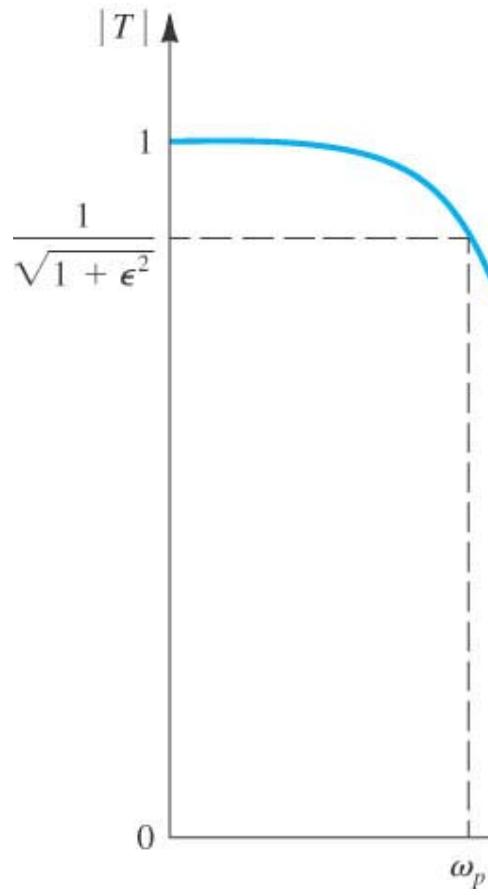


# The Butterworth Filter



$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2} \left(\frac{\omega}{\omega_p}\right)^{2N}}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2}} \text{ when } \omega = \omega_p$$

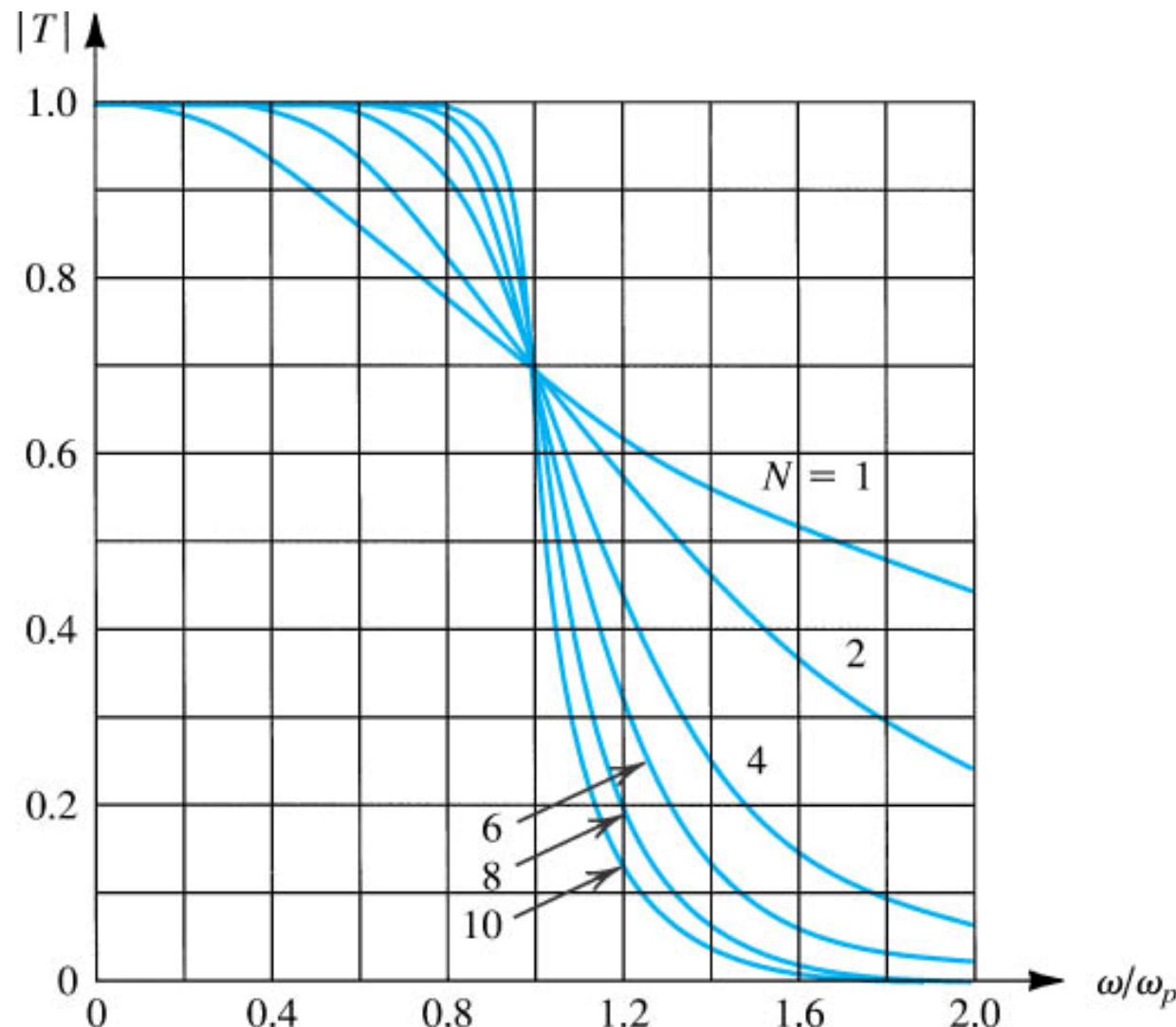
$$A_{\max} = 20 \log \sqrt{1 + \epsilon^2}$$

