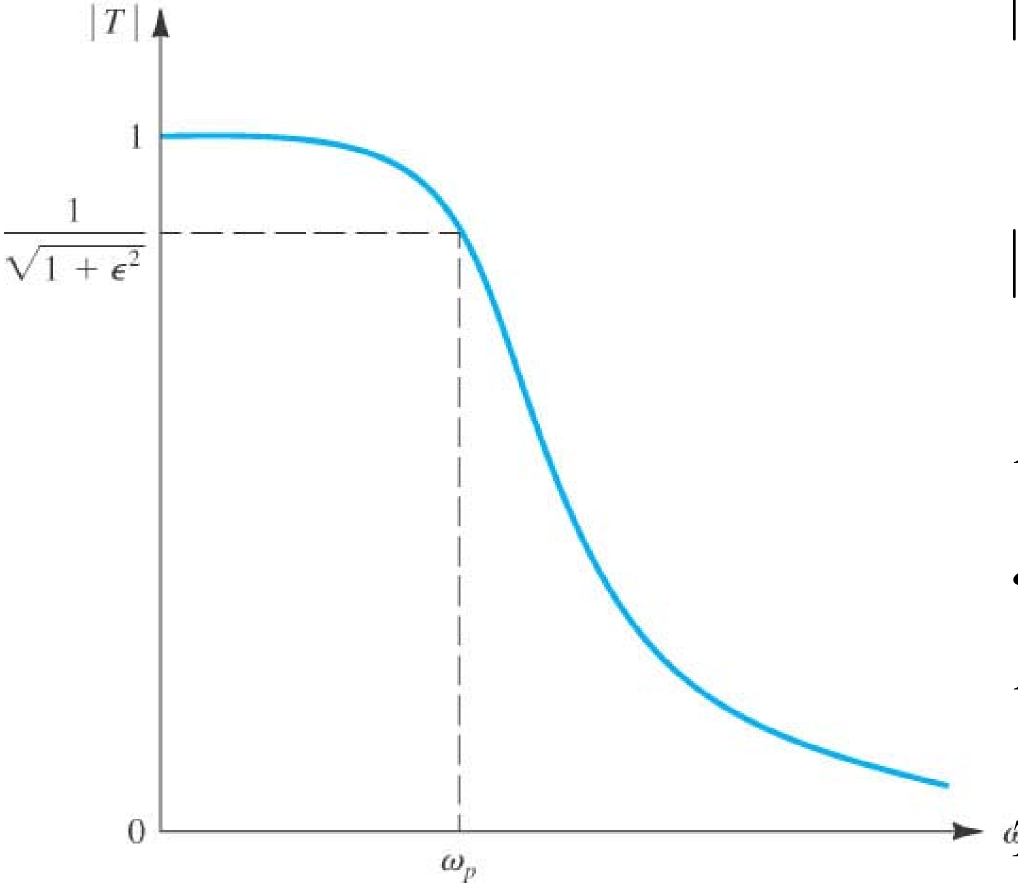
The Butterworth Filter



$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 (\frac{\omega}{\omega_p})^{2N}}}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}} \text{ when } \omega = \omega_p$$

