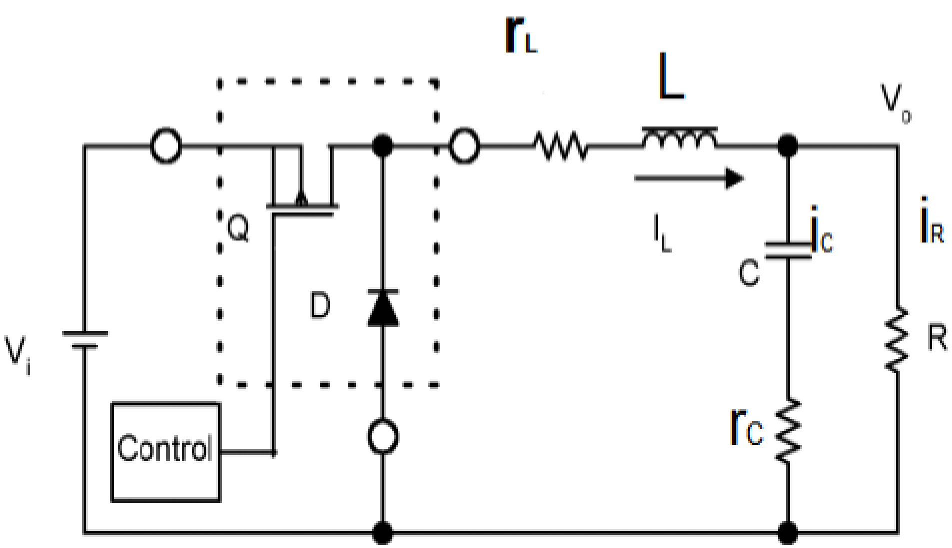
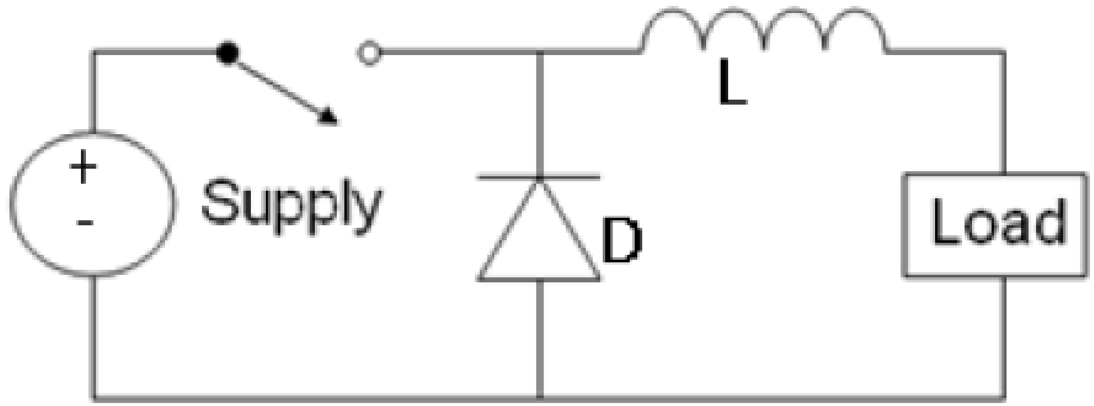
ELG4156
State Space Averaging of DC to DC Converters



Introduction

- A few applications of interest of DC-DC converters are where 5V DC on a personal computer motherboard must be stepped down to 3V, 2V or less for one of the latest CPU chips.
- DC to DC converters are widely used in hybrid cars which is our main focus to alter DC energy from a particular level to other with minimum loss.
- The need for converters is in demand due to the fact that transformers are unable to operate on DC.
- A converter is not producing power. Whatever comes at the output has to come only from input. Efficiency cannot be made equal to 100%.

