ELG4152: Problems from Chapter 11

Problem 1: Consider the system represented in state variable form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

Where

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ -5 & 10 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\mathbf{C} = \begin{bmatrix} 1 & -4 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 \end{bmatrix}$$

Verify that the system is observable. If so, design a full-state observer by placing the observer poles at $s_{1,2} = -1$.

Solution:
The observability matrix is

$$\mathbf{P}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 21 & -36 \end{bmatrix} = 48$$

The system is observable.
The desired poles of the observer are $s_{1,2} = -1$. So the desired characteristic equation is

$$s^2 + 2s + 1$$

The actual characteristic equation is

$$\text{Det } \left[s\mathbf{I} - (\mathbf{A} - \mathbf{LC}) \right] = \text{Det} \begin{bmatrix} s - 1 + L_1 & -4 - 4L_1 \\ 5 + L_2 & s - 10 - 4L_2 \end{bmatrix}$$
$$= s^2 + (L_1 - 4L_2 - 11)\, s + 10L_1 + 8L_2 + 30 = 0$$

Comparing the actual equation with the desired equation yields

$$\mathbf{L} = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} -0.25 \\ -3.3125 \end{bmatrix}$$

Problem 2: Consider the third order system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} u$$
$$y = \begin{bmatrix} 2 & -4 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix}$$

Verify that the system is observable. If so, determine the observer gain matrix required to place the observer poles at $s_{1,2} = -1 \pm 2j$ and $s_3 = -10$.

Answer:

$$\mathbf{L} = \begin{bmatrix} 5.45 \\ 0.48 \\ 1.24 \end{bmatrix}$$

Problem 3: Consider the second order system

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathrm{x} + \begin{bmatrix} 0 \end{bmatrix} u$$

Determine the observer gain matrix required to place the observer poles at $s_{1,2} = -1 \pm 1j$