$$\epsilon = \sqrt{10^{A_{\max}/10} - 1}$$

$$A(\omega_s) = 10 \log [1 + \epsilon^2 (\omega_s / \omega_p)^{2N}]$$

$$T(s) = \frac{K \omega_0^N}{(s - p_1)(s - p_2) \dots (s - p_N)}$$

# Magnitude response for Butterworth filters of various order

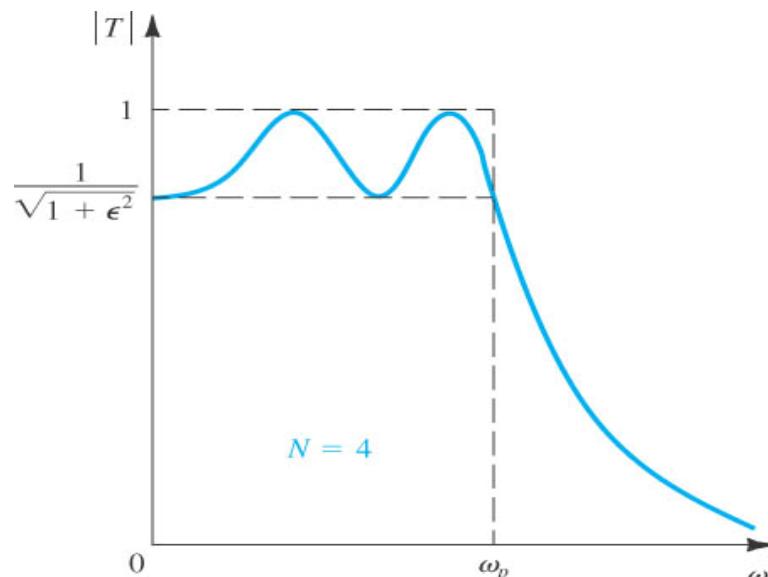


# The Chebyshev Filter

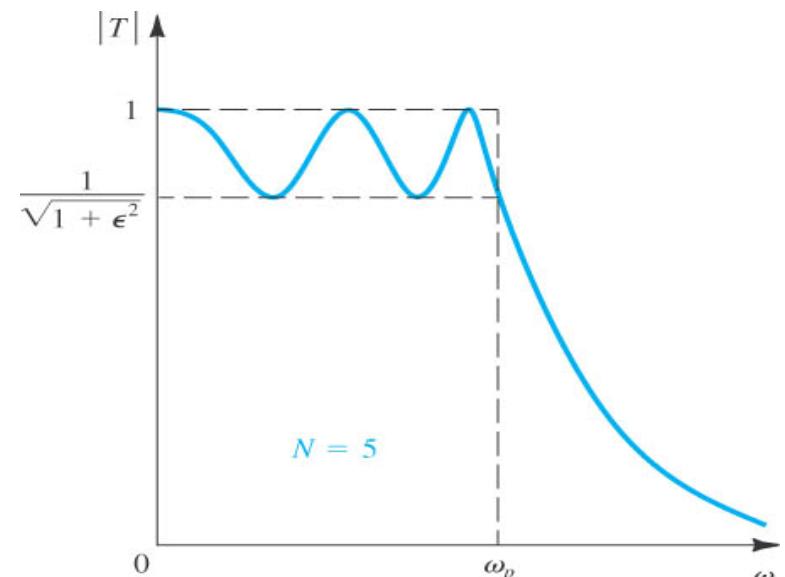
$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cos^2[N \cos^{-1}(\omega/\omega_p)]}} \text{ for } \omega \leq \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 \cosh^2[N \cosh^{-1}(\omega/\omega_p)]}} \text{ for } \omega \geq \omega_p$$