Buck Converter

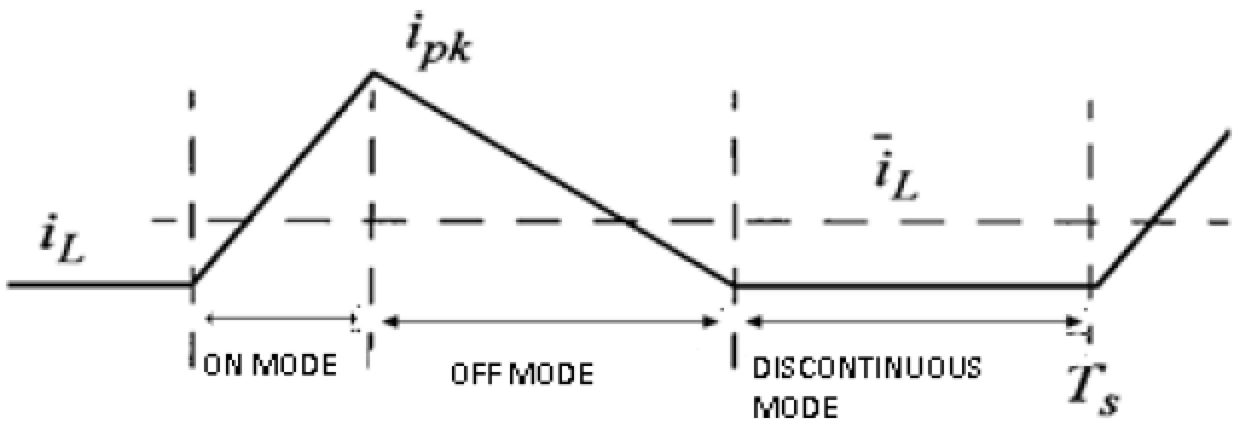


When the switch is closed, the voltage across the inductor is $V_L = V_i - V_o$. The current through inductor linearly rises. The diode does not allow current to flow through it, since it is reverse-biased by voltage V.

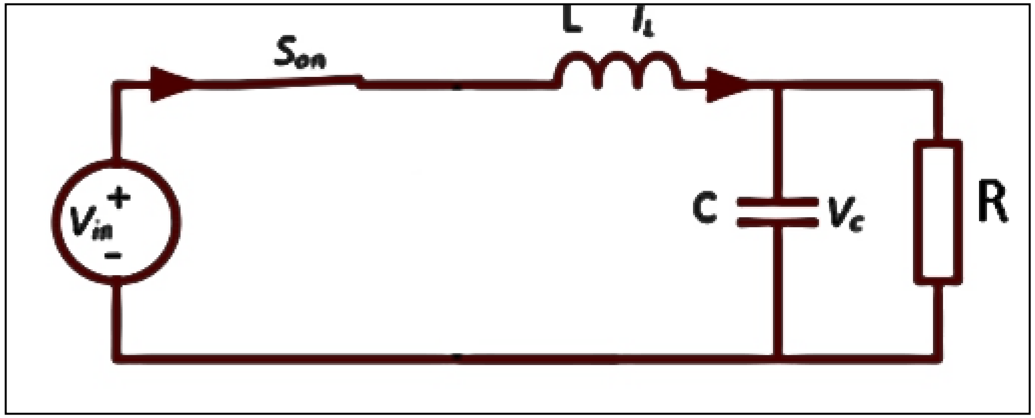
For Off state, diode is forward biased and voltage is $V_L = - V_o$ across inductor. The inductor current which was rising in ON case, now decreases.

Buck Modes of Operation

$d_1 T_S = ON\ Period\ time$
$d_2 T_S = OFF\ Period\ time$
$T_S = Total\ time\ period\ for\ one\ cycle$
$i_{pk} = peak\ value\ of\ inductor\ current\ after\ ON\ period$
$\bar{i_L} = Average\ value\ of\ current$
$V_{in} = input\ voltage$



Buck Converter During ON Mode
Write the State Space Model



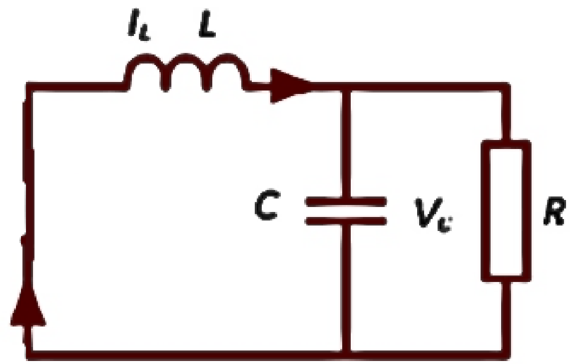
From KVL $\quad v_{in} - L\frac{di_L}{dt} - v_C = 0$

From KCL $\quad \frac{v_c}{R} + C\frac{dv_C}{dt} - i_L = 0$

Write

$$-\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \quad \\ \quad \end{bmatrix} v_{in} \; ; V_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

Buck Converter During Off Mode
Write State Space Model



From KVL $\quad v_C + L\frac{di_L}{dt} = 0$

Write

From KCL $\quad i_L - \frac{v_C}{R} - \frac{Cdv_C}{dt} = 0$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in} \; ; V_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ V_C \end{bmatrix}$$

During Discontinuous Conduction Mode

From KVL $\frac{di_L}{dt} = 0$

From KCL $\frac{v_c}{R} + C\frac{dv_C}{dt} = 0$

Write

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} \quad & \quad \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in} \; ; v_o = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

Buck Modelling Analysis

- Averaging
- Inductor current analysis
- Duty-ratio constraint.

