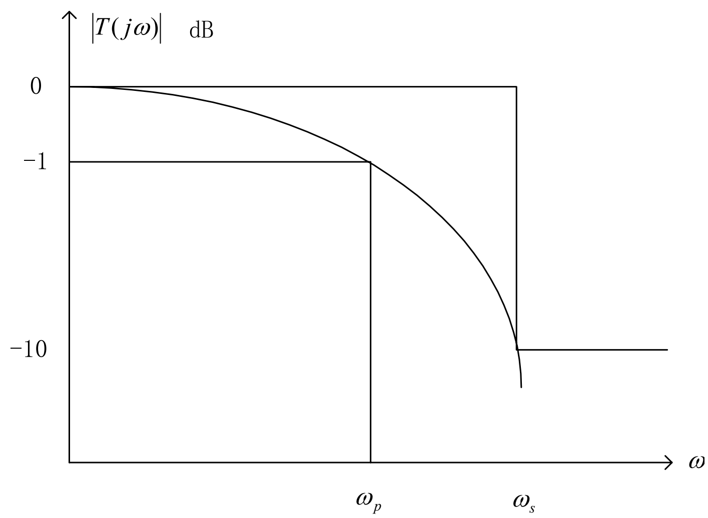
ELG4135: Tutorial on Chapter 12

Q1 (12.5)

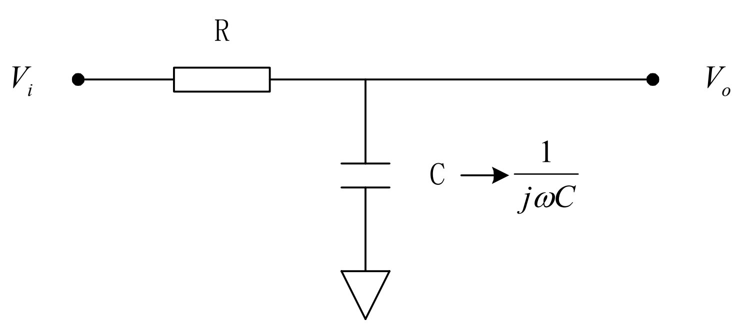
A low-pass filter is specified to have $A_{max} = 1$ dB and $A_{min} = 10$ dB. It is found that its specifications can be just met with a single-time-constant RC circuit having a time constant of 1 s and a dc transmission of unity. What must ω_p and ω_s of this filter be? What is the selectivity factor?

Solution:

The frequency response of the low-pass filter can be plotted as



An R-C low-pass filter can be plotted as



The voltage transfer function $T(j\omega)$ is then

$$T(j\omega) = \frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

Given $\tau = RC = 1$ s,

$$T(j\omega) = \frac{1}{1 + j\omega}$$

The magnitude of $T(j\omega)$ is then

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

At the pass-band edge:

$|T(j\omega_p)| = 10^{\frac{-1}{20}}$. (Note: $10^{\frac{-1}{20}}$ is the linear scale of -1 dB loss.)

ie. $\dfrac{1}{\sqrt{1+\omega_p{}^2}} = 10^{\frac{-1}{20}}$

=> $\omega_p = 0.5088$ rad/s

At the stop-band edge:

$|T(j\omega_s)| = 10^{\frac{-10}{20}}$. (Note: $10^{\frac{-10}{20}}$ is the linear scale of -10 dB loss.)

ie. $\dfrac{1}{\sqrt{1+\omega_s{}^2}} = 10^{\frac{-10}{20}}$

=> $\omega_s = 3$ rad/s

=>Selectivity= $\dfrac{\omega_s}{\omega_p}$ =5.9

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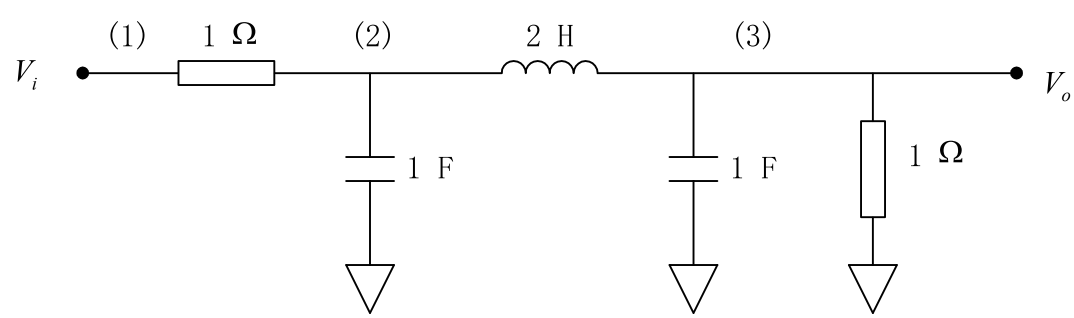
Q2 (12.11)

