

Filters and Tuned Amplifiers

The word filter refers to the process of removing undesired portions of the frequency spectrum.

The word active filters implies the use of one or more active devices, usually op amps.

An example of the application of op amps in the area of active filters is Butterworth filter.

Filter Design

- Can we use a simple passive filter (i.e., only passive components such as resistors, capacitors and inductors; no operational amplifiers).
- The advantages of a passive filter are that it is quite simple to design and implement. It also provides a simple single pole or two pole filter whose electrical response can be easily calculated. For a single pole low-pass filter, $f_c = 1 / (2 \pi RC)$ the filter roll-off is 6 dB per octave or 20 dB/decade. However, this filter does have some noticeable drawbacks:
 - It is very sensitive to the component value tolerances.
 - For low frequencies, the values of R and C can be quite large, leading to physically large components.
 - A first or second-order filter may not give adequate roll-off.
 - If gain is required in the circuit, it cannot be added to the filter itself.
 - The filter may have a high output impedance. Since the resistor value is large, to keep the capacitors with reasonable values, the next stage device can see a significant source impedance.
 - An op amp can be added at the output, but why add the op amp here when it can be used to improve the filter's performance in addition to lowering the output impedance?

Two-Port Network

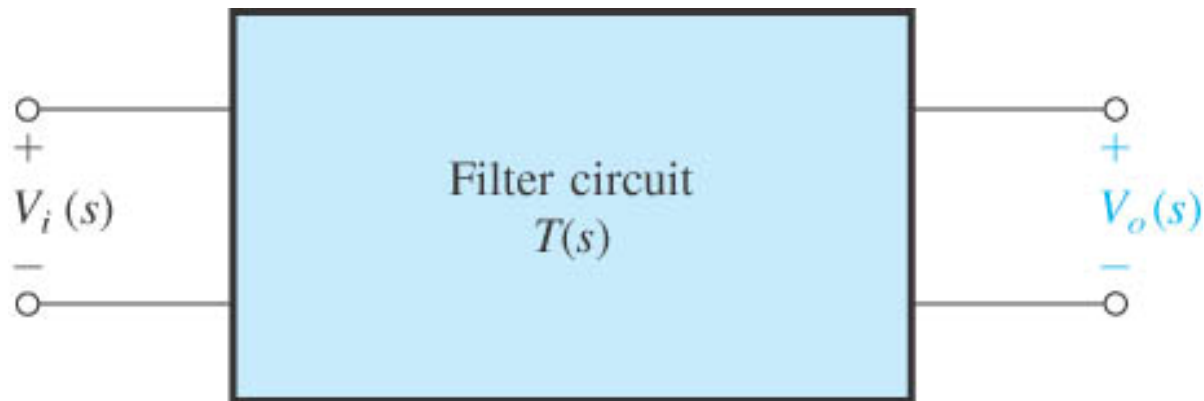
$$\text{Transfer function } T(s) = \frac{V_o(s)}{V_i(s)}$$

Filter transmiss

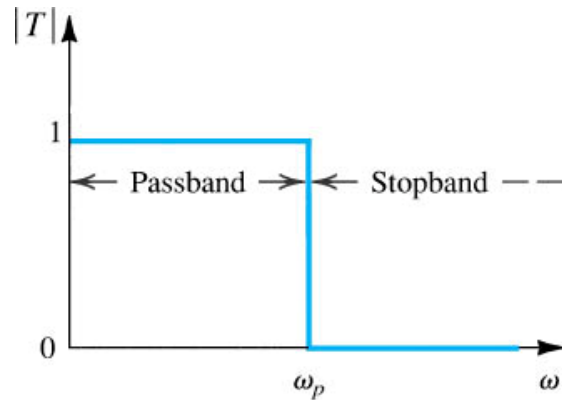
$$\text{ion } T(j\omega) = |T(j\omega)|e^{j\phi(\omega)}$$