State space averaging techniques are employed to get a set of equations that describe the system over one switching period.

$$\dot{\bar{X}} = [A_1 d_1 + A_2 d_2 + A_3(1 - d_1 - d_2)]\bar{x} + [B_1 d_1 + B_2 d_2 + B_3(1 - d_1 - d_2)]u$$

$$\overline{i_L} = \frac{i_{pk}}{2} . (d_1 + d_2)$$

The Final Model

The state space averaged model for the above equation is

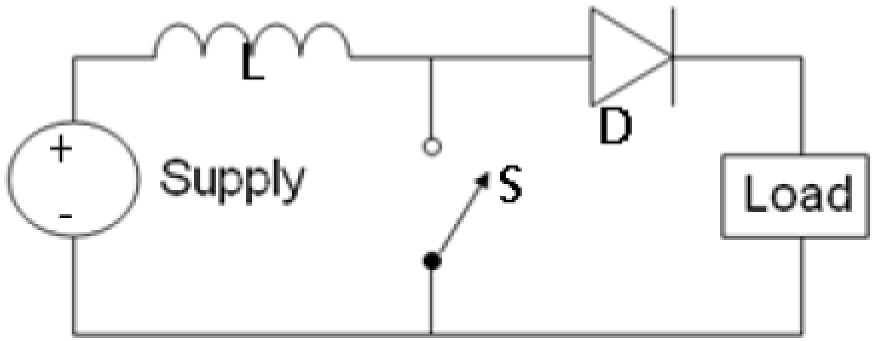
Write

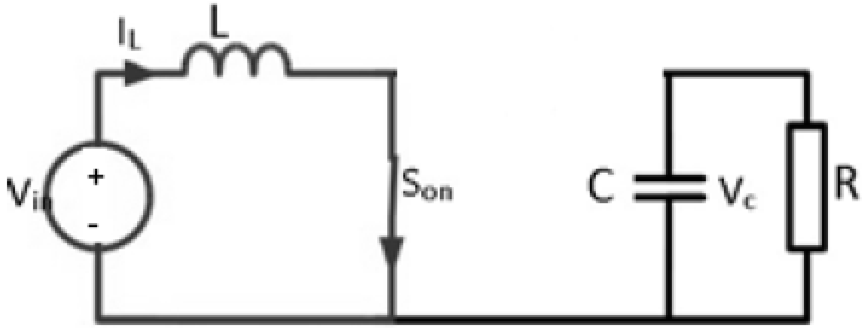
$$\frac{d}{dt}\begin{bmatrix}\bar{i}_L \\ \bar{v}_C\end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix}\bar{i}_L \\ \bar{v}_C\end{bmatrix} + \begin{bmatrix}\frac{d_1}{L} \\ 0\end{bmatrix} V_{in}$$

$$\frac{d}{dt}\begin{bmatrix}\bar{i}_L \\ \bar{v}_C\end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} K \begin{bmatrix}\bar{i}_L \\ \bar{v}_C\end{bmatrix} + \begin{bmatrix}\frac{d_1}{L} \\ 0\end{bmatrix} V_{in} \qquad K = \begin{bmatrix}\frac{1}{d1+d2} & 0 \\ 0 & 1\end{bmatrix}$$

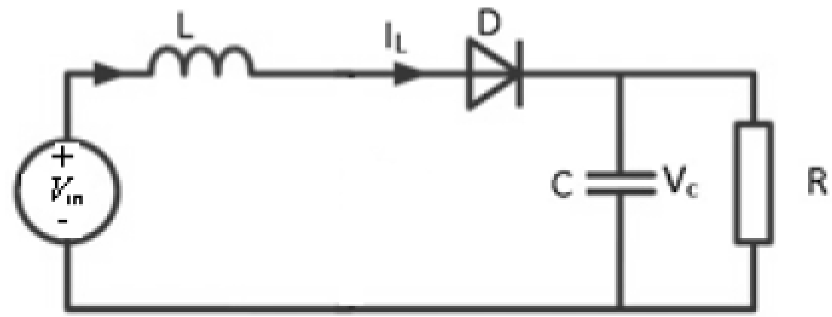
$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix}\bar{i}_L \\ \bar{v}_C\end{bmatrix} + \begin{bmatrix}\frac{d1}{L} \\ 0\end{bmatrix} V_{in}$$

Boost Converter
Write the State Space Model





On



Off