$$|T(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2}} \text{ when } \omega = \omega_p$$



(a)



(b)

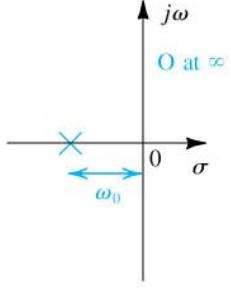
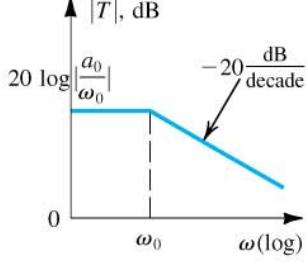
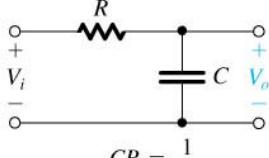
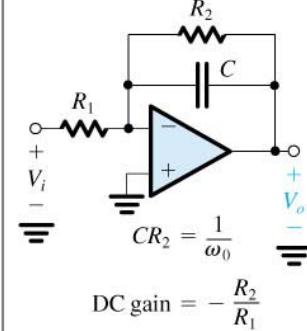
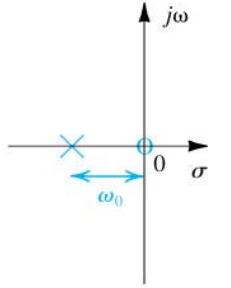
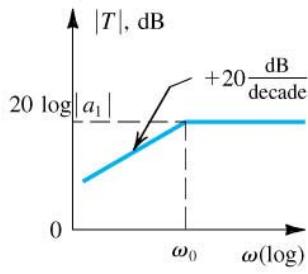
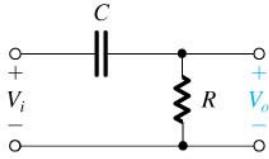
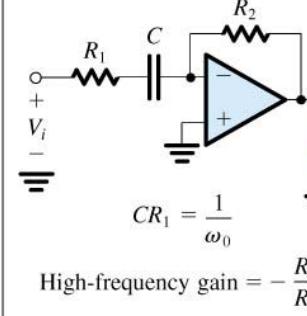
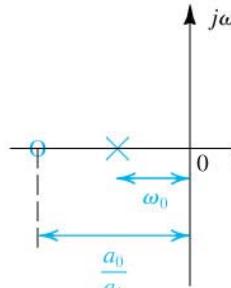
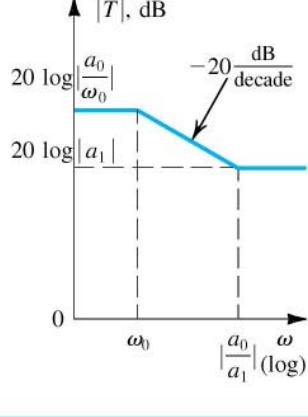
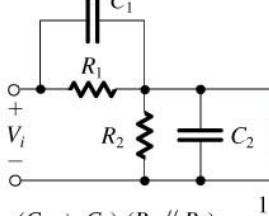
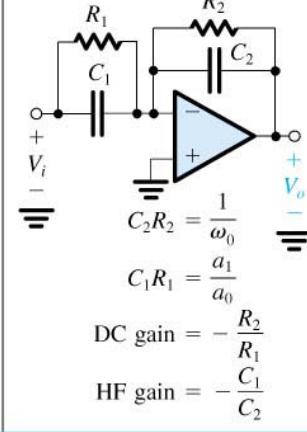
$$A_{\max}=10\log(1+\varepsilon^2)$$