$$\text{Gain function } G(\omega) = 20 \log |T(j\omega)|$$

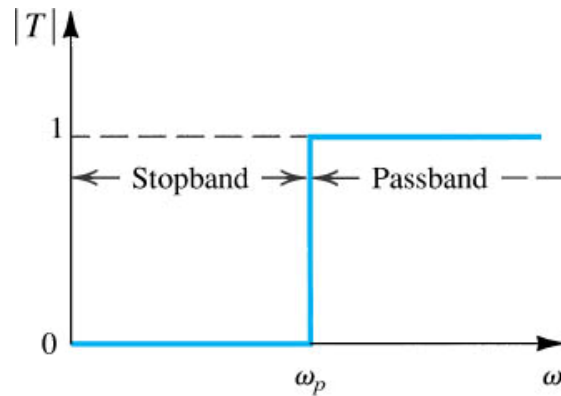
$$\text{Attenuation function } A(\omega) = -20 \log |T(j\omega)|$$



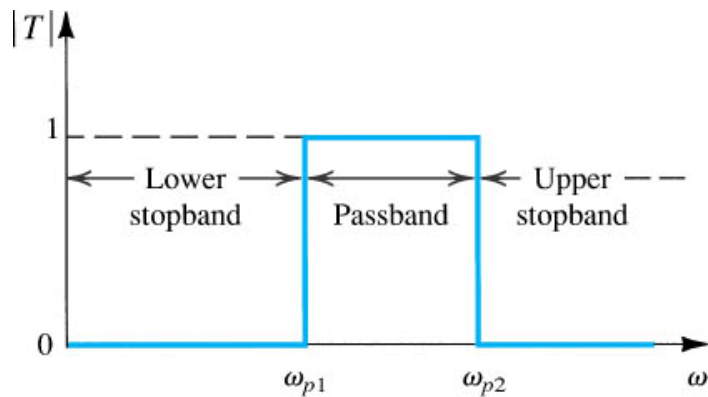
Transmission Characteristics



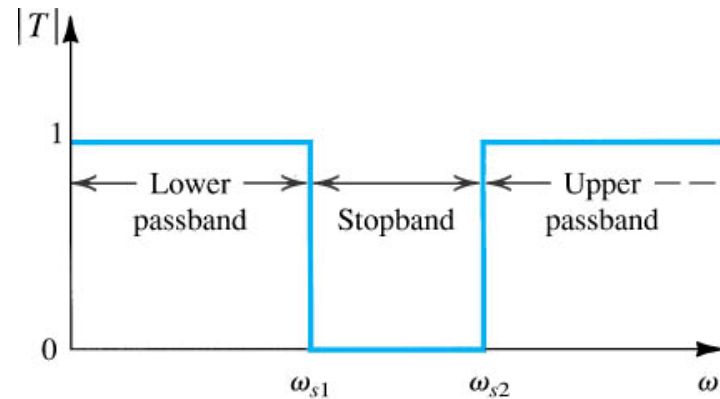
(a) Low-pass (LP)



(b) High-pass (HP)

