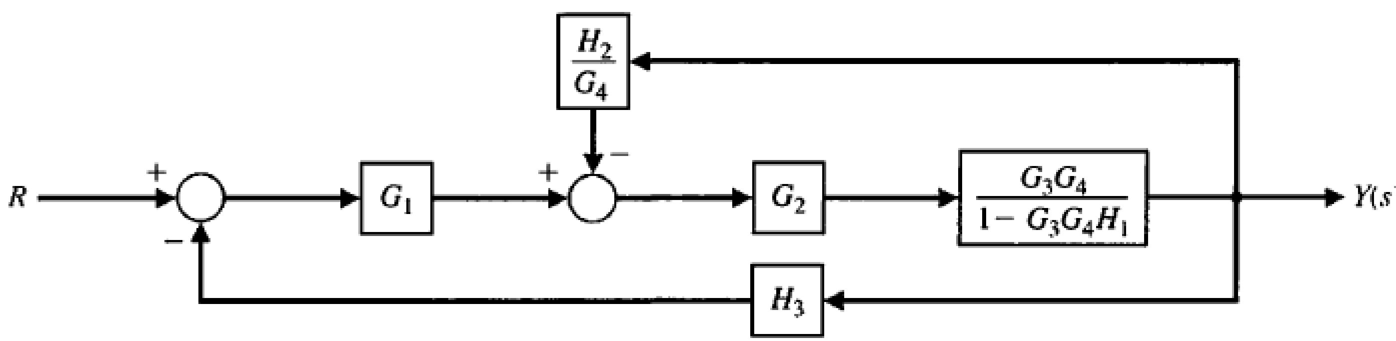


Multiple-loop feedback control system

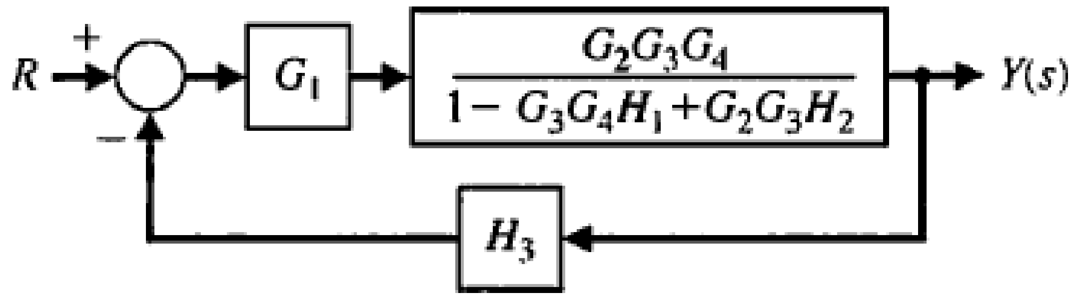


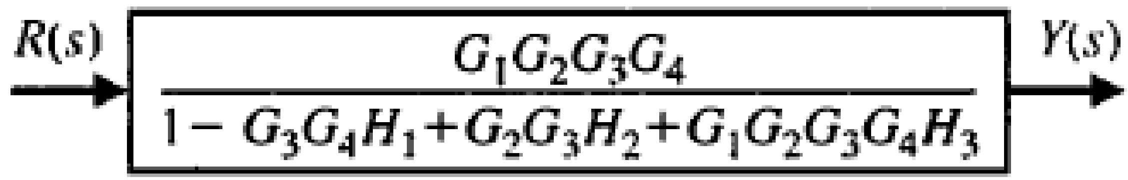


R
+
−
G_1
+
−
$\frac{H_2}{G_4}$
G_2
+
+
G_3
G_4
$Y(s)$
H_1
H_3



R
+
−
G_1
+
−
$\frac{H_2}{G_4}$
G_2
$\frac{G_3G_4}{1-G_3G_4H_1}$
$Y(s)$
H_3

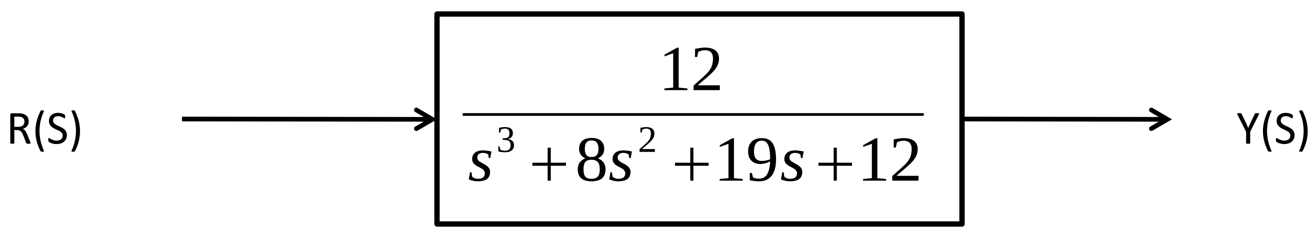

R
+
−
G_1
$\frac{G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}$
$Y(s)$
H_3