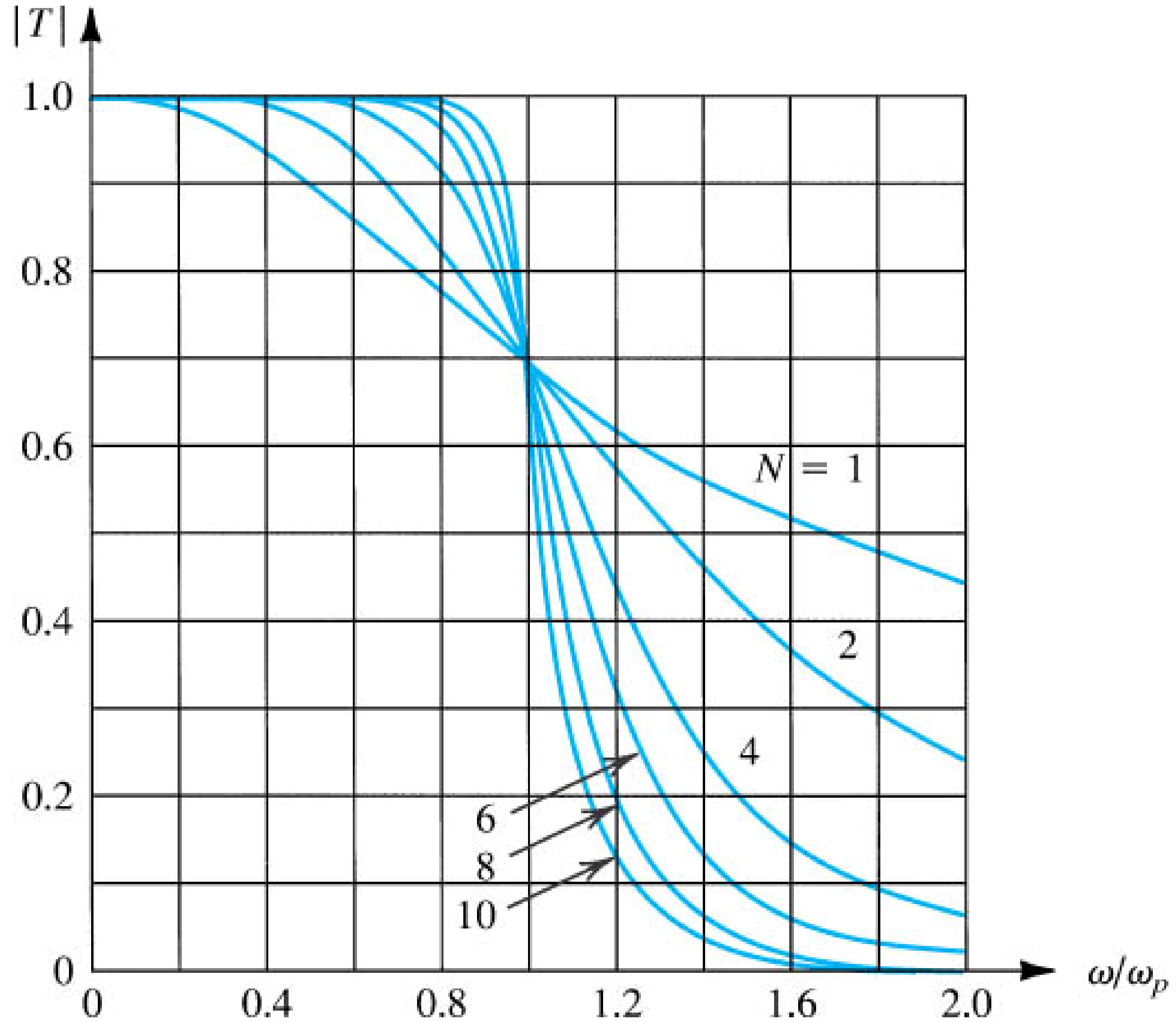
$$A_{\max} = 20 \log \sqrt{1+\varepsilon^2}$$