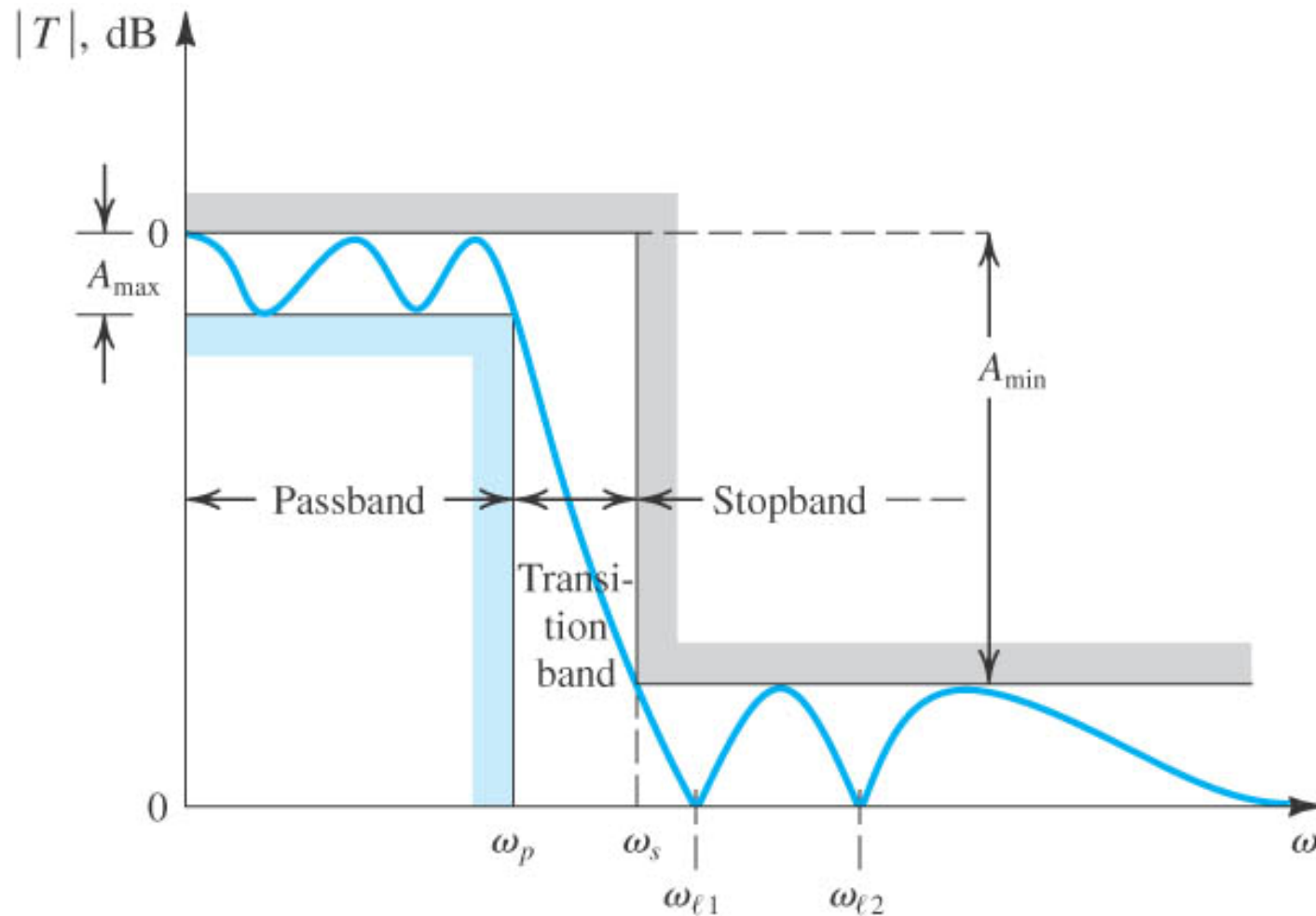


(c) Bandpass (BP)