$R(s)$
$\frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}$
$Y(s)$

A system represented by the following figure

(a) determine the partial fraction expansion and y(t) for a ramp input, r(t) = t, t>=0

(b)Determine the impulse response of the system y(t)



(a) *given* $R(s)=1/s^2$, *the partial expansion is*

$$Y(s)=\frac{12}{s^2(s+1)(s+3)(s+4)}=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+1}+\frac{D}{s+3}+\frac{E}{s+4}$$

$$A=\lim_{s\to 0}[\frac{12}{s^2(s+1)(s+3)(s+4)}s^2]=\lim_{s\to 0}[\frac{12}{(s+1)(s+3)(s+4)}]=1$$

$$B=\lim_{s\to 0}[\frac{d}{ds}\frac{12}{s^2(s+1)(s+3)(s+4)}s^2]=\lim_{s\to 0}[\frac{d}{ds}\frac{12}{(s+1)(s+3)(s+4)}]=-\frac{19}{12}$$

$$C=\lim_{s\to -1}[\frac{12}{s^2(s+1)(s+3)(s+4)}(s+1)]=2$$

$$D=\lim_{s\to -3}[\frac{12}{s^2(s+1)(s+3)(s+4)}(s+3)]=-\frac{2}{3}$$

$$E=\lim_{s\to -4}[\frac{12}{s^2(s+1)(s+3)(s+4)}(s+4)]=\frac{1}{4}$$

$$Y(s)=\frac{1}{s^2}-\frac{19/12}{s}+\frac{2}{s+1}-\frac{2/3}{s+3}+\frac{1/4}{s+4}$$

Therefore, using Laplace Transform Table, we have

$$y(t)=t-\frac{19}{12}+2e^{-t}-\frac{2}{3}e^{-3t}+\frac{1}{4}e^{-4t},\quad t\geq 0$$

(*b*) *given* $R(s) = 1$, *the partial expansion is*

$$Y(s) = \frac{12}{(s+1)(s+3)(s+4)} = \frac{2}{s+1} - \frac{6}{s+3} + \frac{4}{s+4}$$

Therefore, using Laplace Transform Table, we have

$$y(t) = 2e^{-t} - 6e^{-3t} + 4e^{-4t}, \quad t \geq 0$$

$$\frac{5}{s^2(s+1)^3} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+1)^3} + \frac{D}{(s+1)^2} + \frac{E}{(s+1)}$$

$$A = \lim_{s \to 0}\left[\left(\frac{5}{s^2(s+1)^3}\right)s^2\right] = \lim_{s \to 0}\left[\frac{5}{(s+1)^3}\right] = 5$$

$$B = \lim_{s \to 0}\left[\frac{d}{ds}\left(\frac{5}{s^2(s+1)^3}\right)s^2\right] = \lim_{s \to 0}\left[\frac{d}{ds}\left(\frac{5}{(s+1)^3}\right)\right] = \lim_{s \to 0}\left[\frac{5(-3)}{(s+1)^4}\right] = -15$$

$$C = \lim_{s \to -1}\left[\left(\frac{5}{s^2(s+1)^3}\right)(s+1)^3\right] = \lim_{s \to -1}\left[\frac{5}{s^2}\right] = 5$$

$$D = \lim_{s \to -1}\left[\frac{1}{1!}\frac{d}{ds}\left(\frac{5}{s^2(s+1)^3}\right)(s+1)^3\right] = \lim_{s \to -1}\left[\frac{1}{1!}\frac{d}{ds}\frac{5}{s^2}\right] = \lim_{s \to -1}\left[\frac{1}{1!}\frac{-2(5)}{s^3}\right] = 10$$

$$E = \lim_{s \to -1}\left[\frac{1}{2!}\frac{d}{ds}^2\left(\frac{5}{s^2(s+1)^3}\right)(s+1)^3\right] = \lim_{s \to -1}\left[\frac{1}{2!}\frac{d}{ds}^2\frac{5}{s^2}\right] = \lim_{s \to -1}\left[\frac{1}{2!}\frac{30}{s^4}\right] = 15$$

$$\frac{5}{s^2(s+1)^3} = \frac{5}{s^2} + \frac{-15}{s} + \frac{5}{(s+1)^3} + \frac{10}{(s+1)^2} + \frac{15}{(s+1)}$$