$$\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$$

$$A(\omega_s) = 10 \log[1+\varepsilon^2 (\omega_s / \omega_p)^{2N}]$$

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)..(s-p_N)}$$

Magnitude response for Butterworth filters of various order

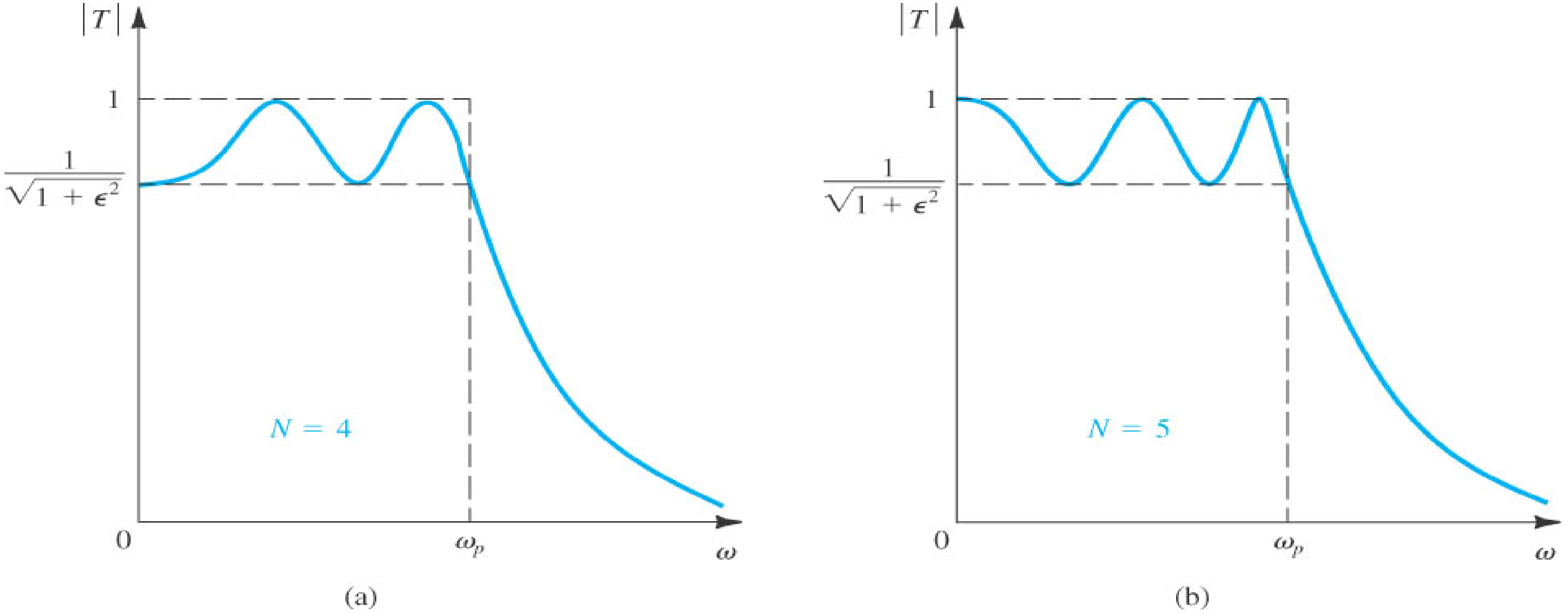


The Chebyshev Filter

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 \cos^2[N \cos^{-1}(\omega/\omega_p)]}} \text{ for } \omega \le \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 \cosh^2[N \cosh^{-1}(\omega/\omega_p)]}} \text{ for } \omega \ge \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}} \text{ when } \omega = \omega_p$$