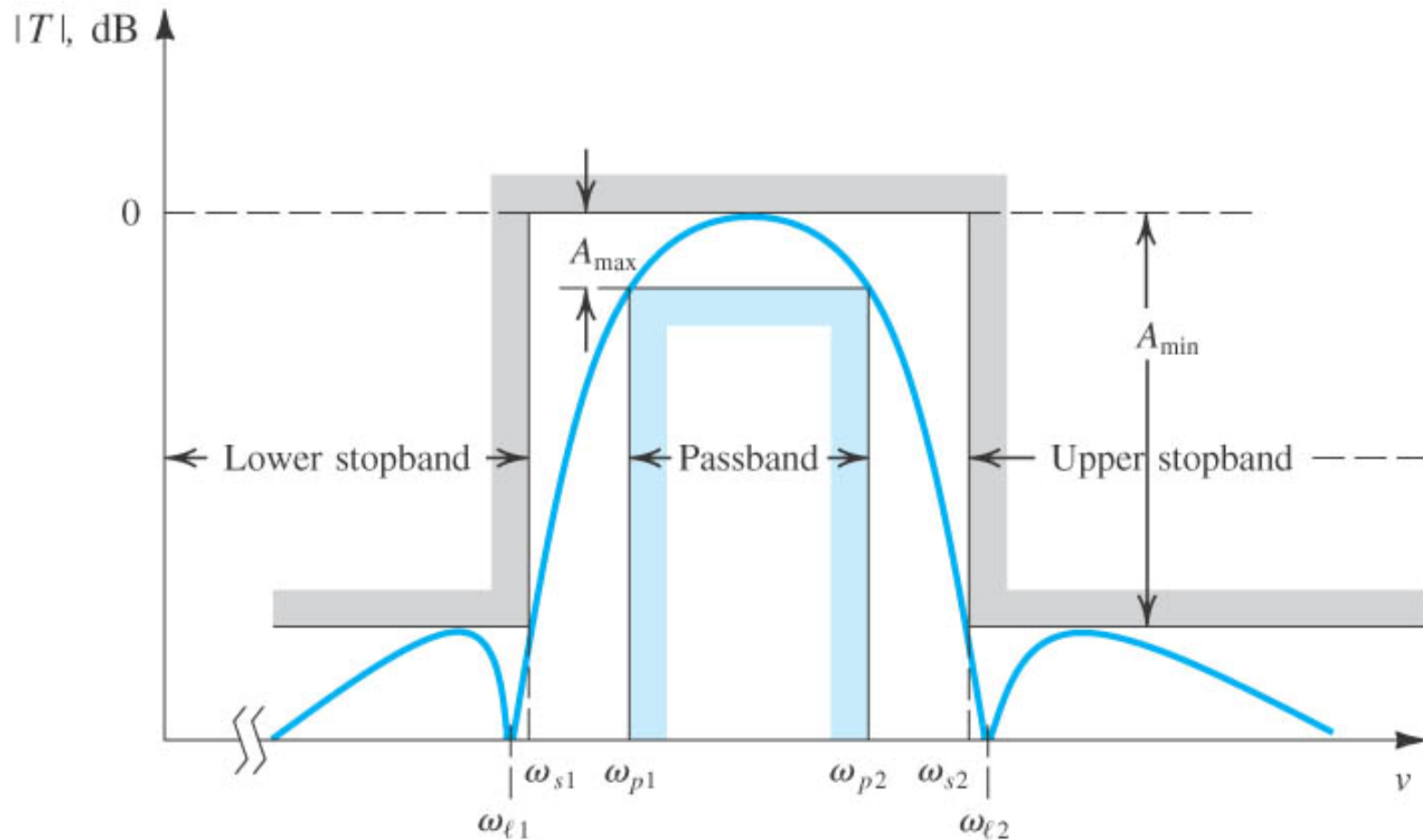


(d) Bandstop (BS)

Specifications of the Transmission Characteristics



Transmission Characteristics for Band Pass Filter



Exercise 12.2: If the magnitude of passband transmission is to remain constant to within $\pm 5\%$, and if the stopband transmission is to be greater than 1% of the passband transmission, find A_{\max} and A_{\min} .