Analyze the RLC network of Fig. P11.11 to determine its transfer function $T(s) = \dfrac{V_o(s)}{V_i(s)}$ and hence its poles and zeros.



Solution:
The easiest way to solve the circuit is to use nodal analysis at nodes (1), (2) and (3).

At node (3), $\Sigma I = 0$

$$\frac{V_o}{1} + \frac{V_o}{1/s} + \frac{V_o - V_1}{2s} = 0,$$

where V_1 is the voltage at node (2).

-> $V_1 = V_o(2s^2 + 2s + 1)$ (a)

At node (2), $\Sigma I = 0$

$$\frac{V_1 - V_i}{1} + \frac{V_1}{1/s} + \frac{V_1 - V_o}{2s} = 0,$$

-> $V_1(2s^2 + 2s + 1) = V_o + 2sV_i$ (b)

Substitute (a) to (b)

$$V_o(2s^2 + 2s + 1)^2 = V_o + 2sV_i$$

$$\Rightarrow T(s) = \frac{V_o(s)}{V_i(s)} = \frac{0.5}{s^3 + 2s^2 + 2s + 1}$$

=>There is no zero.

=>Poles are given by.

$$s^3 + 2s^2 + 2s + 1 = 0$$

$$(s+1)(s^2 + s + 1) = 0$$

$$s = -1,\ s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

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Q3 (12.12)

Determine the order N of the Butterworth filter for which $A_{maz} = 1$ dB, $A_{\min} \geq 20$ dB, and the selectivity ratio $\frac{\omega_s}{\omega_p} = 1.3$. What is the actual value of minimum stopband attenuation realized? If $A_{\min}$ is to be exactly 20 dB, to what value can $A_{\max}$ be reduced?

Solution:
From equation (12.15)

$$A(\omega_s) = 10\log[1+\varepsilon^2(\frac{\omega_s}{\omega_p})^{2N}] = A_{\min}$$

$$1+\varepsilon^2(\frac{\omega_s}{\omega_p})^{2N} = 10^{\frac{A_{\min}}{10}}$$

$$(\frac{\omega_s}{\omega_p})^{2N} = \frac{10^{\frac{A_{\min}}{10}} - 1}{\varepsilon^2}$$

$$\log[(\frac{\omega_s}{\omega_p})^{2N}] = \log[\frac{10^{\frac{A_{\min}}{10}} - 1}{\varepsilon^2}]$$

$$N = \frac{\log[\frac{10^{\frac{A_{\min}}{10}} - 1}{\varepsilon^2}]}{\log[(\frac{\omega_s}{\omega_p})^2]} \qquad \text{(a)}$$

From equation (11.14), as $A_{maz} = 1$ dB,

$$\varepsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$$

Substitute $A_{\min} \geq 20$ dB, ε, and the selectivity ratio $\frac{\omega_s}{\omega_p} = 1.3$ into (a)

$$N \approx 11.3$$

=> Choose N=12

The actual value of stopband attenuation can be calculated using N=12 as

$$A(\omega_s) = 10\log[1+\varepsilon^2(\frac{\omega_s}{\omega_p})^{2N}] = 27.35 \text{ dB}$$

If $A_{\min}$ is to be exactly 20 dB, from equation (12.15)

$$\varepsilon^2 = \frac{10^{\frac{A_{\min}}{10}} - 1}{(\frac{\omega_s}{\omega_p})^{2N}} = 0.1824$$, with $A_{\min} = 20$ dB and N=12

From equation (11.13),

$$A_{\max} = 10\log(1+\varepsilon^2) = 0.73 \text{ dB}$$

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Q4 (12.14)
Find the natural modes of a Butterworth filter with a 1-dB bandwidth of 10^3 rad/s and N=5.