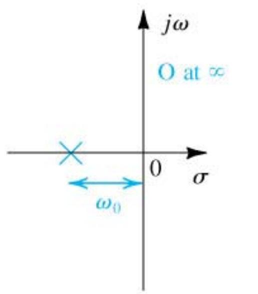
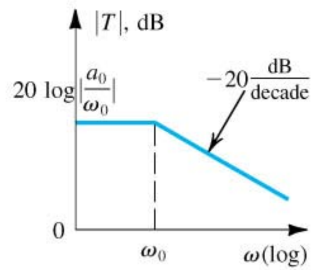
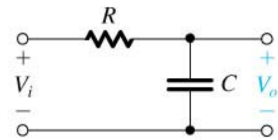
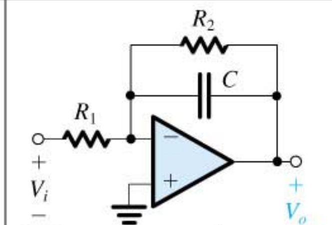
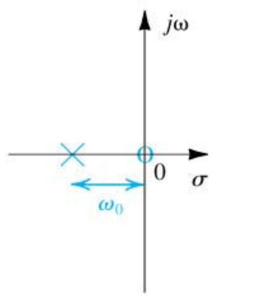
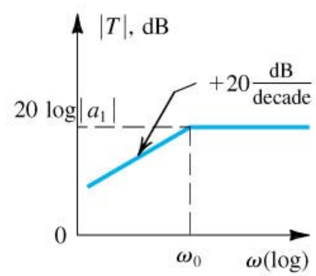
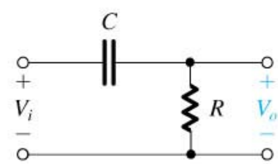
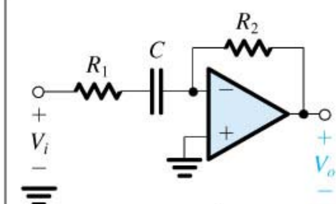
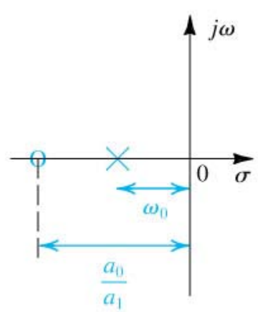
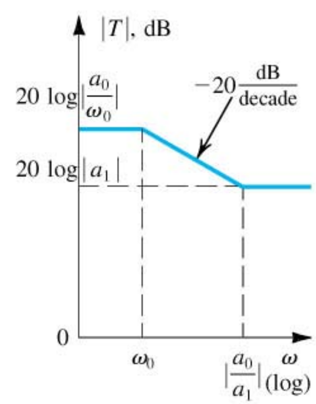
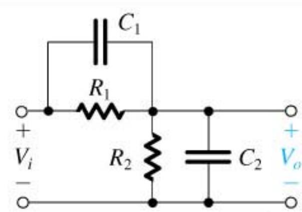
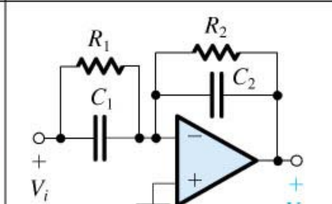
$$A_{\max} = 10\log(1+\varepsilon^2)$$

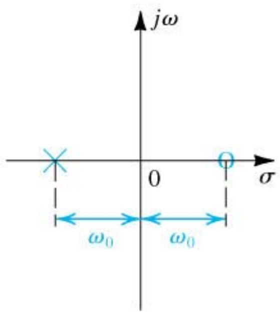
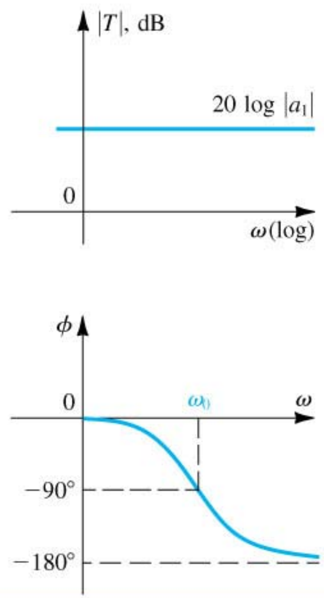
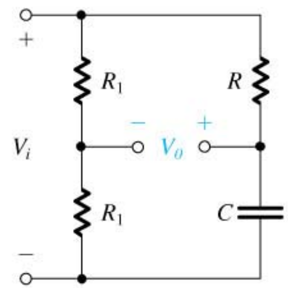
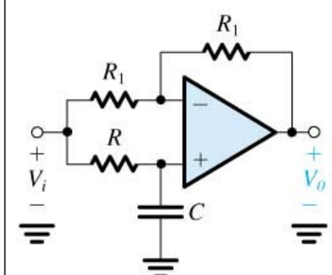
$$\varepsilon = \sqrt{10^{A_{\max}/10} - 1}$$

$$A(\omega_s) = 10\log[1+\varepsilon^2 \cosh^2(N\cosh^{-1}(\omega_s/\omega_p))]$$