$$A_{\max} = 20 \log 1.05 - 20 \log 0.95 = 0.9 \text{ dB}$$

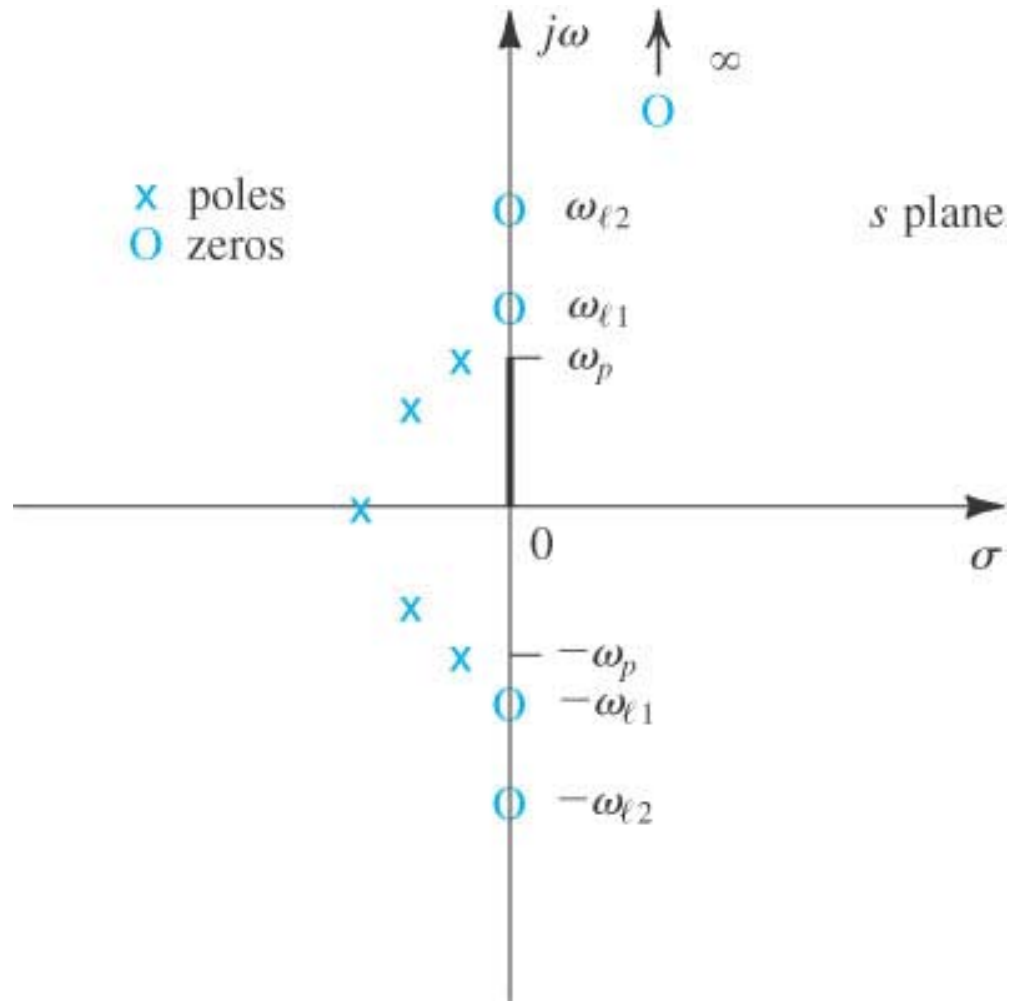
$$A_{\min} = 20 \log \left(\frac{1}{0.01} \right) = 40 \text{ dB}$$