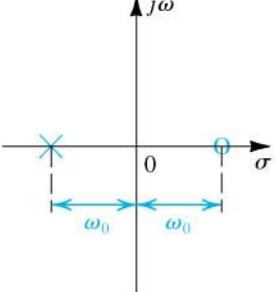
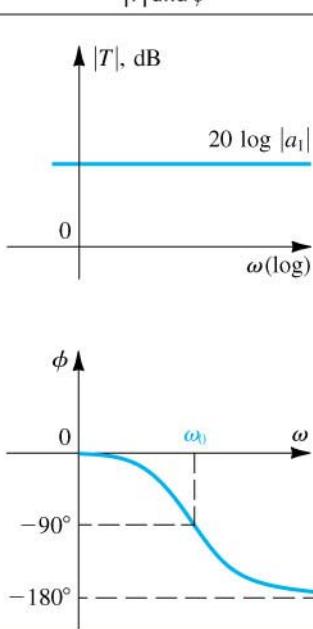
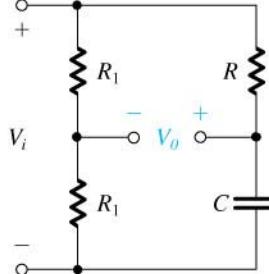
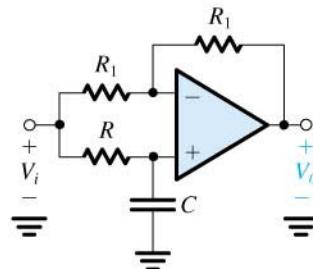
$$\varepsilon=\sqrt{10^{A_{\max}/10}-1}$$

$$A(\omega_S)=10\log[1+\varepsilon^2\cosh^2(N\cosh^{-1}(\omega_S/\omega_p))]$$

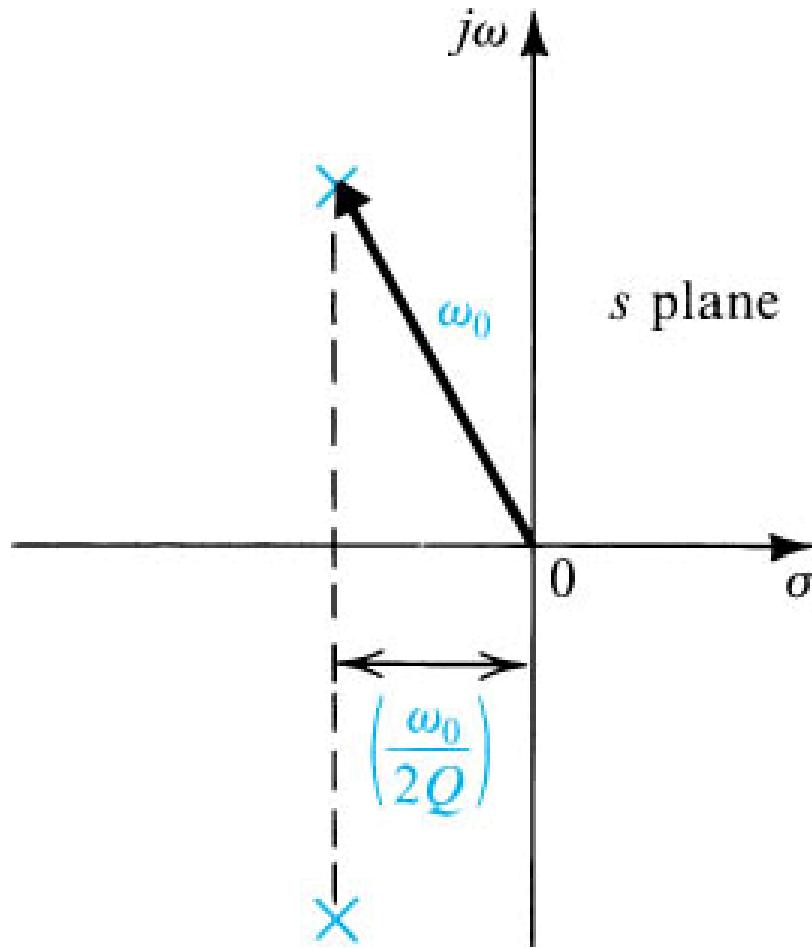
$$T(s) = \frac{K\omega_p^N}{\varepsilon 2^{N-1}(s-p_1)(s-p_2)...(s-p_N)}$$