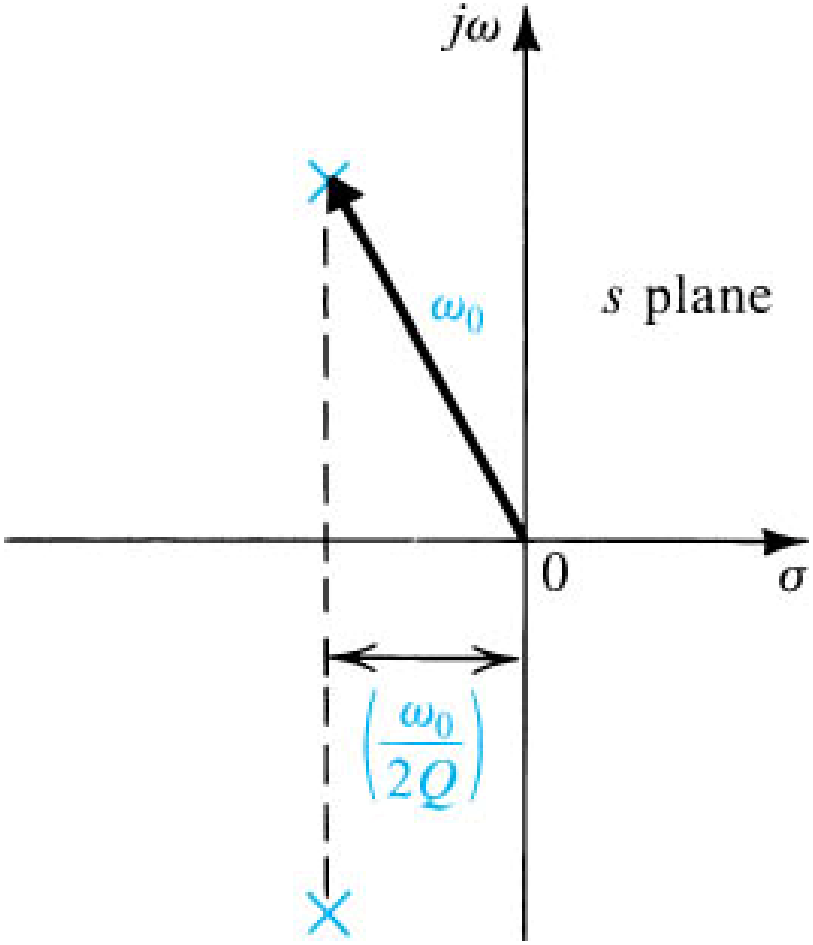
$$T(s) = \frac{K\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)..(s-p_N)}$$

First Order Filter Functions

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\|T\|$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \frac{a_0}{s + \omega_0}$			 $CR = \frac{1}{\omega_0}$ DC gain $= 1$	 $CR_2 = \frac{1}{\omega_0}$ DC gain $= -\frac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \frac{a_1 s}{s + \omega_0}$			 $CR = \frac{1}{\omega_0}$ High-frequency gain $= 1$	 $CR_1 = \frac{1}{\omega_0}$ High-frequency gain $= -\frac{R_2}{R_1}$
(c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain $= \frac{R_2}{R_1 + R_2}$ HF gain $= \frac{C_1}{C_1 + C_2}$	 $C_2 R_2 = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$

$T(s)$	Singularities	$\lvert T\rvert$ and ϕ	Passive Realization	Op Amp–RC Realization
All pass (AP) $T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$

Second Order Filter Functions



<table>
<tr><th>Filter Type and T(s)</th><th>s-Plane Singularities</th><th>|T|</th></tr>
<tr><td>

(a) Low pass (LP)

$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

DC gain $= \frac{a_0}{\omega_0^2}$

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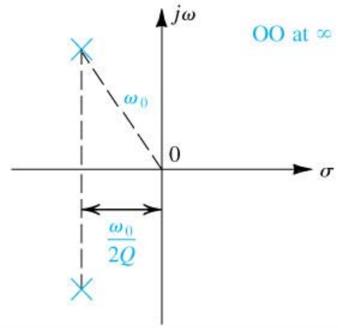

