DGDnotes: about Chapter 11: State variable model

Hints about assignment 1:

E11.5 and E11.6:

1. How to convert an time domain equation to compact matrix expression:

For example: if $y = ax_1(t) + bx_2(t) + \ldots$, $y = Cx = \begin{bmatrix} a & b & \cdots \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$.

2. How to determine whether a system is controllable or observable:

eq 11.2 and 11.3 in your text book.

P11.12:

A dc motor has a transfer function $G(s) = \dfrac{10}{s^2(s+1)(s^2+2s+2)}$. Determine whether this system is controllable and observable.

1. we need to know the matrix A,B,C in the state variable model to determine whether the system is controllable and observable. So first, we have to convert the given transfer function into state variable model. The conversion may be not unique, varying according to the states you choose.

$$G(s) = \frac{10}{s^2(s+1)(s^2+2s+2)} = \frac{10}{s^5+3s^4+4s^3+2s^2} = \frac{10s^{-5}}{1+3s^{-1}+4s^{-2}+2s^{-3}}$$

use the phase variable format or input feed forward format of signal flow graph in Chapter 3.4, we can get the matrix A,B,C, and D for the state space representation.

2. use eq 11.2 and 11.3 in your text book to check whether the system is controllable and observable

P11.13:

A feedback system has a plant transfer function $\dfrac{Y(s)}{R(s)} = G(s) = \dfrac{K}{s(s+70)}$. It is desired that the velocity error constant K_v be 35 and the overshoot to a step input be approximately 4% so that ξ is $\frac{1}{\sqrt{2}}$. The setting time (2% criterion) desired is 0.11 second. Design an appropriate state variable feedback system.

1. when we are asked to design a state variable feedback system, the first thing we should have is the state variable model of the system. So in this problem we need convert s domain transfer function to time domain state space representation.

   $\dot{x} = Ax + Bu$

   $y = Cx$

2. define the feedback, usually we use negative feedback, the general format is

   $u = -Kx = \begin{bmatrix} -k_1 & -k_2 & \cdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$. In some case the format of the feedback already defined in the problem. In this case just use the given format.

   $\dot{x} = Ax + Bu = Ax - B(-K)x = (A - BK)x = Hx$

   $y = Cx$

3. check what information is given about the performance (performance specifications).

a) In this problem, given velocity error constant K_v be 35. According to chapter 5.7,

$Kv = \lim_{s \to 0} sG(s)$, where $G(s) = \frac{K}{s(s+70)}$. This help us get *K*.

b) the problem gives us the parameters for overshoot, setting time. According to

Chapter 5.3, the percentage overshoot, P.O. is defined as $P.O. = 100e^{-\xi\pi / \sqrt{1-\xi^2}}$.

the setting time is defined as $Ts = 4\tau = \frac{4}{\xi\omega_n}$. These parameter are in s domain characteristic equation. So we need to get the closed-loop characteristic equation of the feedback system.

In general, $G(s) = \frac{Y(s)}{X(s)} = C[sI - A]^{-1}B = \frac{C \cdot matrix \cdot B}{\Delta}$, the characteristic equation is $\Delta = \det[sI - A]$. In this problem the closed-loop feedback system has $\Delta = \det[sI - H] = s^2 + 2\xi\omega_n s + \omega_n^2$. It is going to a function of K=[k1 k2].

And $Ts = 4\tau = \frac{4}{\xi\omega_n} = 0.11$, $P.O. = 100e^{-\xi\pi/\sqrt{1-\xi^2}}$ =4, given ξ is $\frac{1}{\sqrt{2}}$, we can get ω_n. Then we get the parameter K.

Done.

AP11.2

A system has a plant $G(s) = \frac{3s^2 + 4s - 2}{s^3 + 3s^2 + 7s + 5}$. Add state variable feedback so that the closed-loop poles are s=-4,-4, and –5.

1. convert the transfer function to state space representation.

   $\dot{x} = Ax + Bu$

   $y = Cx$

2. add the state variable feedback $u = -Kx = \begin{bmatrix} -k_1 & -k_2 & \cdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix}$.

   $\dot{x} = Ax + Bu = Ax - B(-K)x = (A - BK)x = Hx$

3. given the closed-loop poles are –4, -4, -5. It means the characteristic equation of the closed loop transfer function is (s+4)(s+4)(s+5) and the characteristic equation= $\Delta = \det[sI - H] = f(K)$, then we get K.

AP11.3:

A system has a matrix differential equation $\dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u$.

What values for b1 and b2 are required so that the system is controllable?

To be controllable, the determinant of the controllability matrix Pc should not be zero. $Pc = \begin{bmatrix} B & AB \end{bmatrix}$, $\det(Pc) = f(b_1, b_2) \neq 0$

AP11.5:

A system and it state variable feedback is define in figure AP11.5. determine the parameters K2, k3 to keep the poles of the closed-loop system between –3 and –6, also select Kp so that the steady-state error for a step input is equal to zero.

1. find the transfer function from block diagram ;
2. according to the steady error from chapter 4.5,

   steady-error= $\lim_{s\to 0} sE(s) = 0$. Where

$$E(s) = R(s) - Y(s) = R(s) - G(s)R(s) = (1 - G(s))R(s) \text{ and } R(s) = \frac{1}{S}.$$

3. to keep the roots of the characteristic equation are three real root and lying between –3 to –6, the characteristic equation=(s-s1)(s-s2)(s-s3), then we can choose the appropriate k2,k3.

DP11.4:

DP11.4 A high performance helicopter has a model as

$$\frac{d^2\theta}{dt^2} = -\sigma_1 \frac{d\theta}{dt} - \alpha_1 \frac{dx}{dt} + n\delta$$

$$\frac{d^2x}{dt^2} = g\theta - \alpha_2 \frac{d\theta}{dt} - \sigma_2 \frac{dx}{dt} + g\delta \; .$$

The goal is to control the pitch angle θ by adjust the rotor angle δ, where *x* is the translation in the horizontal direction.

Given: σ_1 =0.415 α_1 =1.43 n=6.27

σ_2 =0.0198 α_2 =0.0111 g=9.8

the state variable representation of the system;

As stated in the problem, the input of the system is δ, the output of the system is θ. To be clear in notation, we use *z* instead *x* for the translation in the horizontal direction. Then the given system is $\frac{d^2\theta}{dt^2} = -\sigma_1 \frac{d\theta}{dt} - \alpha_1 \frac{dz}{dt} + n\delta$

$$\frac{d^2z}{dt^2} = g\theta - \alpha_2 \frac{d\theta}{dt} - \sigma_2 \frac{dz}{dt} + g\delta$$

a) Set the state variable as $\mathrm{x} = \begin{bmatrix} \theta \\ \dot{\theta} \\ \dot{z} \end{bmatrix}$, $\dot{\mathrm{x}} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \ddot{z} \end{bmatrix}$, $y = \theta = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathrm{x}$.

b) From Chapter 3 Eq 3.71, we knew that transfer function of a system is

$G(s) = \dfrac{Y(s)}{X(s)} = C[sI - A]^{-1}B$.

c) let feedback as $\delta = \begin{bmatrix} -k_1 & -k_2 & -k_3 \end{bmatrix}\mathrm{x}$, the characteristic equation is

$\Delta = \det[sI - H] = s^2 + 2\xi\omega_n s + \omega_n{}^2$, using the given performance specifications,

we can get the solution for K, where Ts=$\dfrac{4}{\zeta\omega_{\mathrm{n}}} < 1.5$, P.O=$100e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$<20.