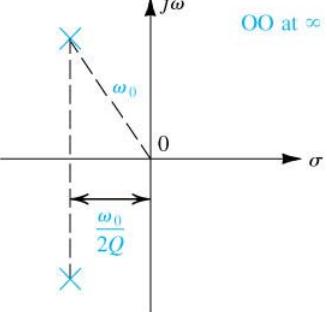
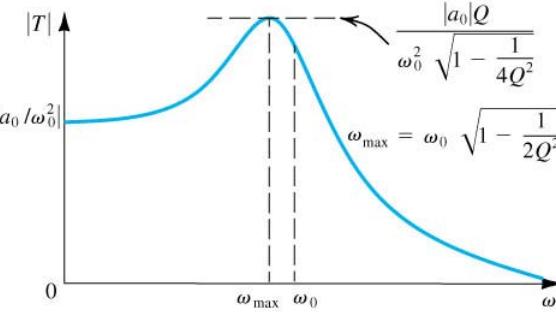
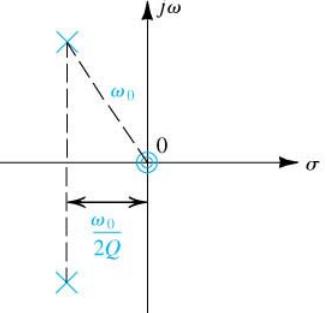
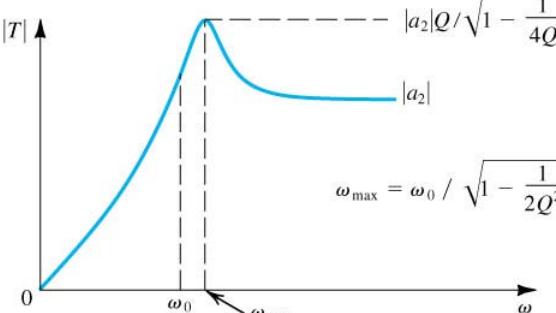
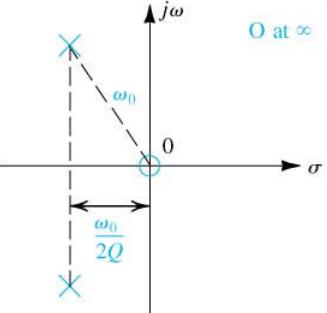
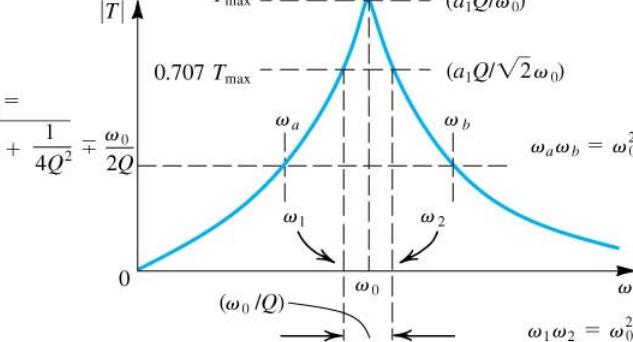
# First Order Filter Functions