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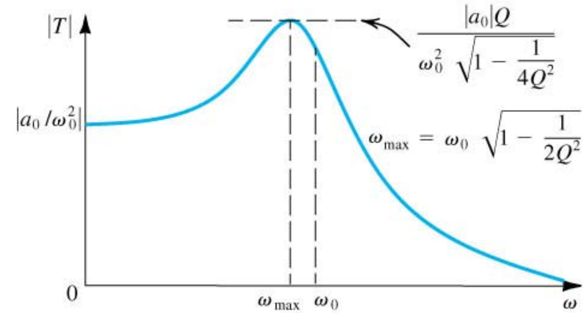

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<tr><td>

(b) High pass (HP)

$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

High-frequency gain $= a_2$

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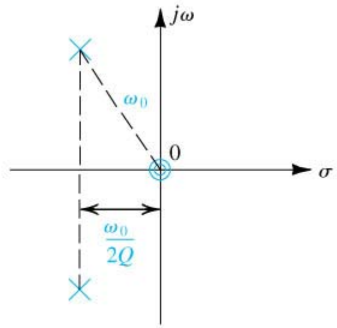

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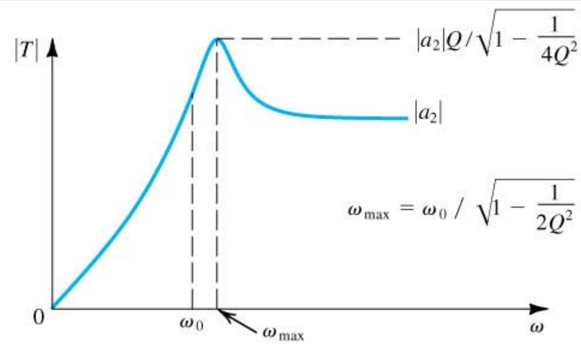

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<tr><td>

(c) Bandpass (BP)

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Center-frequency gain $= \frac{a_1 Q}{\omega_0}$

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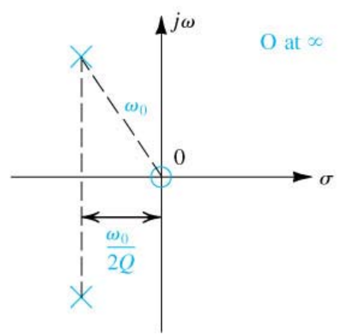

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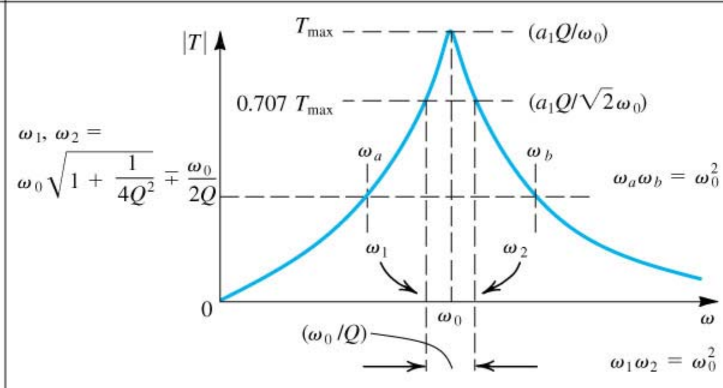