The Filter Transfer Function

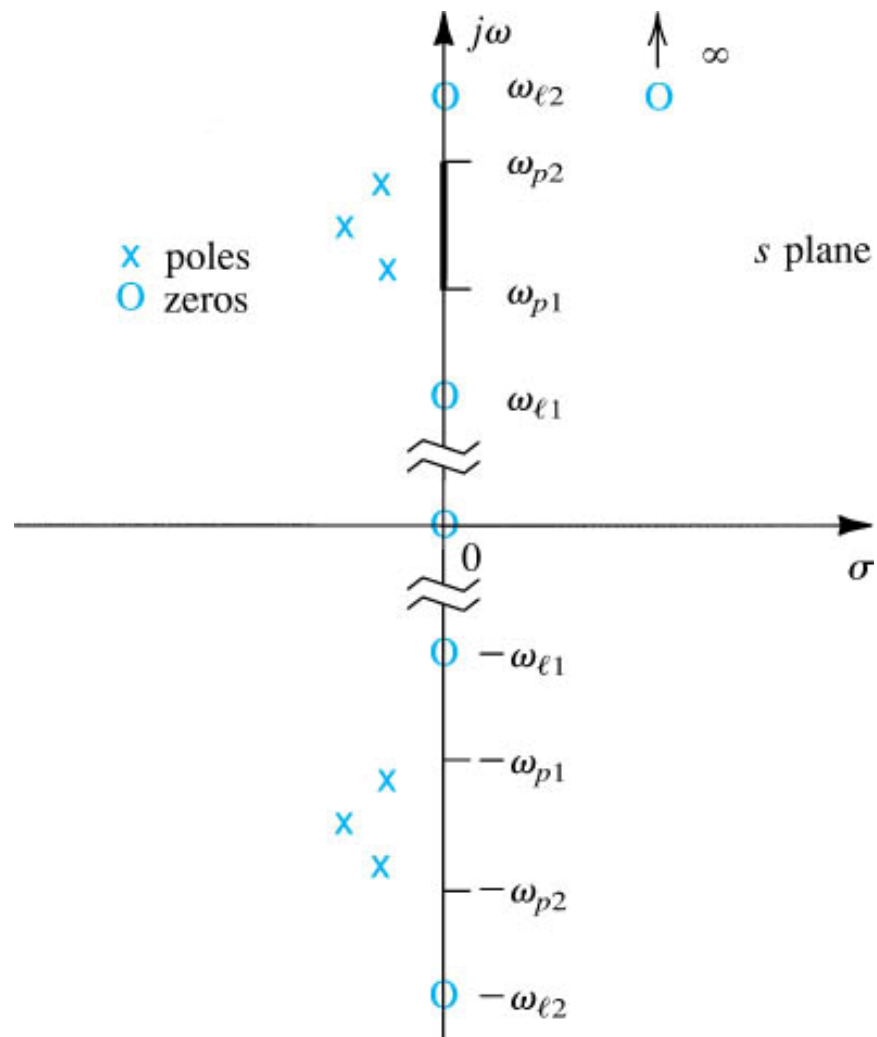
$$T(s) = \frac{a_M s^M + a_{M-1} s^{M-1} + \dots + a_0}{s^N + b_{N-1} s^{N-1} + \dots + b_0}$$

$$T(s) = \frac{a_M (s - z_1)(s - z_2) \dots (s - z_M)}{(s - p_1)(s - p_2) \dots (s - p_M)}$$