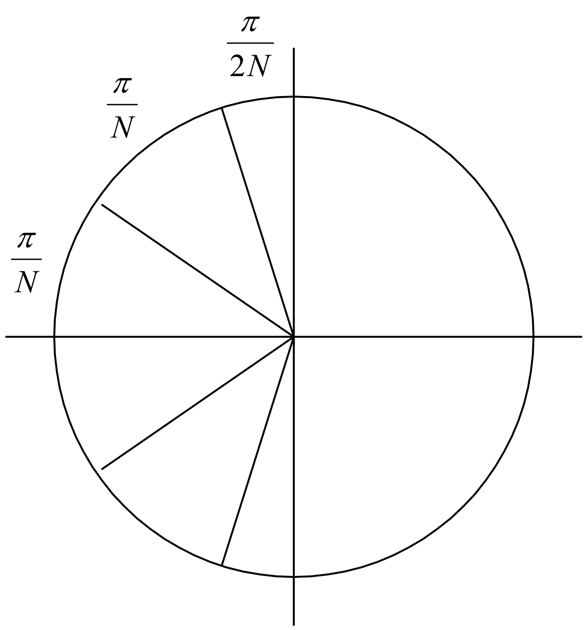
Solution:
From equation (11.14), as $A_{maz} = 1$ dB,

$$\varepsilon = \sqrt{10^{\frac{A_{\max}}{10}} - 1} = \sqrt{10^{\frac{1}{10}} - 1} = 0.5088$$

And $\omega_p = 10^3$ rad/s, N=5, the solutions can be found graphically. (The method is shown in Fig. 11.10)

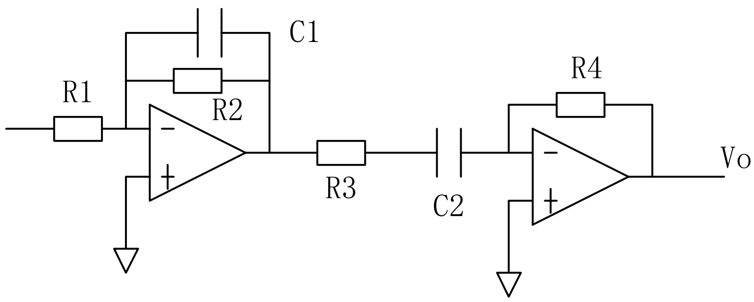
The radius of the circle is $\omega_0 = \omega_p (\frac{1}{\varepsilon})^{1/N} = 873.59$

$$P_1 = \omega_0 e^{\pm j(\frac{\pi}{2} + \frac{\pi}{2N})} = -269.96 \pm j830.84$$
$$P_2 = \omega_0 e^{\pm j(\frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi}{N})} = -706.75 \pm j513.49$$
$$P_3 = \omega_0 e^{\pm j(\pi)} = -873.59$$



Q5 (12.22)
By cascading a first-order op amp-RC low-pass circuit with a first-order op amp-RC high-pass circuit one can design a wideband bandpass filter. Provide such a design for the case the midband gain is 12 dB and the 3-dB bandwidth extends from 100 Hz to 10 kHz. Select appropriate component values under the constraint that no resistors higher than 100 kohms are to be used, and the input resistance is to be as high as possible.

Solution:



Gain=$10^{12/20} = 3.98 \approx 4$

Want Ri=R1 large, so R1=100 kOhm

Total gain=Alp*Ahp=4

Alp=-R2/R1 => R2=-Alp*R1 and R2<100 kOhm, so

Alp=-1, R2=100 kOhm

From Fig. 11.14, $R_2 C_1 = \dfrac{1}{\omega_{0,lp}}$,

$$C_1 = \frac{1}{R_2 \omega_{0,lp}} = \frac{1}{2\pi(10\times10^3)100\times10^3} = 0.159 \text{ nF}$$

Ahp=-4, R4=100 kOhm, R3=25 kOhm

From Fig. 11.14, $R_3 C_2 = \dfrac{1}{\omega_{0,hp}}$,

$$C_1 = \frac{1}{R_2 \omega_{0,lp}} = \frac{1}{2\pi\times(100)\times25\times10^3} = 63.7 \text{ nF}$$