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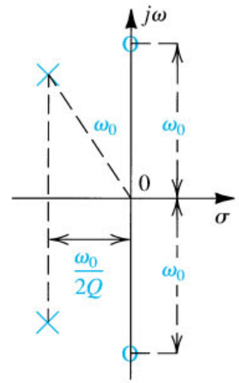
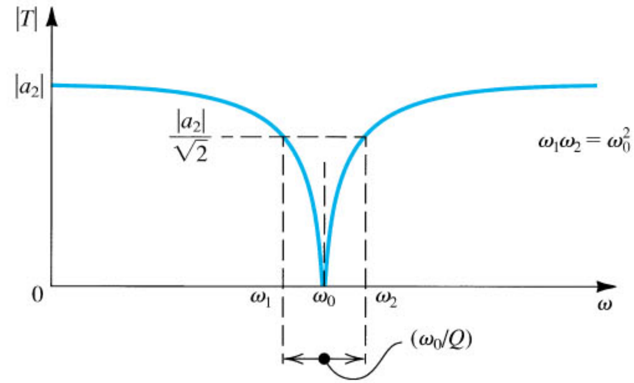
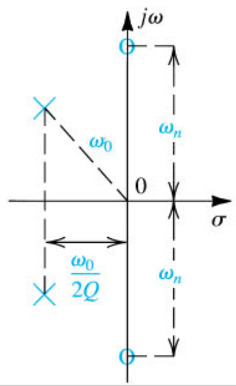
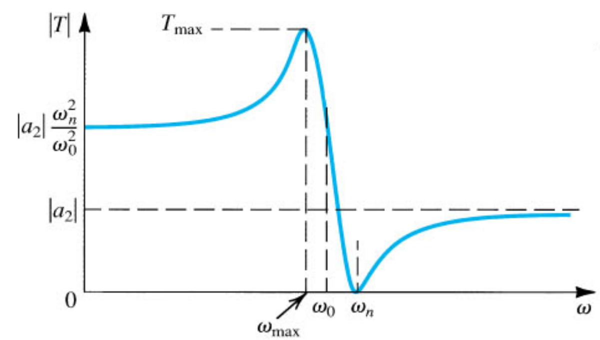
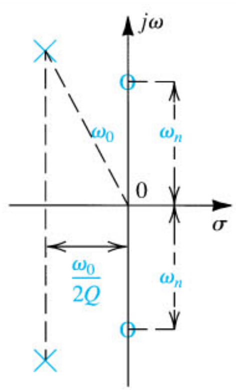
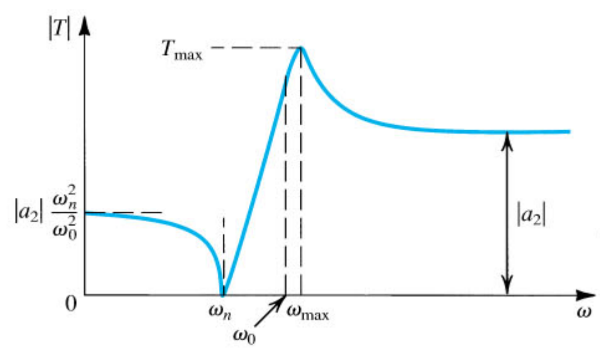
Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(d) Notch $T(s)=a_2\dfrac{s^2+\omega_0^2}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}$ DC gain = High-frequency gain = a_2		
(e) Low-pass notch (LPN) $T(s)=a_2\dfrac{s^2+\omega_n^2}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}$ $\omega_n \geq \omega_0$ DC gain = $a_2\dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain = a_2		$\omega_{max}=\omega_0\sqrt{\dfrac{\left(\dfrac{\omega_n^2}{\omega_0^2}\right)\left(1-\dfrac{1}{2Q^2}\right)-1}{\dfrac{\omega_n^2}{\omega_0^2}+\dfrac{1}{2Q^2}-1}}$
(f) High-pass notch (HPN) $T(s)=a_2\dfrac{s^2+\omega_n^2}{s^2+s\dfrac{\omega_0}{Q}+\omega_0^2}$ $\omega_n \leq \omega_0$ DC gain = $a_2\dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain = a_2		$T_{max}=\dfrac{\lvert a_2\rvert\,\lvert\omega_n^2-\omega_{max}^2\rvert}{\sqrt{(\omega_0^2-\omega_{max}^2)^2+\left(\dfrac{\omega_0}{Q}\right)^2\omega_{max}^2}}$

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain $= a_2$