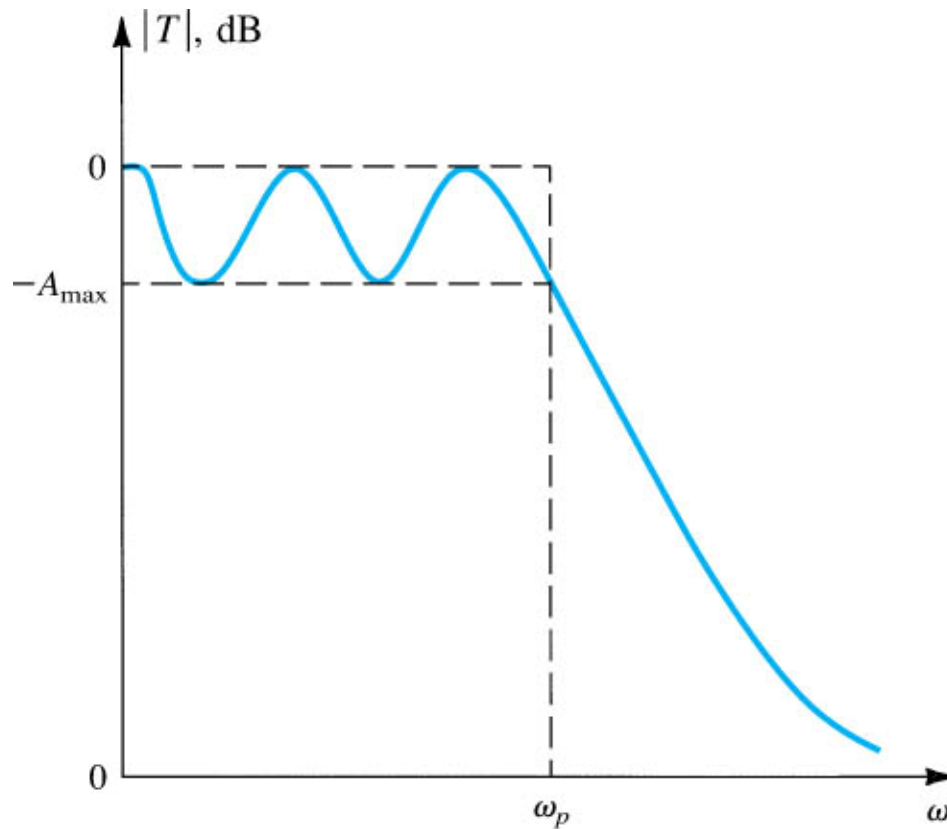
Pole-Zero Pattern for Low Pass Filter



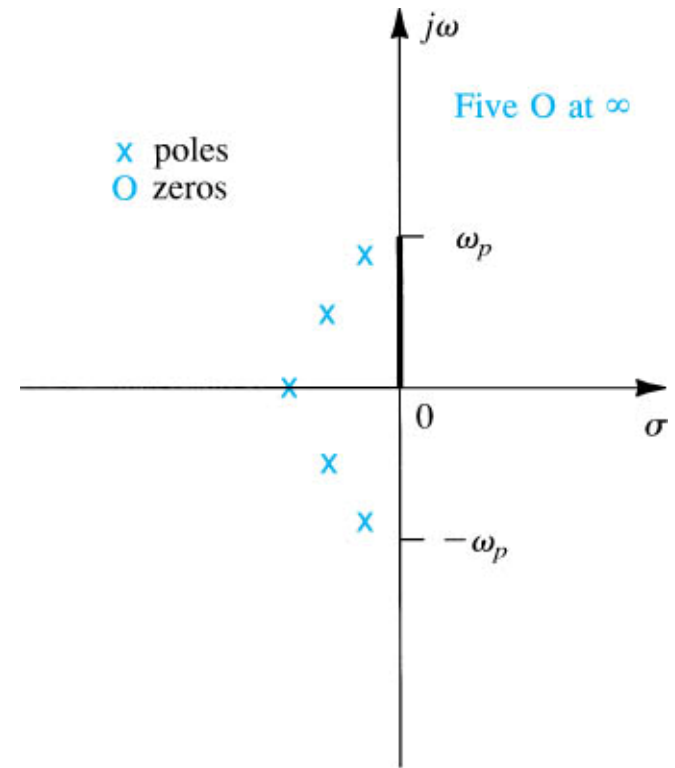
Pole-Zero Pattern for Bandpass Filter



Transmission Characteristics of Fifth Order Low Pass Filter



(a)



(b)

Exercise 12.3: A second order filter has its poles at

$s = -(1/2) \pm j(\sqrt{3}/2)$. The transmission is zero at $\omega = 2$ rad/s and is unity at DC. Find the transfer function.

$$\begin{aligned} T(s) &= k \frac{(s + j2)(s - j2)}{(s + 1/2 + j\sqrt{3}/2)(s + 1/2 - j\sqrt{3}/2)} \\ &= k \frac{(s^2 + 4)}{s^2 + s + 1/4 + 3/4} = k \frac{(s^2 + 4)}{s^2 + s + 1} \end{aligned}$$

$$T(0) = 4k = 1$$

$$k = 1/4$$

$$T(s) = \frac{1}{4} \frac{s^2 + 4}{s^2 + s + 1}$$