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $ T $	Passive Realization	Op Amp-RC Realization
(a) Low pass (LP)	$T(s) = \frac{a_0}{s + \omega_0}$ 		 $CR = \frac{1}{\omega_0}$ DC gain = 1	 $CR_2 = \frac{1}{\omega_0}$ DC gain = $-\frac{R_2}{R_1}$
(b) High pass (HP)	$T(s) = \frac{a_1 s}{s + \omega_0}$ 		 $CR = \frac{1}{\omega_0}$ High-frequency gain = 1	 $CR_1 = \frac{1}{\omega_0}$ High-frequency gain = $-\frac{R_2}{R_1}$
(c) General	$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$ 		 $(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain = $\frac{R_2}{R_1 + R_2}$ HF gain = $\frac{C_1}{C_1 + C_2}$	 $C_2 R_2 = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain = $-\frac{R_2}{R_1}$ HF gain = $-\frac{C_1}{C_2}$

$T(s)$	Singularities	$ T $ and $\phi$	Passive Realization	Op Amp-RC Realization
All pass (AP)  $T(s) = -a_1 \frac{s - \omega_0}{s + \omega_0}$ $a_1 > 0$	 <p>The plot shows the complex plane with the real axis (<math>\sigma</math>) and imaginary axis (<math>j\omega</math>). A pole is marked with a 'x' at <math>s = -\omega_0</math> and a zero is marked with a circle at <math>s = \omega_0</math>. Two double-headed arrows below the real axis indicate the distance from the origin to each singularity, labeled <math>\omega_0</math>.</p>	 <p>The top plot shows magnitude <math> T </math> in dB versus frequency <math>\omega</math> on a logarithmic scale. The gain is flat at <math>20 \log  a_1 </math> dB. The bottom plot shows phase <math>\phi</math> versus frequency <math>\omega</math>. The phase starts at 0° at low frequencies, passes through -90° at <math>\omega_0</math>, and continues towards -180° at high frequencies.</p>	 <p>Circuit diagram of a passive realization using resistors <math>R_1</math> and <math>R</math>, and capacitor <math>C</math>. The input voltage <math>V_i</math> is split: one path goes through <math>R_1</math> to ground, and the other path goes through <math>R</math> and <math>C</math> in series before reaching the output. The output voltage <math>V_o</math> is taken across <math>R</math>.</p> <p> <math>CR = 1/\omega_0</math>  Flat gain (<math>a_1</math>) = 0.5 </p>	 <p>Circuit diagram of an op amp-RC realization. It uses an operational amplifier (op-amp) with a feedback network consisting of resistor <math>R_1</math> and capacitor <math>C</math>. The non-inverting input is connected to ground through resistor <math>R</math>. The inverting input is connected to the non-inverting input through resistor <math>R_1</math>. The output voltage <math>V_o</math> is taken from the inverting input node.</p> <p> <math>CR = 1/\omega_0</math>  Flat gain (<math>a_1</math>) = 1 </p>

# Second Order Filter Functions



Filter Type and $T(s)$	$s$ -Plane Singularities	$ T $
(a) Low pass (LP)  $T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$ DC gain = $\frac{a_0}{\omega_0^2}$		
(b) High pass (HP)  $T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain = $a_2$		
(c) Bandpass (BP)  $T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain = $\frac{a_1 Q}{\omega_0}$		

Filter Type and $T(s)$	s-Plane Singularities	$ T $
(d) Notch	$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ <p>DC gain = <math>a_2</math> High-frequency gain = <math>a_2</math></p>	
(e) Low-pass notch (LPN)	$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ <p><math>\omega_n \geq \omega_0</math> DC gain = <math>a_2 \frac{\omega_n^2}{\omega_0^2}</math> High-frequency gain = <math>a_2</math></p>	<p><math>\omega_{max} = \omega_0 \sqrt{\frac{(\omega_n^2)(1 - \frac{1}{2Q^2}) - 1}{\frac{\omega_n^2}{\omega_0^2} + \frac{1}{2Q^2} - 1}}</math></p>
(f) High-pass notch (HPN)	$T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$ <p><math>\omega_n \leq \omega_0</math> DC gain = <math>a_2 \frac{\omega_n^2}{\omega_0^2}</math> High-frequency gain = <math>a_2</math></p>	<p><math>T_{max} = \frac{ a_2  \sqrt{ \omega_n^2 - \omega_{max}^2 }}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\frac{\omega_0}{Q}\right)^2 \omega_{max}^2}}</math></p>

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s \frac{\omega_0}{Q} + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain =  $a_2$

